DESIGN AIDS
AND EXAMPLES

In Accordance with
The Egyptian Code for Design and
Construction of Concrete Structures ECCS 203-2001

Limit States Design Method of Concrete Structures

مساعدات التصميم مع أمثلة
طبقاً للكود المصرى
لتصميم وتنفيذ المنشات الخرسانية كود ٢٠٣-٢٠٠١

تصميم العناصر الخرسانية
بطريقه الحدود القصوى

اصدار ٢٠٠٦

نسخة رقم ٦ ادارة التحرير

اللجنة الدائمة للكود المصري
كود رقم ٢٠٣

قرار وزارى

رقم (٩٨) لسنة ٢٠٠١

بشأن تحديث الكود المصرى

لتصميم وتنفيذ المنشآت الخرسانية

وزير الإسكان والمرافق والمجتمعات العمرانية

- بعد الإطلاع على القانون رقم ٦ لسنة ١٩٦٤ فى شأن أسس تصميم وشروط تنفيذ الأعمال الإنشائية وأعمال البناء.
- وعلى القرار الوزارى رقم ١٠٩٥ لسنة ١٩٦٩ فى شأن أسس تصميم وشروط تنفيذ أعمال الخرسانة المسلحة فى المبانى.
- وعلى القرار الجمهورى رقم ٤٦ لسنة ١٩٧٧ فى شأن الهيئة العامة لمركز بحوث الإسكان والبناء والتخطيط العمرانى.
- وعلى القرار الوزارى رقم ٢٠٨ لسنة ١٩٩٥ بشأن الكود المصرى لتصميم وتنفيذ المنشآت الخرسانية المسلحة .
- وعلى القرار الوزارى رقم ٤٩٢ لسنة ١٩٩٦ بتشكيل اللجنة الرئيسية لأسس تصميم وشروط تنفيذ الأعمال الإنشائية وأعمال البناء..
- وعلى القرار الوزارى رقم ٤٩٣ لسنة ١٩٩٦ والمتضمن تشكيل اللجنة الدائمة لأسس تصميم وشروط تنفيذ المنشآت الخرسانية المسلحة والقرارات المكملة رقم ٦٩ لسنة ١٩٩٨ ورقم ١٤١ لسنة ١٩٩٨.
- وعلى المذكرة المقدمة من كل من السيد الأستاذ الدكتور / رئيس اللجنة الدائمة لأسس تصميم وتنفيذ المنشآت الخرسانية المسلحة والسيدة الأستاذ الدكتور / رئيس مجلس إدارة مركز بحوث الإسكان والبناء .

قـــــــــــــرر

مادة (١) : تحديث الكود المصرى لتصميم وتنفيذ المنشآت الخرسانية المسلحة الصادر بالقرار الوزارى رقم ٢٠٨ لسنة ١٩٩٥ طبقاً لما هو وارد بالكود المرفق.

مادة (٢) : تتولى اللجنة الدائمة لأسس تصميم وشروط تنفيذ المنشآت الخرسانية المسلحة اقتراح التعديلات التى تراها لازمة بهدف التحديث كلما دعت الحاجة لذلك وتصير التعديلات بعد إصدارها جزءاً لا يتجزأ من الكود.

مادة (٣) : يتولى مركز بحوث الإسكان والبناء العمل على تنفيذ ما جاء بالكود المصرى لتصميم وتنفيذ المنشآت الخرسانية المسلحة ونشره والتدريب عليه.

مادة (٤) : ينشر هذا القرار فى الوقائع المصرية ويعتبر نافذاً من تاريخ نشره.

وزير الإسكان والمرافق والمجتمعات العمرانية

صدر فى ٢١ / ٤ / ٢٠٠١

استاذ دكتور مهندس / محمد ابراهيم سليمان

1-MATERIAL STRENGTH AND STRESS – STRAIN RELATIONSHIPS

1.1- Introduction

The Egyptian code of 1969 for design and construction of reinforced concrete structures was based solely on the working stress design method.

The pioneers who initiated the first edition of the Egyptian code worked out limits which suits adequately local conditions and design procedures which cope successfully with the more developed methods adopted by some other codes not only in that era but also for a long duration after that.

The Egyptian code of 1989, 1995 and 2001 incorporates mainly the limit state design method for reinforced concrete structures. It emphasizes not only the strength but also the serviceability limit states chapters (3) and (4). Working stress design method chapter (5) is also concisely presented in the code as for a transition period.

Also main considerations have been given in this code and its revisions to cover local conditions, practices, quality control, materials and industries. Basic material strengths covered in the ECCS 203-2001 revision include the following:

1.2 Types and Grades of Reinforcement Bars

The types of steel permitted for use as reinforcement bars clause (2.2.5) of the code and their characteristic strength table 2.4 (specified minimum yield stress or 0.2 percent proof stress) are as follows:

Mechanical properties of steel reinforcement

Type of Steel	Egyptian Standard	Yield strength or 0.2% proof stress (N/mm^2)
1-Mild steel (plain bars)(ϕ)	1988/262	
240/350		240
280/450		280
II-High strength steel	1988/262	
[360/520] (Hot rolled deformed bars) ϕ		360
[400/600] (Cold-worked deformed bars) Φ		400
III-Hard drawn steel welded wire Fabric #	1988/262	
plain, deformed or indented cold drawn to be of grade 450/520	1990/1618	
		450*

* Clause 4.2.1.1 of the ECCS 203-2001 limits this value to 300 N/mm^2 and 400 N/mm^2 for plain and deformed or indented wires respectively.

The characteristic yield or 0.20% proof stress is the value of yield stress below which not more than 5% of the test material may be expected to fail. ECCS 203-2001 clause (2.2.5) and Table (2.4) present the mechanical properties for locally produced steel reinforcement.

Taking the above values into consideration most of the design charts and tables have been prepared for the four grades of steel reinforcement, having characteristic strength f equal to 240 N/mm^2 , 280 N/mm^2, 360 N/mm2 and 400 N/mm^2.

1.3 Grades of Concrete

The following grades of concrete can be used for reinforced concrete work as presented in the ECCS 203- 2001 clause (2.3.2) and tables (2.14) and (5-1):

f_{cu}: 18,20,25,30,35,40 and 45 .

The number in the grade designation refers to the characteristic strength f_{cu} in N/mm^2. The characteristic strength being defined as the compressive strength of cubes with 150 mm side length at the age of 28 days below which no more than five percent of the test results are expected to fail.

The code clause (2.5.2) states that the concrete grade f_{cu} should not be less than 18 N/mm^2 for reinforced concrete members when using working stress method and 15 N/mm^2 for plain concrete works. In addition clause (3.1.1) states that f_{cu} should not be less than 20 N/mm^2 when using limit strength design method.

1.4- Stress – Strain Relationship for concrete

The code permits the use of any appropriate curve for the relationship between the compressive stress and the strain distribution in concrete, subject to the condition that it results in the prediction of strength substantial agreement with test results mentioned in clause (4.2.1.1) of the code. An acceptable stress – strain curve given in fig. (4-2) of the code will form the basis for the design aids. The compressive strength of concrete in the structure is assumed to be 0.67 f_{cu} with a value of 1.5 for the partial safety factor γ_c for concrete clause (3.2.1.2) and equation (3.15-a) of the code. The maximum compressive stress in concrete for design purpose is 0.446 f_{cu}. For eccentric compression equation (3.16.a) of the code gives γ_c equal to:

$$\gamma_c = 1.5\left(\frac{7}{6} - \frac{e/t}{3}\right) \geq 1.5$$

$$where \ \frac{e}{t} \geq 0.05$$

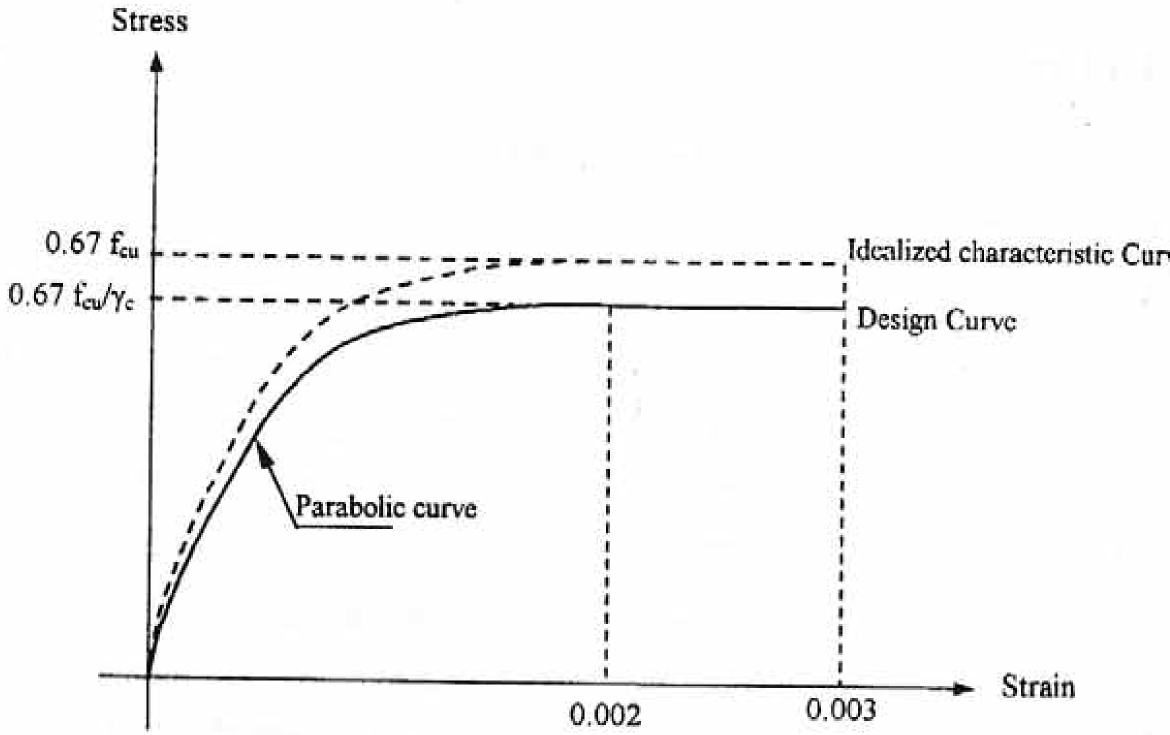


Figure (1-1) Idealized short-term characteristic and design concrete stress-strain curves.

1.5- Stress – Strain Relationship for Steel Reinforcement

The idealized stress – strain curve given in Figure (4.1) of the code which is reproduced in Figure (2-1) will form the basis for this design aids.

For mild steel and high yield hot rolled or cold formed steel, the stress is proportional to the strain up to the yield point or the 0.2% proof stress and thereafter the strain increases at constant stress, Figure (2-1).

The design yield stress (or 0.2 percent proof stress) of steel reinforcement is equal to (f_y/γ_s), where a value of 1.15 is assumed for the partial safety factor γ_s. Thus design stress (f_y/γ_s) becomes equal to 0.87 f_y. Furthermore, the stress – strain relationship for steel in tension and compression is assumed to be the same.

For section subjected to eccentric compression forces, Equation (3-16-b) in the code gives γ_s equal to

$$\gamma_s = 1.15\left(\frac{7}{6} - \frac{e/t}{3}\right) \geq 1.15$$

$$where\ \frac{e}{t} \geq 0.05$$

The modulus of elasticity of steel E_s is taken as 200000 N/mm^2 for all types of reinforcing steel as stated in the ECCS 203-2001 clause (4.2.1).

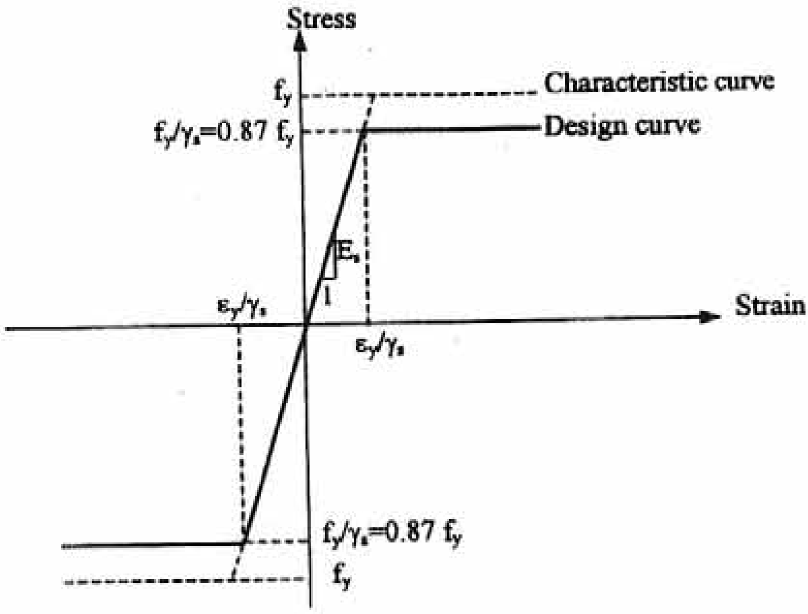


Figure 2-1 Idealized and design stress-strain curves for steel reinforcement.

2. FLEXURAL MEMBERS

2.1-General Considerations:

All members subject to flexure must be designed to satisfy strength requirements and the serviceability requirements of deflection and crack control. For basic design assumptions, refer to chapter four in the ECCS 203-2001.

The ultimate limit flexure strength of cross-section ($M_{ultimate\ limit}$) must be equal to or greater than the required strength (M_u). Where M_u = Load factor . $M_{working}$.(see chapter three in the code).

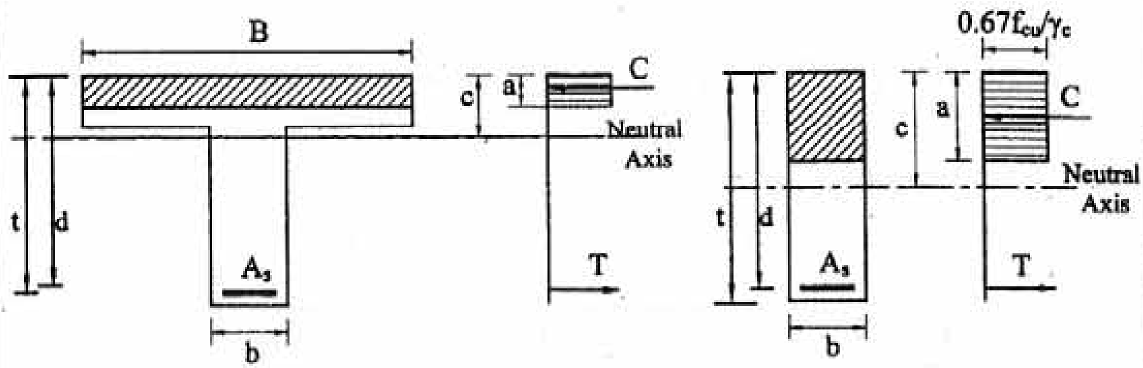
For design and investigation of sections subjected to simple bending, the strength reduction factor for concrete is (γ_c=1.5) and for steel is (γ_s=1.15).

2.2-Sections Subject to Simple Bending with Tension RFT. only:

2.2.1) Rectangular Sections

This includes beams, Slabs and T-sections with ($a \le t_s$):

For the design of rectangular sections (beams and slabs) and T-section with ($a \le t_s$) with tension reinforcement only, the conditions of equilibrium are:



The force equilibrium:

$$T = C \quad \text{or} \quad A_s \frac{f_y}{\gamma_s} = 0.67 \frac{f_{cu}}{\gamma_c} a.b^*$$

where, b^* = (b)web for R – sec.

= (B) flange for T – sec.

= unit width for slabs

$$a = A_s \frac{f_y}{\gamma_s} \frac{1}{0.67 \frac{f_{cu}}{\gamma_c} b^*}$$ .... code eqn.(4 – 2)

$$\frac{a}{2d} = \frac{As}{b^* d} \frac{f_y}{\gamma_s} \frac{\gamma_c}{1.34 f_{cu}} = \frac{\mu}{1.34} \frac{f_y}{\gamma_s} \frac{\gamma_c}{f_{cu}}$$

where, $\mu = \frac{A_s}{b.d}$

The moment equilibrium:

$$M_u = (T \text{ or } C).(d - \frac{a}{2}) = A_s \frac{f_y}{\gamma_s}(d - \frac{a}{2})$$ .... code eqn.(4 – 1)

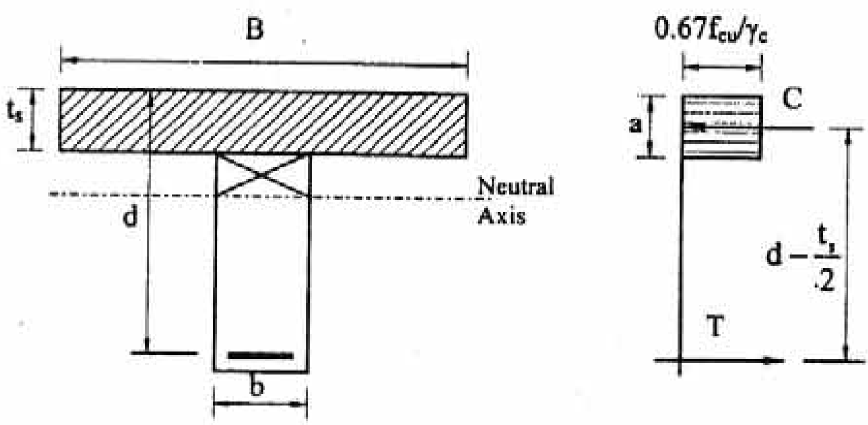
$$M_u = A_s \frac{f_y}{\gamma_s} d(1 - \frac{a}{2d}) = A_s.d \frac{f_y}{\gamma_s}(1 - \frac{\mu}{1.34} \frac{f_y}{\gamma_s} \frac{\gamma_c}{f_{cu}})$$

$$\frac{M_u}{bd^2} = \frac{A_s}{bd} \frac{f_y}{\gamma_s}(1 - \frac{\mu}{1.34} \frac{f_y}{\gamma_s} \frac{\gamma_c}{f_{cu}}) = \mu \frac{f_y}{\gamma_s}(1 - \frac{\mu}{1.34} \frac{f_y}{\gamma_s} \frac{\gamma_c}{f_{cu}})$$

For simple bending : $\gamma_c = 1.5$ & $\gamma_s = 1.15$

$$R_u = \frac{M_u}{bd^2} = \mu \frac{f_y}{1.15}(1 - 0.973 \mu \frac{f_y}{f_{cu}})$$ .... (4 – 1 – a)

2.2.2) T-section with $a \geq t_s$



It is assumed that $a \geq t_s$ and the part of the web below the flange which is subjected to compressive stress can be neglected, then:

$$M_u = 0.67 \frac{f_{cu}}{\gamma_c} B.t_s(d - \frac{t_s}{2}) = A_s \frac{f_y}{\gamma_s}(d - \frac{t_s}{2})$$ .... code eqn.(4 – 8)

Ultimate Limit Design Charts for Simple Bending

CHARTS (2-1) and (2-2)

$$R_u = \frac{M_u}{b.d^2}$$

$$A_s = \mu.b.d$$

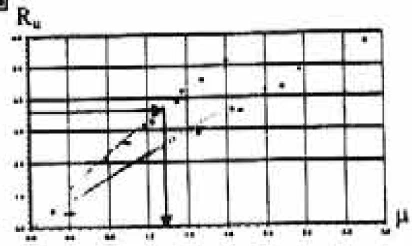


CHART (2-3)

$$d = C_1\sqrt{\frac{M_u}{f_{cu}.b}}$$

Assume: $C1 = 3-4$ (*for Beams*)

$= 4-5$ (*for Slabs*)

$$A_s = \frac{M_u}{f_y.d.j}$$

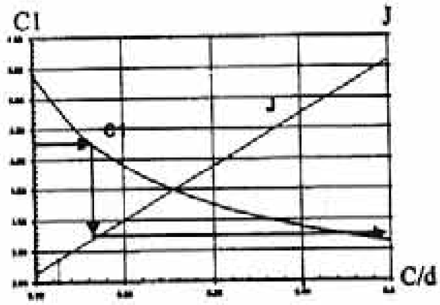


CHART (2-4)

$$d = C_1\sqrt{\frac{M_u}{f_{cu}.b}} \quad \text{or} \quad R_1 = \frac{M_u}{f_{cu}.b.d^2} = \frac{1}{C_1^{\,2}}$$

$$A_s = \omega\frac{f_{cu}.b.d}{f_y}$$

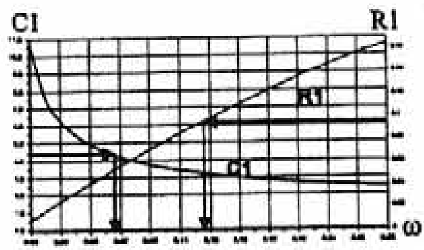
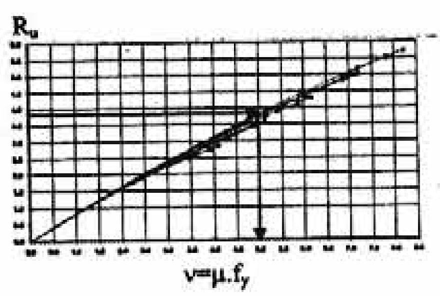


CHART (2-5)

$$R_u = \frac{M_u}{b.d^2}$$

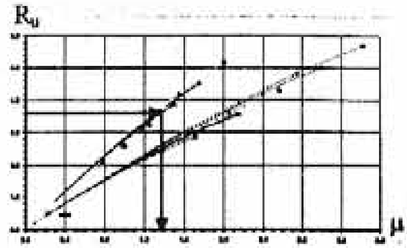
$$\nu = \mu.f_y \Rightarrow \mu = \frac{\nu}{f_y}$$

$$A_s = \mu.b.d$$



2.3-Design aids for flexure in the form of charts and tables

i) Charts (2-1) and (2-2)



Recalling Equation No. (4 - 1 - a)

$$R_u = \frac{M_u}{bd^2} = \mu \frac{f_y}{1.15}(1 - 0.973\mu \frac{f_y}{f_{cu}})$$

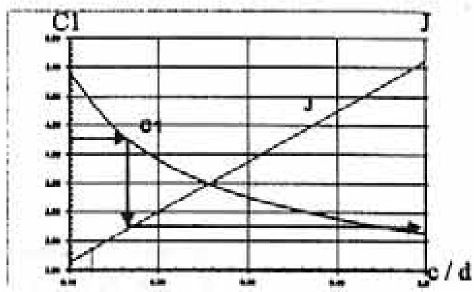
Thus, $R_u = \frac{M_u}{b.d^2}$ *is a function of* μ, f_y *and* f_{cu}.

For given values of f_{cu} and f_y curves can be developed for μ *versus* R_u

The same relation can also be expressed *in table format*

ii) Chart (2-3)

Design chart (C) represents equation (4-1) in the following alternative form:



$$d = C_1\sqrt{\frac{M_u}{f_{cu}\ b}} \quad \& \quad A_s = \frac{M_u}{f_y.j.d}$$

$$j = \frac{M_u}{A_s.f_y.d} = \frac{A_s \frac{f_y}{\gamma_s}(d - \frac{a}{2})}{A_s.f_y.d} = \frac{1}{\gamma_s}(1 - \frac{a}{2d})$$

i.e. $j = \frac{1}{\gamma_s}(1 - 0.4\frac{c}{d})$ ....(4 − 1 − b)

and $C_1 = d\sqrt{\frac{f_{cu}.b}{M_u}} = d\sqrt{\frac{f_{cu}.b}{0.67\frac{f_{cu}}{\gamma_c}ab(d - \frac{a}{2})}} = 1/\sqrt{\frac{0.67}{\gamma_c}\frac{a}{d}(1 - \frac{a}{2d})}$

i.e. $C_1 = 1/\sqrt{0.3573\frac{c}{d}(1 - 0.4\frac{c}{d})}$ ....(4 − 1 − c)

Where C_1 and j are functions of c/d (a=0.80c)

For different values of c/d calculate C1 & j values and thus design chart (C) can be easily drawn.

Assume: C1 equal to [3- 4 (for beams)] and [4 - 5 (for slabs)]

iii) Chart (2-4)

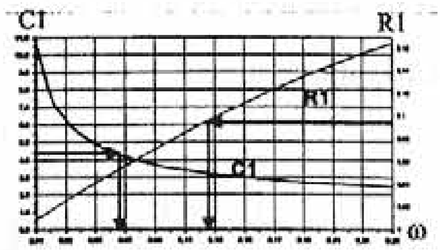
One curve for all concrete grades and types of steel can be developed as follows:

Assume $\omega = \mu \dfrac{f_y}{f_{cu}}$ & $R_1 = \dfrac{M_u}{f_{cu}\, b d^2} = \dfrac{\omega}{\gamma_s}(1 - 0.973\omega)$ .... $(4-1-d)$

Design chart (C) gives one single curve for

R_1 and $C_1 = \dfrac{1}{\sqrt{R_1}}$, versus ω , where

$d = C_1 \sqrt{\dfrac{M_u}{f_{cu}\ b}}$ & $A_s = \omega \dfrac{f_{cu}}{f_y} b d$



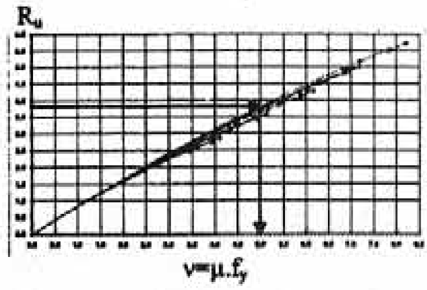
iv) Chart (2-5)

Equation No. (4 - 1 - a) gives

$R_u = \dfrac{M_u}{b.d^2}$ as a function of μ, f_y & f_{cu}

For given values of f_{cu} & f_y a curve can be

developed for $(v = \mu.f_y)$ versus $R_u = \dfrac{M_u}{b.d^2}$



Alternatively a computer program using the pervious equations for the design of flexure members can be developed.

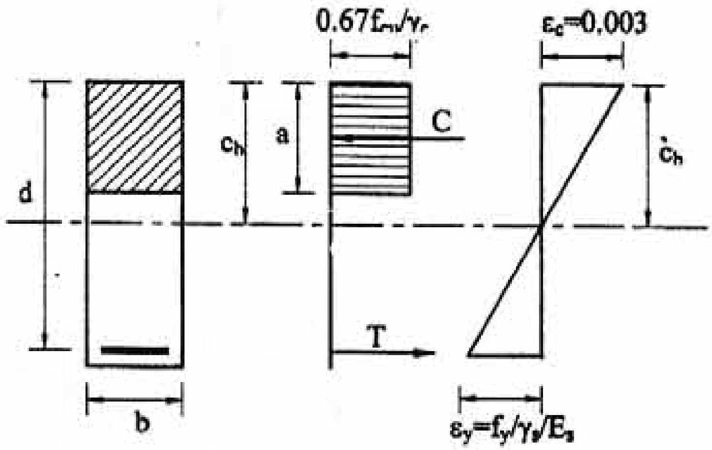
2.4- Modes of Failure

2.4.1- Brittle failure

In this case the concrete on compression side reaches the **maximum** compressive strain before the reinforcement reaches its yield strain.

at failure $\Rightarrow$ $f_s < f_y \Rightarrow \varepsilon_c = 0.003$

2.4.2- Balanced section:



Balanced section is the section at which the concrete on compression side reaches its ultimate strain ($\varepsilon_c = 0.003$) at the same time when the steel in tension side reaches its yield strain ($\varepsilon_y = f_y/\gamma_s/E_s$). This condition gives the position of the neutral axis as follows:

$$\left(\frac{c}{d}\right)_{balanced} = \frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_y} = \frac{0.003}{0.003 + f_y/\gamma_s/E_s} = \frac{600}{600 + f_y/\gamma_s}$$

2.4.3- Ductile failure

This mode of failure occurs when the reinforcement reaches its yield strain before the concrete at compression side reaches the maximum compressive strain.

at failure $\Rightarrow$ $f_s = f_y \Rightarrow \varepsilon_c < 0.003$

2.5- Maximum ultimate moment using tension steel only and maximum RFT:

To avoid brittle failure the code states that:

$$\left(\frac{c}{d}\right)_{max.} = 0.67\left(\frac{c}{d}\right)_{balanced}$$

So, the maximum ultimate limit moment of singly reinforced rectangular section is found from the following equations:

$$\mu^*_{max.} = \frac{A_s}{b.d} = 0.67\frac{f_{cu}}{\gamma_c}\frac{\gamma_s}{f_y}\frac{a_{max.}}{d} = 0.536\frac{f_{cu}}{\gamma_c}\frac{\gamma_s}{f_y}\frac{c_{max.}}{d}$$

$$R_{max.} = \frac{M_{u\ max.}}{\frac{f_{cu}}{\gamma_c}bd^2} = 0.536\frac{c_{max.}}{d}(1-0.4\frac{c_{max.}}{d})$$

$M^*_{u\ max.}$ = max. ultimate limit moment of singly reinforced rectangular sec.

Maximum values for [c/d, μ, R, ω, R_1 and C_1]

Steel type	c/d Balanced	c/d max.	$R_{max.}$	$\mu_{max.}$	$\omega_{max.}$	$R_{1\ max.}$	$C_{1\ min.}$
240/350	0.742	0.50	0.214	$8.56 \times 10^{-4} f_{cu}$	0.204	0.142	2.651
280/450	0.711	0.48	0.206	$7.00 \times 10^{-4} f_{cu}$	0.196	0.138	2.693
360/520	0.657	0.44	0.194	$5.00 \times 10^{-4} f_{cu}$	0.180	0.129	2.780
400/600	0.633	0.42	0.187	$4.31 \times 10^{-4} f_{cu}$	0.172	0.124	2.830
450/520	0.605	0.40	0.180	$3.65 \times 10^{-4} f_{cu}$	0.164	0.120	2.890

Maximum values for [c/d , μ , R , ω , R_1 and C_1] ***(Moment redistribution ±10%)***

Steel type	c/d Balanced	c/d max.	$R_{max.}$	$\mu_{max.}$	$\omega_{max.}$	$R_{1\ max.}$	$C_{1\ min.}$
240/350	0.597	0.40	0.180	$6.85 \times 10^{-4} f_{cu}$	0.164	0.120	2.887
280/450	0.567	0.38	0.173	$5.58 \times 10^{-4} f_{cu}$	0.156	0.115	2.940
360/520	0.507	0.34	0.157	$3.88 \times 10^{-4} f_{cu}$	0.139	0.105	3.090
400/600	0.477	0.32	0.150	$3.29 \times 10^{-4} f_{cu}$	0.132	0.100	3.160
450/520	0.447	0.30	0.142	$2.74 \times 10^{-4} f_{cu}$	0.124	0.095	3.250

Maximum values for [$\mu_{max.}$ (%)] for the different f_{cu} (N/mm^2) values

Steel type	20	25	30	35	40
240/350	1.712	2.14	2.568	2.996	3.424
280/450	1.40	1.75	2.10	2.45	2.80
360/520	1.00	1.25	1.50	1.75	2.00
400/600	0.862	1.0775	1.293	1.5085	1.724
450/520	0.73	0.9125	1.095	1.2775	1.46

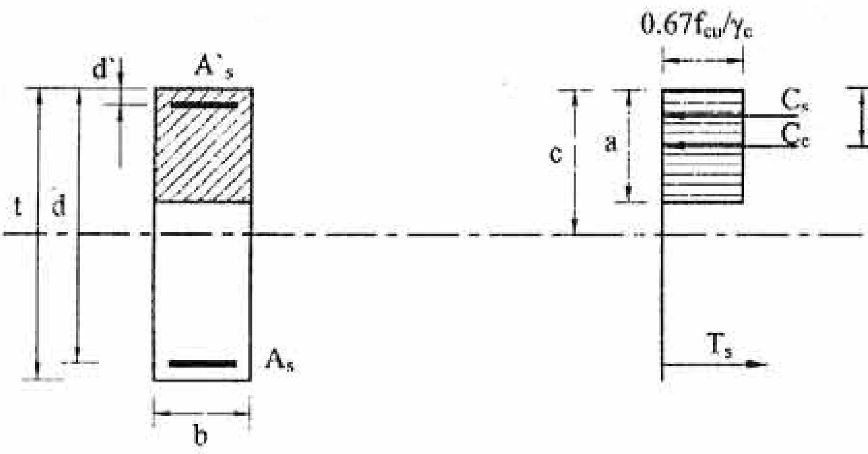
Maximum values for [$\mu_{max.}$ (%)] for the different f_{cu} (N/mm^2) values ***(Moment redistribution ± 10%)***

Steel type	20	25	30	35	40
240/350	1.37	1.7125	2.055	2.3975	2.74
280/450	1.116	1.395	1.674	1.953	2.232
360/520	0.776	0.97	1.164	1.358	1.552
400/600	0.658	0.8225	0.987	1.1515	1.316
450/520	0.548	0.685	0.822	0.959	1.096

2.6- Design of Doubly Reinforced Sections Subjected to Simple Bending

According to clause (4-2-1-2) of the code when $\mu_{max.}$ and $R_{max.}$ are reached the max. ultimate moment of the section can be increased by using steel rft. In the compression zone.

For a given moment M_u^* greater than the maximum moment of singly reinforced section $M_{umax.}$ then add steel area A_s' at both compression and tension side, thus:



Maximum values for [$\mu_{max.}$ (%)] for the different f_{cu} (N/mm^2) values

Steel type	20	25	30	35	40
240/350	1.712	2.14	2.568	2.996	3.424
280/450	1.40	1.75	2.10	2.45	2.80
360/520	1.00	1.25	1.50	1.75	2.00
400/600	0.862	1.0775	1.293	1.5085	1.724
450/520	0.73	0.9125	1.095	1.2775	1.46

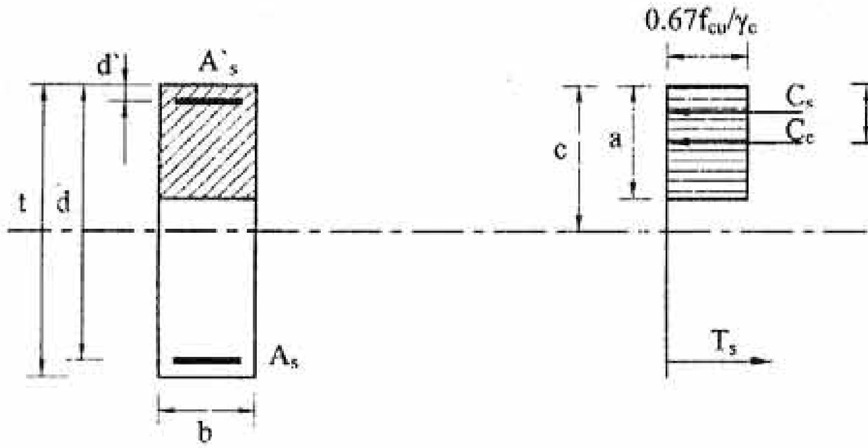
Maximum values for [$\mu_{max.}$ (%)] for the different f_{cu} (N/mm^2) values ***(Moment redistribution ± 10%)***

Steel type	20	25	30	35	40
240/350	1.37	1.7125	2.055	2.3975	2.74
280/450	1.116	1.395	1.674	1.953	2.232
360/520	0.776	0.97	1.164	1.358	1.552
400/600	0.658	0.8225	0.987	1.1515	1.316
450/520	0.548	0.685	0.822	0.959	1.096

2.6- Design of Doubly Reinforced Sections Subjected to Simple Bending

According to clause (4-2-1-2) of the code when $\mu_{max.}$ and $R_{max.}$ are reached the max. ultimate moment of the section can be increased by using steel rft. In the compression zone.

For a given moment M_u^* greater than the maximum moment of singly reinforced section $M_{umax.}$ then add steel area A_s' at both compression and tension side, thus:



$$\Delta M = A_s' \frac{f_y}{\gamma_s} (d - d')$$

Note that this realtion assumes that compression reinforcement is yielded.
Refer to code sections 4.2.1.2.d for applicable conditions

If not, $(\frac{f_y}{\gamma_s})$ must be replaced by the actual $f_s \leq \frac{f_y}{\gamma_s}$

$$\& \; M_u^* = M_{u\,max.} + \Delta M$$

$$M_u^* = R_{max.} \frac{f_{cu}}{\gamma_c} . b . d^2 + A_s' \frac{f_y}{\gamma_s} (d - d') \quad \text{........................ } code\ eqn.(4-6)$$

$$\& \; A_s \frac{f_y}{\gamma_s} = 0.67 a_{max.} . b \frac{f_{cu}}{\gamma_c} + A_s' \frac{f_y}{\gamma_s} \quad \text{........................ } code\ eqn.(4-7)$$

$$A_s = \mu_{max.} . b . d + A_s'$$

2.6.1-General notes for sections subjected to simple bending:

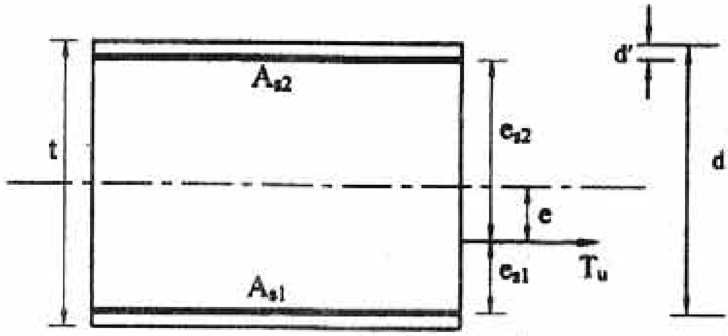
The min. rft. must be checked as $As_{min.}$ = The least of $[\frac{1.1}{f_y}.b.d]$ or $[1.3 A_{s\,req.}]$

But not less than 0.25% b.d for st 240/350
& 0.15% b.d for st 360/520
and in T-section, the value of b is for the web.

The ratio (a/d) used in calculating M_u must not be less than (0.10).

f_y		240	280	360	400
As_{min}	The least of	0.46% b.d	0.40% b.d	0.31% b.d	0.275% b.d
		1.30 As_{req}	1.30 As_{req}	1.30 As_{req}	1.30 As_{req}
	Not less than	0.25% b.d	0.25% b.d	0.15% b.d	0.15% b.d

2.7- Sections subjected to eccentric tensile force:



Axial tension or eccentric tension acting between steel rft. $(e \le t/2 - cover)$

The force is carried by steel reinforcement only: $\left(e < \dfrac{d-d'}{2}\right)$

$i-Calculate \quad e = \dfrac{M_u}{T_u}$

$ii-Caculate \quad e_{s1} = t/2 - e - cover$

$\qquad\qquad\qquad e_{s2} = t/2 + e - cover$

$iii-Calculate \quad A_{s1} = \left(\dfrac{T_u\, e_{s2}}{e_{s1}+e_{s2}}\right) \Big/ \left(\dfrac{f_y}{\gamma_s}\right) \quad \& \quad A_{s2} = \left(\dfrac{T_u\, e_{s1}}{e_{s1}+e_{s2}}\right) \Big/ \left(\dfrac{f_y}{\gamma_s}\right)$

2.8- Solved Examples for Sections Subjected to Flexure

Using Design Charts

Example 1:

Design using chart (2-4):

Given:

M_u= 300 kN.m, b = 300 mm, f_{cu} = 25 N/mm^2, f_y = 360 N/mm^2

Assume $C_1 = 3$ (for beams)

$$d = C_1\sqrt{\frac{M_u}{f_{cu}.b}}$$

$$d = 3\times\sqrt{\frac{300\times10^6}{25\times300}} = 600mm$$

$$R_1 = \frac{M_u}{f_{cu}.b.d^2}$$

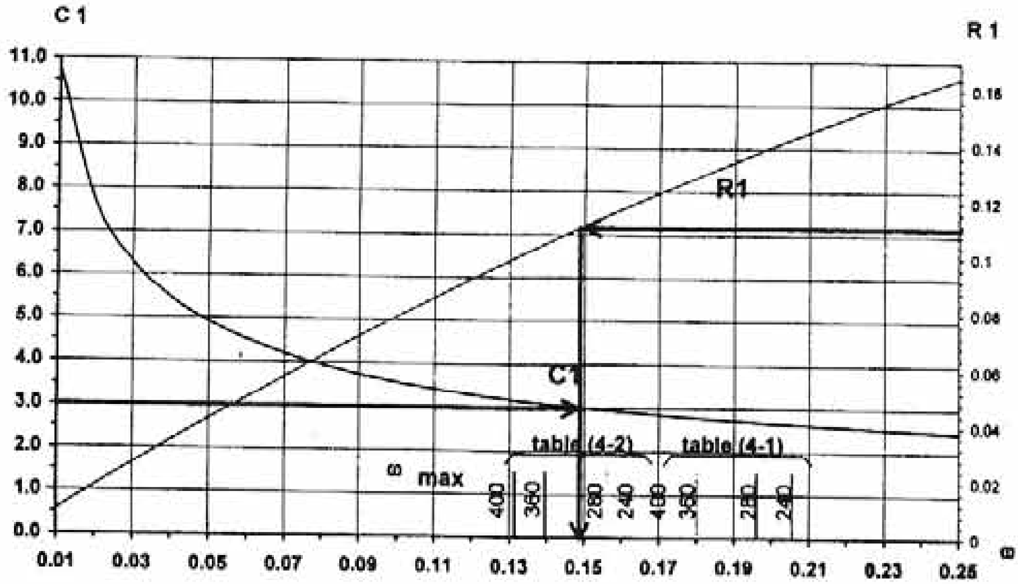
$$R_1 = \frac{300\times10^6}{25\times300\times600^2} = 0.1111$$

$$\textit{from chart } (2-4) \textit{use } R_1 \textit{ to get } \omega = 0.15$$

$$\omega = 0.15 = \mu\frac{f_y}{f_{cu}} = \mu\frac{360}{25} = 14.4\mu$$

$$\mu = \frac{0.15}{14.4} = 0.0104 = \frac{A_s}{b.d} = \frac{A_s}{300\times600}$$

$$A_s = 1875mm^2 = 18.75cm^2$$



Example 2:

Redesign example(1) using chart (2-3):

Given:

M_u= 300 kN.m, b = 300 mm, f_{cu} = N/mm^2, f_y = 360 N/mm^2

Assume $C_1 = 3$ (for beams)

$$d = C_1\sqrt{\frac{M_u}{f_{cu}.b}}$$

$$d = 3\times\sqrt{\frac{300\times10^6}{25\times300}} = 600mm$$

$$from\ chart\ (2-3)\ get\ j = 0.743$$

$$A_s = \frac{M_u}{j.f_y.d} = \frac{300\times10^6}{0.743\times360\times600}$$

$$A_s = 1870mm^2 = 18.70cm^2$$

Example 3:

Redesign example(1) using charts (2-1 and 2-2):

Given:

M_u= 300 kN.m, b = 300 mm, f_{cu} = 25 MPa, f_y = 360 MPa,

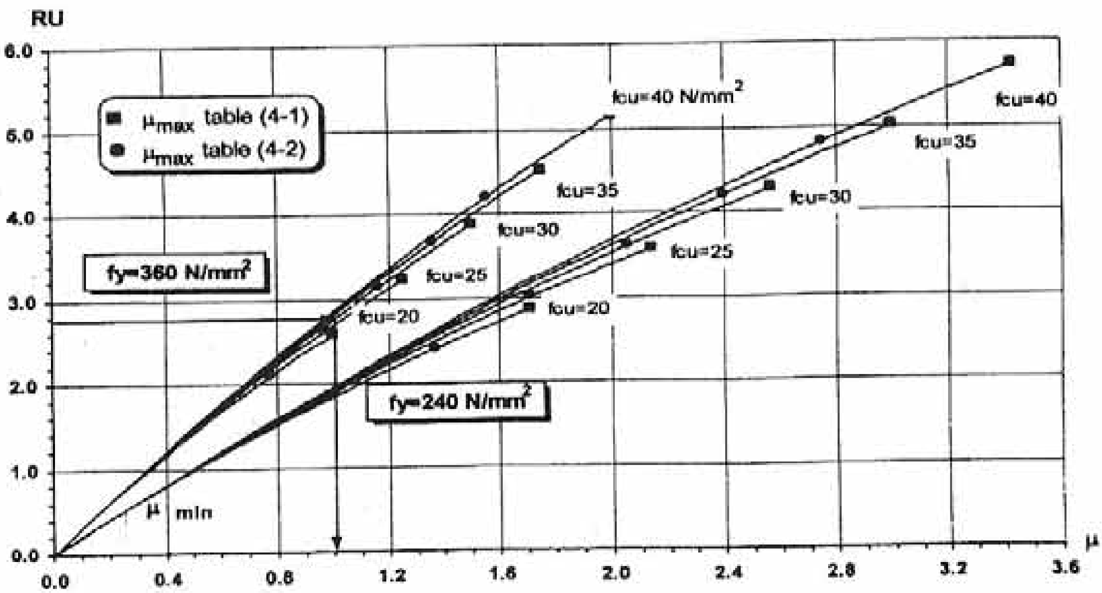
Assume, d=600mm

$$R_u = \frac{M_u}{b.d^2}$$

$$R_u = \frac{300\times10^6}{300\times600^2} = 2.77778 \quad N/mm^2$$

$$\textit{from chart } (2-1) \Rightarrow \mu = 1.038$$

$$A_s = \frac{1.038}{100}\times300\times600 = 1868mm^2 = 18.68cm^2$$



Example 4:

Redesign example(1) using the first principles

Given:

M_u= 300 kN.m, b = 300 mm, f_{cu} = 25 MPa, f_y = 360 MPa,

Assume, $f_s = f_y/\gamma_s$ and d=600mm

*** The force equilibrium:**

$$T = C \Rightarrow A_s \frac{f_y}{\gamma_s} = 0.67 \frac{f_{cu}}{\gamma_c} a.b$$

$$a = A_s \frac{f_y}{\gamma_s} \frac{1}{0.67 \frac{f_{cu}}{\gamma_c} b} = 0.093446 A_s$$

$$\frac{a}{2d} = \frac{0.093446As}{2 \times 600} = 7.7871 \times 10^{-5} A_s \text{ ....(1)}$$

*** The moment equilibrium:**

$$M_u = T.(d - \frac{a}{2}) = A_s \frac{f_y}{\gamma_s}(d - \frac{a}{2})$$

$$M_u = A_s \frac{f_y}{\gamma_s} d(1 - \frac{a}{2d}) \text{ ....(2)}$$

From equations No. (1& 2)

$$M_u = A_s.d \frac{f_y}{\gamma_s}[1 - (7.7871 \times 10^{-5} A_s)]$$

$$300 \times 10^6 = 187826.087A_s - 14.6262A_s^2$$

$$\therefore 14.6262A_s^2 - 187826.087A_s + 300 \times 10^6 = 0$$

$$As = 1869mm^2 = 18.69cm^2$$

$$a = 0.093446A_s = 0.093446 \times 1869 = 174.65mm > 0.1d = 60mm$$

$$c = \frac{a}{0.8} = 218.31mm$$

$$\frac{c}{d} = \frac{218.31}{600} = 0.364 < \left(\frac{C_{max.}}{d} = 0.44\right) \text{......} \textit{ductile failure}$$

*** Check for $A_{s\ min.}$**

$A_{s\ min.}$ = The least of $[\frac{1.1}{f_y}.b.d]$ or $[1.3\,A_{s\ req.}]$

$$\frac{1.1}{f_y}.b.d = 550mm^2 \qquad \& \qquad 1.3A_{sreq.} = 2429.7mm^2$$

A_{smin}=550 mm^2

But not less than 0.15% b.d = 270 mm^2

$\therefore A_{smin.}$ = 550 mm^2

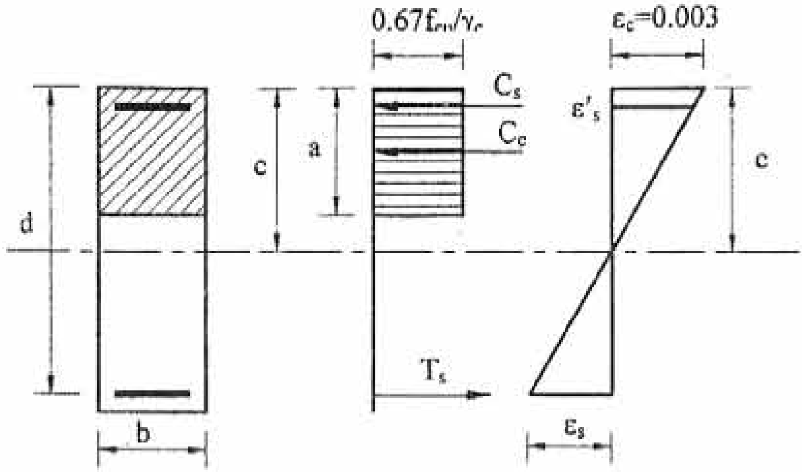
$\therefore A_{s\ design}$ = 1869 mm^2 = 18.69 cm^2

Example 5:

Given: M_u = 600 kN.m

t = 650 mm b = 300 mm d=600mm

f_{cu} = 25 N/mm^2 f_y = 360 N/mm^2 d'=50mm



Design:

$$d = C_1 \sqrt{\frac{M_u}{f_{cu}\, b}} \qquad \rightarrow C_1\text{=2.01 (use compression reinforcement)}$$

$$M_{u\max.} = R_{\max.} \frac{f_{cu}}{\gamma_c} . b . d^2$$

Take $R_{\max}$ *from Table* (4 − 1) *of the code*

$$M_{u\max.} = 0.194 \times \frac{25}{1.5} \times 300 \times 600^2 \times 10^{-6} = 349.20\ kN.m\ < 600\ kN.m$$

$$\Delta M = M_u^* - M_{u\max.} = 600 - 349.20 = 250.80\ kN.m$$

$$\Delta M = 250.80 \times 10^6 = A_s' \frac{f_y}{\gamma_s} (d - d') = A_s' \times \frac{360}{1.15} (600 - 50)$$

$$\therefore A_s' = 1456.67\ mm^2$$

Note:

$$A_s = \mu_{\max.} . b . d + A_s'$$

$$= (5.00 \times 10^{-4} \times 25 \times 300 \times 600) + (1456.67) = 2250 + 1456.67 = 3706.67 mm^2$$

$$\frac{d'}{d} = \frac{50}{600} = 0.083 \le 0.15 \text{ no need to check } \varepsilon'_s$$

Design of section using the charts for doubly reinforced sections:

$$\frac{M_u}{f_{cu}.bt^2} = \frac{600\times10^6}{25\times300\times650^2} = 0.189$$

From the chart for doubly reinforced sections for $f_y = 360\ N/mm^2$, ξ=0.9

Use α=0.40 $\Rightarrow$ $\omega = 0.27$

$\mu = \omega\ .\ f_{cu} / f_y = 0.27 \times 25 / 360 = 0.0187$

$A_s = \mu\ .\ b\ .\ t = 0.0187 \times 300 \times 650 = 3656\ mm^2$

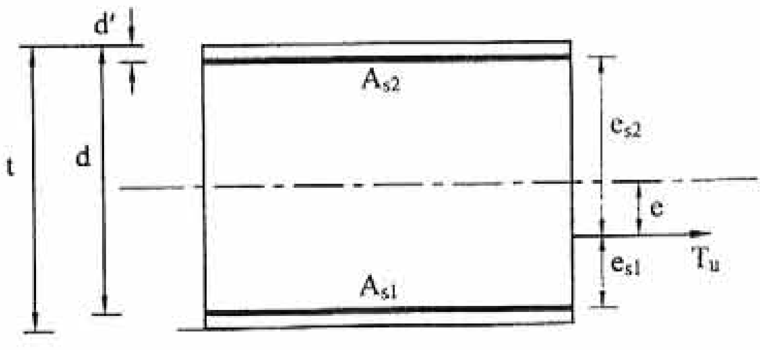
$A_s` = \alpha\ .\ A_s = 0.4 \times 3656 = 1462\ mm^2$

Take $A_s = 8\ \varnothing\ 25$ & $A_s` = 4\ \varnothing\ 25$

Example 6:

Design of unsymmetrical rectangular section subjected to eccentric tension force

Given: T_u = 400 kN, M_u = 100 kN.m, t = 750 mm, b = 250 mm, f_{cu} = 25 N/mm^2, f_y = 240 N/mm^2



$$e = \frac{Mu}{Tu} = \frac{100}{400} = 0.25m = 250mm \quad \Rightarrow \quad e < \frac{d-d`}{2}$$

$$e_{s1} = t/2 - e - cover = 750/2 - 250 - 40 = 85\,mm$$

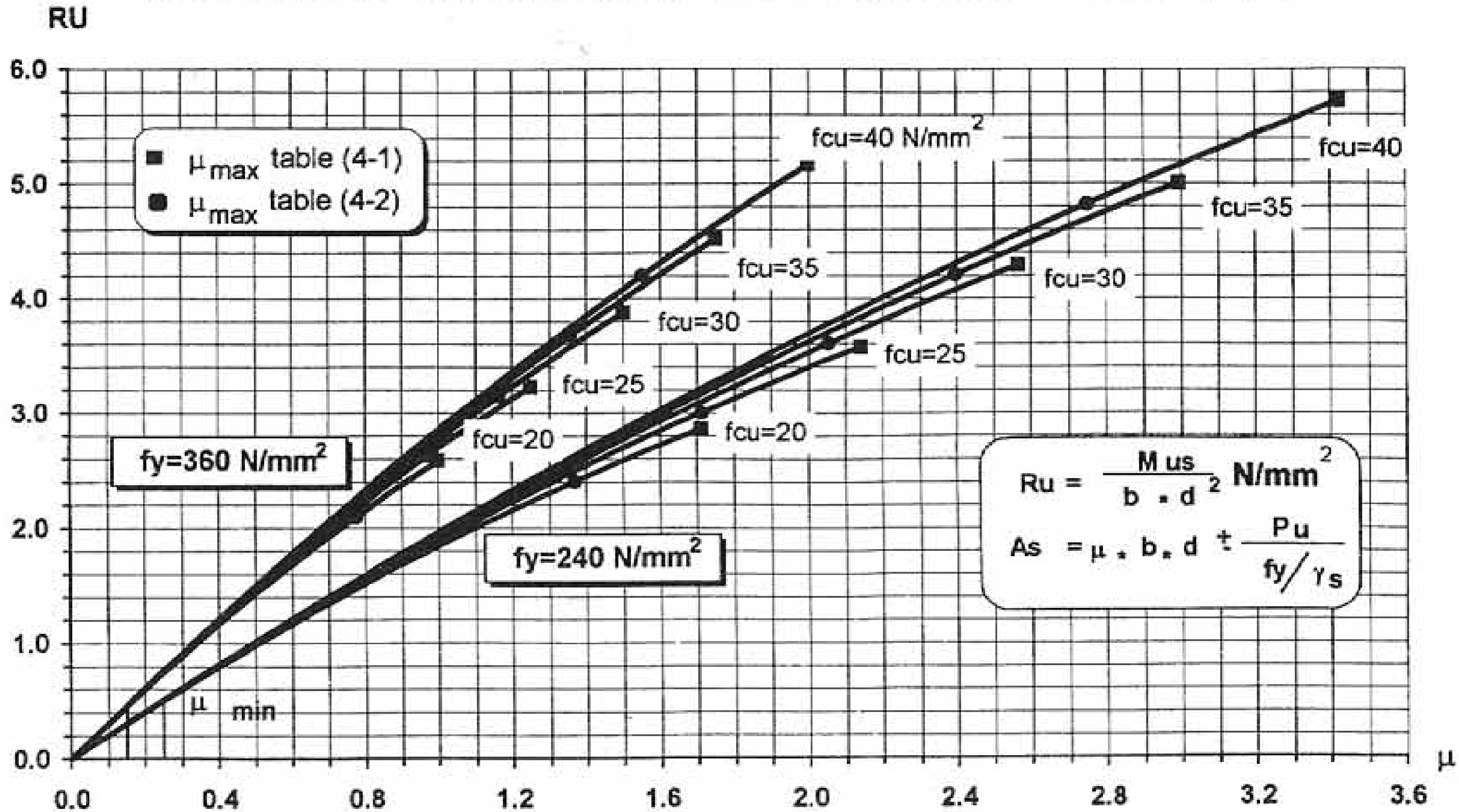
$$e_{s2} = t/2 + e - cover = 750/2 + 250 - 40 = 585\,mm$$

$$A_{s1} = \frac{T_u\, e_{s2}}{e_{s1} + e_{s2}} / \frac{f_y}{\gamma_s} = 1674\,mm^2$$

$$A_{s2} = \frac{T_u\, e_{s1}}{e_{s1} + e_{s2}} / \frac{f_y}{\gamma_s} = 244\,mm^2$$

Take $A_{s1} = 5\ \varnothing\ 22$ & $A_{s2} = 2\ \varnothing\ 16$

CHART (2-1): ULTIMATE LIMIT DESIGN CHARTS FOR SIMPLE BENDING & ECCENTRIC FORCE (TENSION FAILURE) FOR fy = 240 & 360 N/mm²

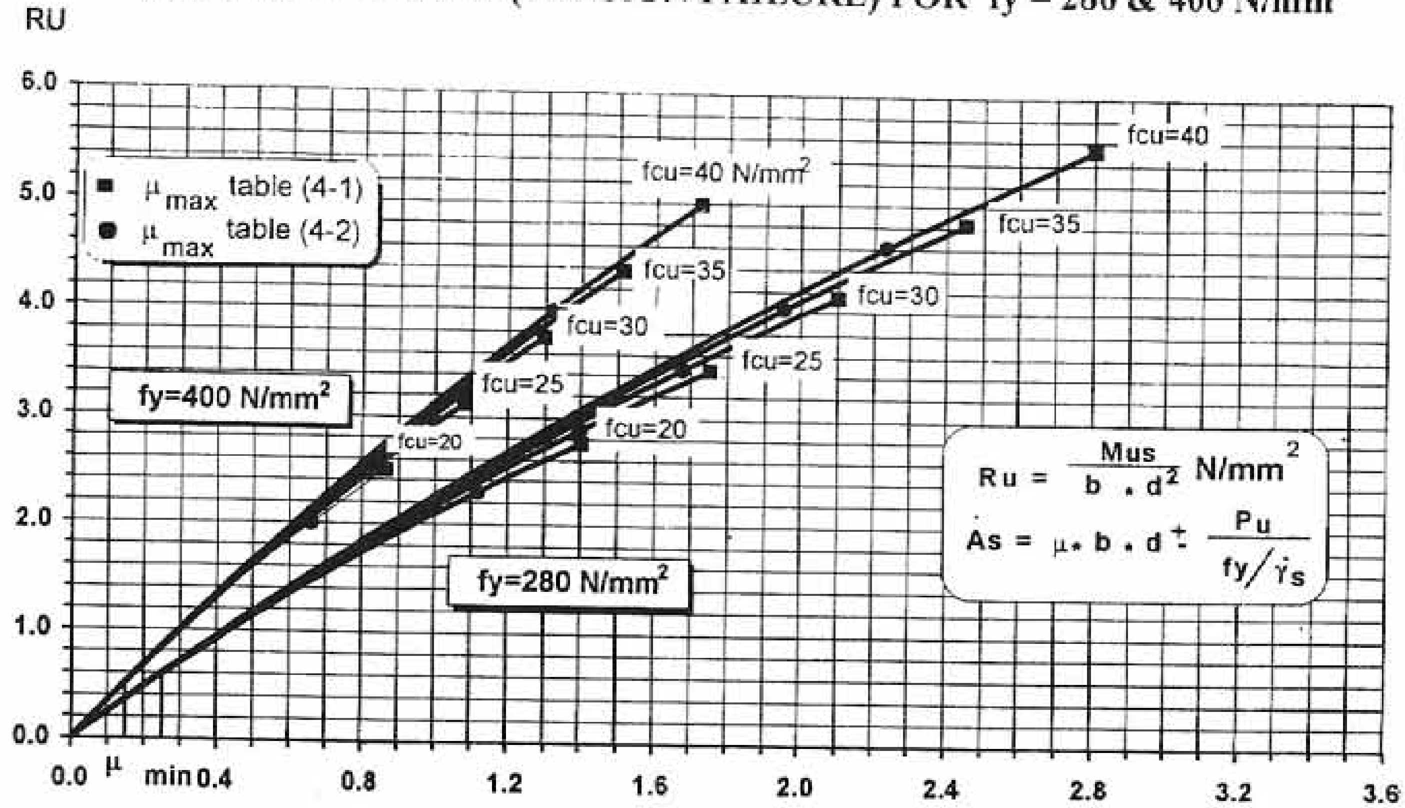


$$Ru = \frac{M_{us}}{b \cdot d^2} \; N/mm^2$$

$$As = \mu \cdot b \cdot d \pm \frac{P_u}{f_y/\gamma_s}$$

$$Ru = \mu \frac{f_y}{1.15} \left[1 - 0.973\, \mu \frac{f_y}{f_{cu}}\right] \qquad \mu = \frac{100\, A_s}{b \cdot d}$$

CHART (2-2): ULTIMATE LIMIT DESIGN CHARTS FOR SIMPLE BENDING & ECCENTRIC FORCE (TENSION FAILURE) FOR $f_y = 280$ & $400\ N/mm^2$



$As_{min.}$ = The least of $\frac{1.1}{f_y}$ b . d & 1.3 $As_{req.}$ But not less than 0.25%.b.d for st. 240, 0.15%.b.d for st. 360

c/d	C1	J
0.1250	4.85	0.826
0.1375	4.64	0.821
0.1500	4.46	0.817
0.1625	4.29	0.813
0.1750	4.15	0.808
0.1875	4.02	0.804
0.2000	3.90	0.800
0.2125	3.79	0.795
0.2250	3.70	0.791
0.2375	3.61	0.786
0.2500	3.53	0.782
0.2625	3.45	0.778
0.2750	3.38	0.773
0.2875	3.32	0.769
0.3000	3.26	0.765
0.3125	3.20	0.760
0.3250	3.15	0.756
0.3375	3.10	0.752
0.3500	3.05	0.747
0.3625	3.00	0.743
0.3750	2.96	0.739
0.3875	2.92	0.734
0.4000	2.89	0.730
0.4125	2.85	0.726
0.4250	2.82	0.721
0.4375	2.78	0.717
0.4500	2.75	0.713
0.4625	2.72	0.708
0.4750	2.70	0.704
0.4875	2.67	0.700
0.5000	2.65	0.695

CHART(2-3):ULTIMATE LIMIT DESIGN CHARTS

FOR SIMPLE BENDING & ECCENTRIC FORCE (TENSION FAILURE)

FOR ALL KINDS OF STEEL AND CONCRETE

$$d = C1\sqrt{\frac{M_u}{f_{cu}\, b}}$$

$$A_s = \frac{M_{us}}{f_y \cdot j \cdot d} \pm \frac{P_u}{f_y/\gamma_s}$$

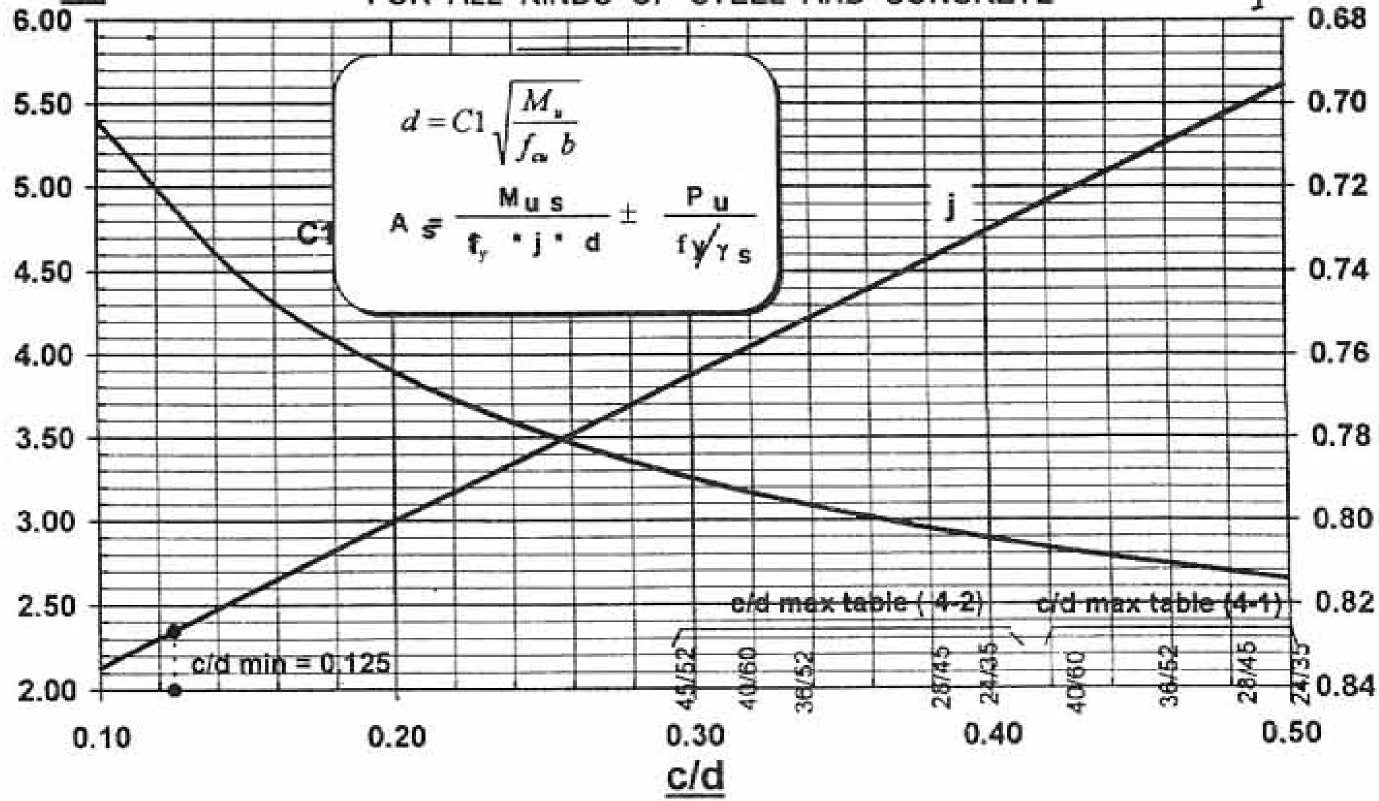
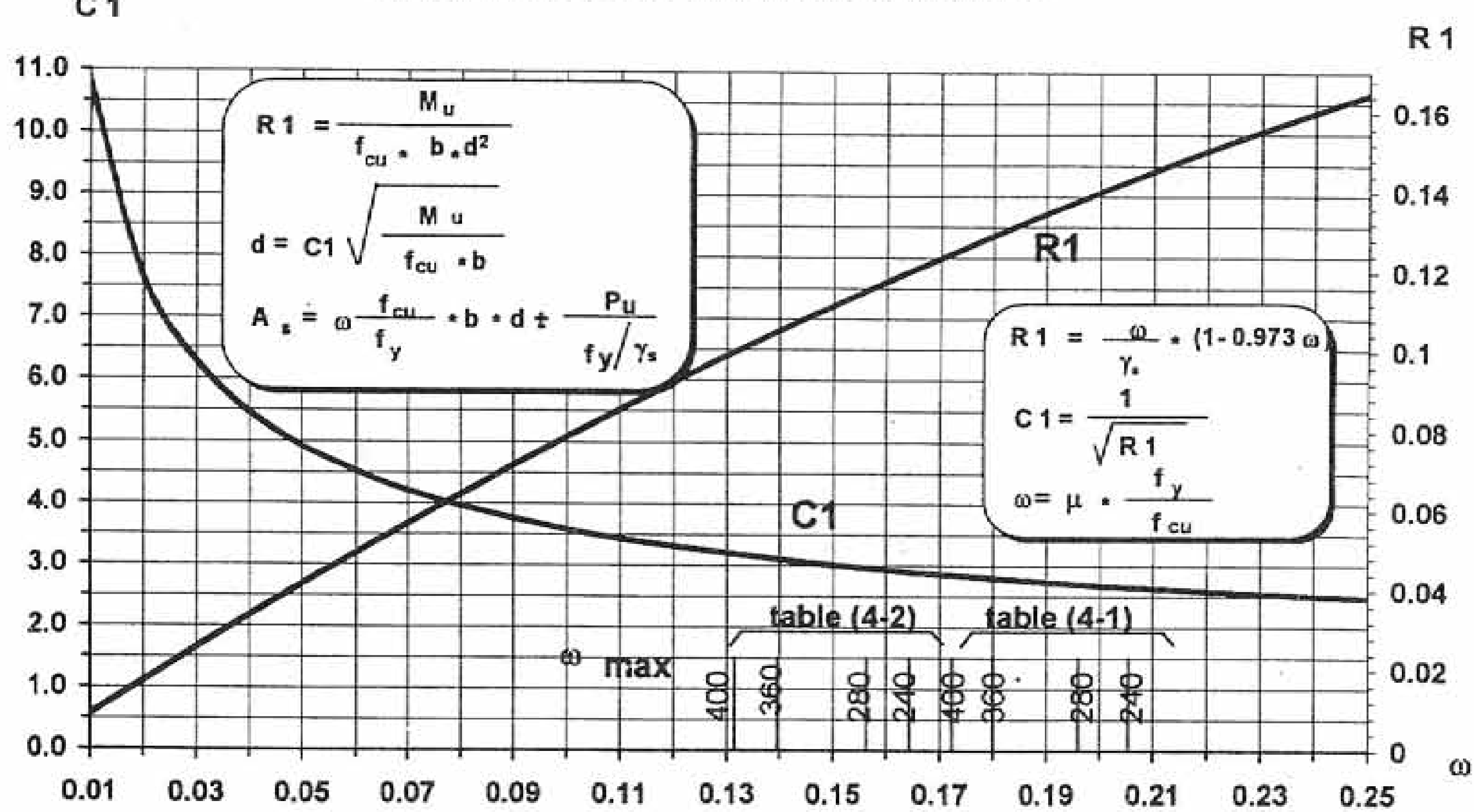


CHART (2-4): ULTIMATE LIMIT DESIGN CHARTS FOR SIMPLE BENDING & ECCENTRIC FORCE (TENSION FAILURE)

FOR ALL GRADES OF CONCRETE AND STEEL



$$R1 = \frac{M_u}{f_{cu} * b * d^2}$$

$$d = C1 \sqrt{\frac{M_u}{f_{cu} * b}}$$

$$A_s = \omega \frac{f_{cu}}{f_y} * b * d \pm \frac{P_u}{f_y / \gamma_s}$$

$$R1 = \frac{\omega}{\gamma_s} * (1 - 0.973\,\omega)$$

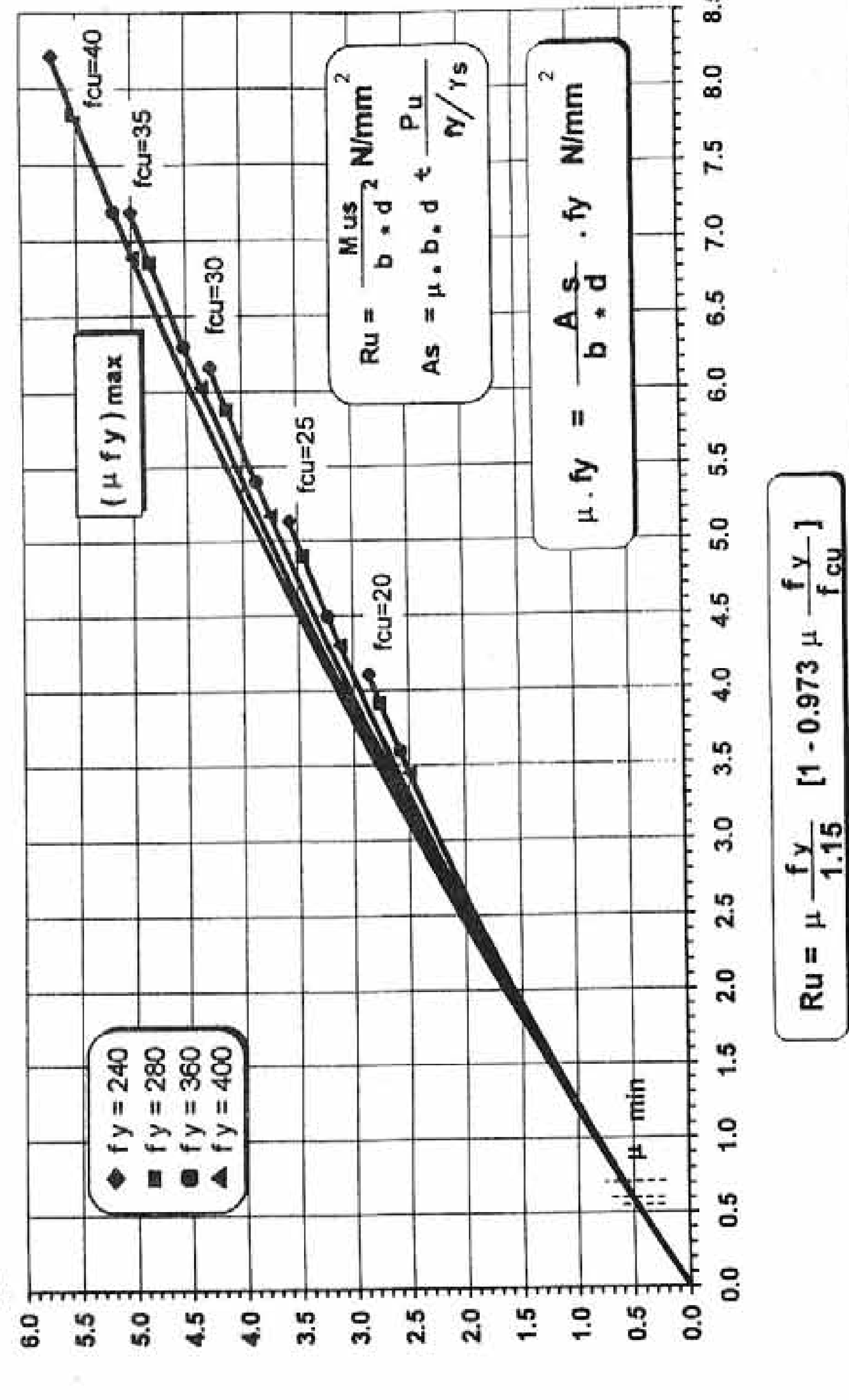
$$C1 = \frac{1}{\sqrt{R1}}$$

$$\omega = \mu * \frac{f_y}{f_{cu}}$$

$As_{min.} =$ The least of $\frac{1.1}{f_y}$ b . d & 1.3 $As_{req.}$ But not less than 0.25%.b.d for st. 240, 0.15%.b.d for st. 360

CHART (2-5):ULTIMATE LIMIT DESIGN CHARTS FOR SIMPLE BENDING & ECCENTRIC FORCE (TENSION FAILURE)

FOR ALL GRADES OF STEEL



TABLE(2-1):ULTIMATE LIMIT DESIGN TABLES

(μ) Values for (fy=240) %					
R_u	fcu=40	fcu=35	fcu=30	fcu=25	fcu=20
0.506					0.250
0.51				0.250	0.252
0.512			0.250	0.251	0.253
0.513		0.250	0.251	0.252	0.253
0.514	0.250	0.250	0.251	0.252	0.254
0.6	0.292	0.293	0.294	0.296	0.298
0.8	0.392	0.394	0.396	0.398	0.402
1	0.493	0.496	0.499	0.503	0.509
1.2	0.596	0.599	0.603	0.610	0.620
1.4	0.699	0.704	0.710	0.719	0.734
1.6	0.804	0.810	0.819	0.831	0.851
1.8	0.911	0.919	0.930	0.946	0.973
2	1.019	1.029	1.043	1.064	1.009
2.2	1.129	1.141	1.159	1.185	1.231
2.4	1.240	1.255	1.277	1.310	1.369
2.6	1.353	1.371	1.398	1.439	1.513
2.8	1.467	1.490	1.522	1.573	1.666
3	1.584	1.611	1.649	1.711	1.712
3.2	1.703	1.734	1.780	1.855	
3.4	1.823	1.860	1.914	2.004	
3.6	1.946	1.989	2.053		
3.8	2.071	2.121	2.196		
4	2.199	2.256	2.345		
4.2	2.329	2.395	2.498		
4.4	2.462	2.538			
4.6	2.598	2.685			
4.8	2.737	2.837			
5	2.880	2.994			
5.2	3.026				
5.4	3.177				
5.6	3.331				

(μ) Values for (fy=360) %					
R_u	fcu=40	fcu=35	fcu=30	fcu=25	fcu=20
0.456					0.168
0.46			0.150	0.150	0.168
0.464	0.150	0.150	0.151	0.151	0.169
0.5	0.162	0.162	0.163	0.163	0.169
0.6	0.195	0.195	0.196	0.197	0.199
0.8	0.262	0.262	0.264	0.265	0.268
1	0.329	0.330	0.332	0.335	0.340
1.2	0.397	0.399	0.402	0.406	0.413
1.4	0.466	0.469	0.473	0.479	0.489
1.6	0.536	0.540	0.546	0.554	0.568
1.8	0.607	0.613	0.620	0.631	0.649
2	0.679	0.686	0.695	0.709	0.733
2.1	0.716	0.723	0.734	0.750	0.821
2.2	0.752	0.761	0.772	0.790	0.912
2.4	0.826	0.837	0.851	0.874	0.913
2.6	0.902	0.914	0.932	0.960	
2.8	0.978	0.993	1.015	1.048	
3	1.056	1.074	1.099	1.141	
3.2	1.135	1.156	1.187	1.236	
3.4	1.215	1.240	1.276		
3.6	1.297	1.326	1.369		
3.8	1.381	1.414	1.464		
4	1.466	1.504			
4.2	1.553	1.597			
4.4	1.642	1.692			
4.6	1.732				
4.8	1.825				
5	1.920				

Assume; b & d

$Ru = Mu / b.d^2$ N/mm^2

From table Find (μ) value

As = (μ/100) . (b.d)

A_{smin} = The least of $\frac{1.1}{f_y}$ b . d & 1.3 $As_{req.}$ But not less than 0.25%.b.d for st. 240 / 0.15%.b.d for st. 360

TABLE(2-2): ULTIMATE LIMIT DESIGN TABLES

(μ) Values for (f_y=280) %					
R_u	fcu=40	fcu=35	fcu=30	fcu=25	fcu=20
0.587					0.25
0.591				0.250	0.251
0.595			0.250	0.251	0.253
0.598		0.250	0.251	0.253	0.254
0.6	0.251	0.251	0.252	0.253	0.255
0.8	0.336	0.337	0.339	0.341	0.345
1	0.423	0.425	0.427	0.431	0.437
1.2	0.511	0.513	0.517	0.523	0.531
1.4	0.599	0.603	0.609	0.616	0.629
1.6	0.690	0.695	0.702	0.712	0.730
1.8	0.781	0.788	0.797	0.811	0.834
2	0.873	0.882	0.894	0.912	0.942
2.2	0.967	0.978	0.993	1.016	1.055
2.304	1.017	1.029	1.046	1.071	1.116
2.4	1.063	1.076	1.095	1.123	1.173
2.6	1.159	1.175	1.198	1.234	1.297
2.759	1.237	1.256	1.283	1.324	1.372
2.8	1.258	1.277	1.305	1.348	1.400
2.88	1.298	1.318	1.348	1.395	
3	1.358	1.380	1.414	1.467	
3.2	1.459	1.486	1.526	1.590	
3.4	1.563	1.594	1.641	1.718	
3.449	1.588	1.621	1.670	1.750	
3.456	1.592	1.625	1.674		
3.6	1.668	1.705	1.760		
3.8	1.775	1.818	1.883		
4	1.885	1.934	2.010		
4.033	1.903	1.953	2.031		
4.138	1.962	2.016	2.100		
4.2	1.996	2.053			
4.4	2.111	2.176			
4.6	2.227	2.302			
4.609	2.232	2.307			
4.8	2.346	2.432			
4.828	2.363	2.450			
5	2.469				
5.2	2.594				
5.4	2.723				
5.518	2.800				

(μ) Values for (f_y=400) %					
R_u	fcu=40	fcu=35	fcu=30	fcu=25	fcu=20
0.505					0.150
0.51			0.150	0.150	0.150
0.514	0.150	0.150	0.151	0.151	0.152
0.55	0.161	0.161	0.162	0.162	0.175
0.6	0.175	0.176	0.177	0.177	0.179
0.8	0.235	0.236	0.237	0.239	0.241
1	0.296	0.297	0.299	0.302	0.306
1.2	0.357	0.359	0.362	0.366	0.372
1.4	0.420	0.422	0.426	0.431	0.440
1.6	0.483	0.486	0.491	0.499	0.511
1.8	0.547	0.551	0.558	0.568	0.584
1.995	0.610	0.616	0.624	0.637	0.655
2	0.611	0.617	0.626	0.638	0.660
2.2	0.677	0.685	0.695	0.711	0.739
2.4	0.744	0.753	0.766	0.786	0.821
2.495	0.776	0.786	0.800	0.823	0.861
2.6	0.812	0.823	0.839	0.864	
2.8	0.880	0.894	0.913	0.944	
2.994	0.948	0.964	0.987	1.024	
3	0.950	0.966	0.990	1.027	
3.119	0.993	1.010	1.036	1.077	
3.2	1.022	1.040	1.068		
3.4	1.094	1.116	1.149		
3.49	1.127	1.151	1.186		
3.6	1.168	1.193	1.232		
3.744	1.222	1.250	1.293		
3.8	1.243	1.273			
3.99	1.316	1.350			
4	1.319	1.354			
4.2	1.398	1.437			
4.365	1.463	1.508			
4.4	1.477				
4.6	1.559				
4.8	1.642				
4.99	1.724				

Assume; b & d

$Ru = Mu / b.d^2$ N/mm^2

From table Find (μ) value

$As = (\mu/100) . (b.d)$

A_{smin} = The least of $\frac{1.1}{f_y}$ b . d & 1.3 $As_{req.}$ But not less than 0.25%.b.d for st. 280 / 0.15%.b.d for st. 400

TABLE (2-3): ULTIMATE LIMIT DESIGN TABLES (f_y=240 MPa and f_y=360 MPa)

R_u	μ (%)				
	f_{cu} (MPa)				
	40	35	30	25	20
0.1	0.048	0.048	0.048	0.048	0.048
0.2	0.096	0.096	0.097	0.097	0.097
0.3	0.145	0.145	0.145	0.146	0.146
0.4	0.194	0.194	0.195	0.195	0.196
0.5	0.243	0.244	0.244	0.245	0.247
0.6	0.292	0.293	0.294	0.296	0.298
0.7	0.342	0.343	0.345	0.347	0.350
0.8	0.392	0.394	0.396	0.398	0.402
0.9	0.443	0.444	0.447	0.450	0.455
1	0.493	0.496	0.499	0.503	0.509
1.1	0.544	0.547	0.551	0.556	0.564
1.2	0.596	0.599	0.603	0.610	0.620
1.3	0.647	0.651	0.656	0.664	0.676
1.4	0.699	0.704	0.710	0.719	0.734
1.5	0.752	0.757	0.764	0.775	0.792
1.6	0.804	0.811	0.819	0.831	0.851
1.7	0.858	0.864	0.874	0.888	0.912
1.8	0.911	0.919	0.930	0.946	0.973
1.9	0.965	0.974	0.986	1.005	1.036
2	1.019	1.029	1.043	1.064	1.100
2.1	1.074	1.085	1.101	1.124	1.165
2.2	1.129	1.141	1.159	1.185	1.231
2.3	1.184	1.198	1.218	1.248	1.299
2.4	1.240	1.255	1.277	1.310	1.369
2.5	1.296	1.313	1.337	1.374	1.440
2.6	1.353	1.371	1.398	1.439	1.513
2.7	1.410	1.430	1.460	1.506	1.588
2.8	1.467	1.490	1.522	1.573	1.666
2.9	1.525	1.550	1.585	1.641	1.745
3	1.584	1.611	1.649	1.711	1.828
3.1	1.643	1.672	1.714	1.782	1.913
3.2	1.703	1.734	1.780	1.855	2.001
3.3	1.763	1.797	1.847	1.929	2.093
3.4	1.823	1.860	1.915	2.005	2.189
3.5	1.884	1.924	1.983	2.082	2.289
3.6	1.946	1.989	2.053	2.162	2.395
3.7	2.009	2.055	2.124	2.243	2.507
3.8	2.071	2.121	2.197	2.327	2.627
3.9	2.135	2.188	2.270	2.413	2.756
4	2.199	2.257	2.345	2.501	2.897
4.1	2.264	2.326	2.421	2.593	3.054
4.2	2.329	2.396	2.499	2.687	3.235
4.3	2.396	2.466	2.578	2.785	3.454
4.4	2.462	2.538	2.659	2.887	3.758
4.5	2.530	2.611	2.742	2.994	
4.6	2.599	2.686	2.826	3.105	
4.7	2.668	2.761	2.913	3.222	
4.8	2.738	2.837	3.002	3.347	
4.9	2.809	2.915	3.093	3.479	
5	2.880	2.994	3.187	3.621	
5.1	2.953	3.075	3.283	3.776	
5.2	3.027	3.157	3.383	3.949	
5.3	3.101	3.240	3.486	4.145	
5.4	3.177	3.326	3.593	4.381	
5.5	3.254	3.413	3.703	4.697	
5.6	3.332	3.502	3.819		
5.7	3.411	3.593	3.940		

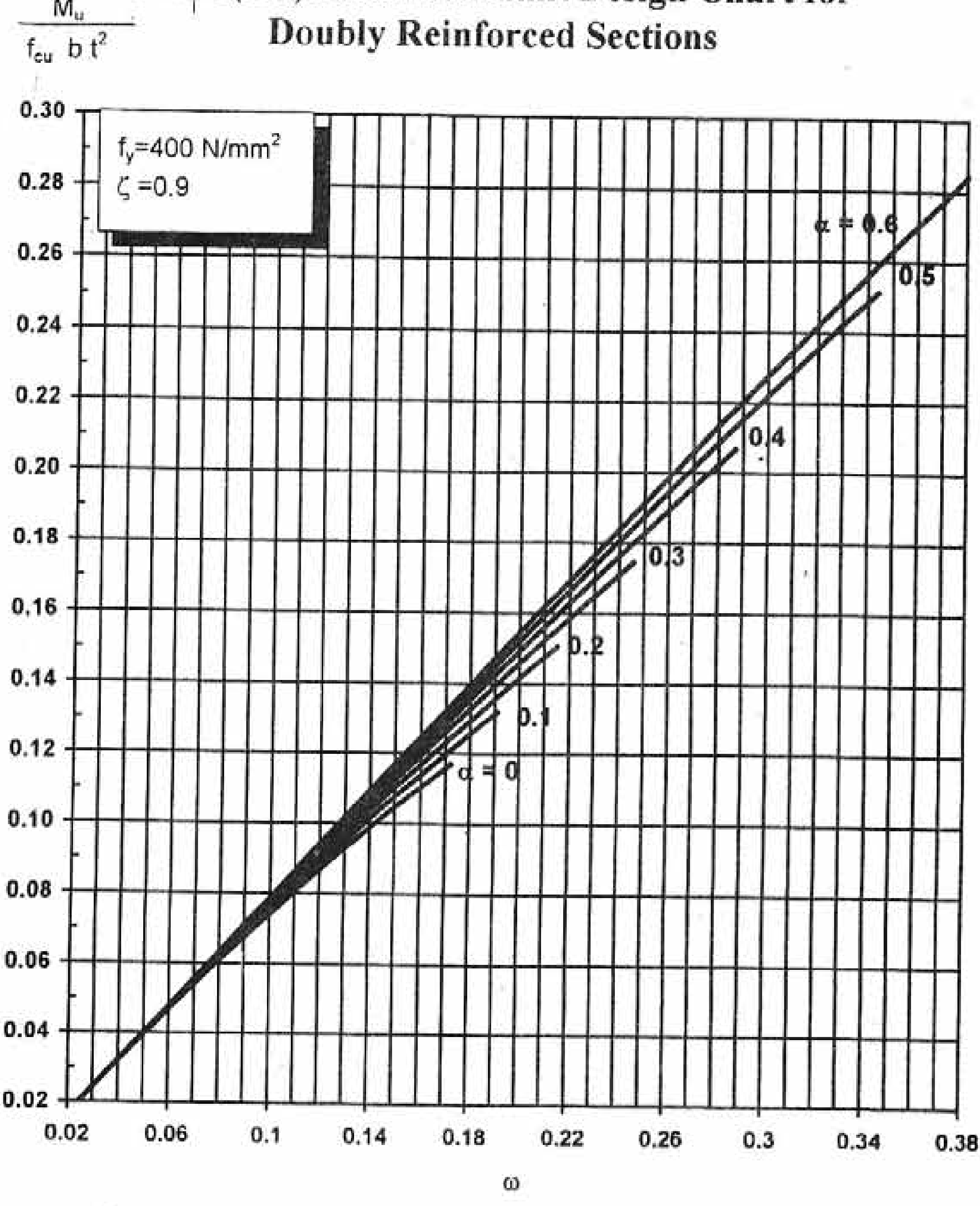
R_u	μ (%)				
	f_{cu} (MPa)				
	40	35	30	25	20
0.1	0.032	0.032	0.032	0.032	0.032
0.2	0.064	0.064	0.064	0.064	0.065
0.3	0.097	0.097	0.097	0.097	0.097
0.4	0.129	0.129	0.130	0.130	0.131
0.5	0.162	0.162	0.163	0.163	0.164
0.6	0.195	0.195	0.196	0.197	0.199
0.7	0.228	0.229	0.230	0.231	0.233
0.8	0.262	0.262	0.264	0.265	0.268
0.9	0.295	0.296	0.298	0.300	0.304
1	0.329	0.330	0.332	0.335	0.340
1.1	0.363	0.365	0.367	0.371	0.376
1.2	0.397	0.399	0.402	0.406	0.413
1.3	0.432	0.434	0.438	0.443	0.451
1.4	0.466	0.469	0.473	0.479	0.489
1.5	0.501	0.505	0.509	0.517	0.528
1.6	0.536	0.540	0.546	0.554	0.568
1.7	0.572	0.576	0.583	0.592	0.608
1.8	0.607	0.613	0.620	0.631	0.649
1.9	0.643	0.649	0.657	0.670	0.690
2	0.679	0.686	0.695	0.709	0.733
2.1	0.716	0.723	0.734	0.750	0.776
2.2	0.752	0.761	0.772	0.790	0.821
2.3	0.789	0.799	0.812	0.832	0.866
2.4	0.827	0.837	0.851	0.874	0.913
2.5	0.864	0.875	0.891	0.916	0.960
2.6	0.902	0.914	0.932	0.960	1.009
2.7	0.940	0.954	0.973	1.004	1.588
2.8	0.978	0.993	1.015	1.049	1.666
2.9	1.017	1.033	1.057	1.094	1.745
3	1.056	1.074	1.100	1.141	1.828
3.1	1.095	1.115	1.143	1.188	1.913
3.2	1.135	1.156	1.187	1.237	2.001
3.3	1.175	1.198	1.231	1.929	2.093
3.4	1.216	1.240	1.276	2.005	2.189
3.5	1.256	1.283	1.322	2.082	2.289
3.6	1.297	1.326	1.369	2.162	2.395
3.7	1.339	1.370	1.416	2.243	2.507
3.8	1.381	1.414	1.464	2.327	2.627
3.9	1.423	1.459	1.513	2.413	2.756
4	1.466	1.504	2.345	2.501	2.897
4.1	1.509	1.550	2.421	2.593	3.054
4.2	1.553	1.597	2.499	2.687	3.235
4.3	1.597	1.644	2.578	2.785	3.454
4.4	1.642	1.692	2.659	2.887	3.758
4.5	1.687	1.741	2.742	2.994	
4.6	1.732	2.686	2.826	3.105	
4.7	1.778	2.761	2.913	3.222	
4.8	1.825	2.837	3.002	3.347	
4.9	1.872	2.915	3.093	3.479	
5	1.920	2.994	3.187	3.621	
5.1	1.969	3.075	3.283	3.776	
5.2	2.018	3.157	3.383	3.949	
5.3	3.101	3.240	3.486	4.145	
5.4	3.177	3.326	3.593	4.381	
5.5	3.254	3.413	3.703	4.697	
5.6	3.332	3.502	3.819		
5.7	3.411	3.593	3.940		

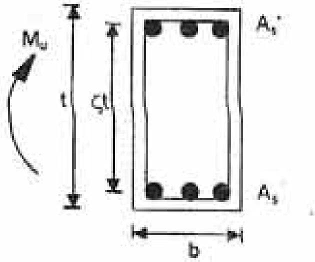
TABLE (2-4): ULTIMATE LIMIT DESIGN TABLES (f_y=280 MPa and f_y=400 MPa)

R_u	μ (%)				
	f_{cu} (MPa)				
	40	35	30	25	20
0.1	0.041	0.041	0.041	0.041	0.041
0.2	0.083	0.083	0.083	0.083	0.083
0.3	0.124	0.124	0.125	0.125	0.125
0.4	0.166	0.166	0.167	0.167	0.168
0.5	0.208	0.209	0.209	0.210	0.211
0.6	0.251	0.251	0.252	0.253	0.255
0.7	0.293	0.294	0.295	0.297	0.300
0.8	0.336	0.337	0.339	0.341	0.345
0.9	0.379	0.381	0.383	0.386	0.390
1	0.423	0.425	0.427	0.431	0.437
1.1	0.467	0.469	0.472	0.477	0.484
1.2	0.511	0.513	0.517	0.523	0.531
1.3	0.555	0.558	0.563	0.569	0.580
1.4	0.599	0.603	0.609	0.616	0.629
1.5	0.644	0.649	0.655	0.664	0.679
1.6	0.690	0.695	0.702	0.712	0.730
1.7	0.735	0.741	0.749	0.761	0.781
1.8	0.781	0.788	0.797	0.811	0.834
1.9	0.827	0.835	0.845	0.861	0.888
2	0.873	0.882	0.894	0.912	0.942
2.1	0.920	0.930	0.943	0.964	0.998
2.2	0.967	0.978	0.993	1.016	1.055
2.3	1.015	1.027	1.044	1.069	1.114
2.4	1.063	1.076	1.095	1.123	1.173
2.5	1.111	1.125	1.146	1.178	1.234
2.6	1.159	1.175	1.198	1.234	1.297
2.7	1.208	1.226	1.251	1.290	1.362
2.8	1.258	1.277	1.305	1.348	1.428
2.9	1.308	1.329	1.359	1.407	1.496
3	1.358	1.381	1.414	1.467	1.567
3.1	1.408	1.433	1.469	1.528	1.640
3.2	1.459	1.486	1.526	1.590	1.715
3.3	1.511	1.540	1.583	1.653	1.794
3.4	1.563	1.594	1.641	1.718	1.876
3.5	1.615	1.649	1.700	1.785	1.962
3.6	1.668	1.705	1.760	1.853	2.053
3.7	1.722	1.761	1.821	1.923	2.149
3.8	1.776	1.818	1.883	1.994	2.252
3.9	1.830	1.876	1.946	2.068	2.362
4	1.885	1.934	2.010	2.144	2.483
4.1	1.941	1.993	2.075	2.222	2.618
4.2	1.997	2.053	2.142	2.303	2.773
4.3	2.053	2.114	2.210	2.388	2.960
4.4	2.111	2.176	2.279	2.475	3.221
4.5	2.169	2.238	2.350	2.566	
4.6	2.227	2.302	2.422	2.662	
4.7	2.287	2.366	2.497	2.762	
4.8	2.347	2.432	2.573	2.868	
4.9	2.407	2.499	2.651	2.982	
5	2.469	2.567	2.731	3.104	
5.1	2.531	2.636	2.814	3.237	
5.2	2.594	2.706	2.900	3.385	
5.3	2.658	2.778	2.988	3.553	
5.4	2.723	2.851	3.079	3.755	
5.5	2.789	2.925	3.174	4.026	
5.6	2.856	3.002	3.274		
5.7	2.923	3.080	3.377		

R_u	μ (%)				
	f_{cu} (MPa)				
	40	35	30	25	20
0.1	0.029	0.029	0.029	0.029	0.029
0.2	0.058	0.058	0.058	0.058	0.058
0.3	0.087	0.087	0.087	0.087	0.088
0.4	0.116	0.117	0.117	0.117	0.118
0.5	0.146	0.146	0.147	0.147	0.148
0.6	0.175	0.176	0.177	0.177	0.179
0.7	0.205	0.206	0.207	0.208	0.210
0.8	0.235	0.236	0.237	0.239	0.241
0.9	0.266	0.267	0.268	0.270	0.273
1	0.296	0.297	0.299	0.302	0.306
1.1	0.327	0.328	0.330	0.334	0.339
1.2	0.357	0.359	0.362	0.366	0.372
1.3	0.388	0.391	0.394	0.398	0.406
1.4	0.420	0.422	0.426	0.431	0.440
1.5	0.451	0.454	0.459	0.465	0.475
1.6	0.483	0.486	0.491	0.499	0.511
1.7	0.515	0.519	0.524	0.533	0.547
1.8	0.547	0.551	0.558	0.568	0.584
1.9	0.579	0.584	0.592	0.603	0.621
2	0.611	0.617	0.626	0.638	0.660
2.1	0.644	0.651	0.660	0.675	0.699
2.2	0.677	0.685	0.695	0.711	0.739
2.3	0.710	0.719	0.731	0.749	0.780
2.4	0.744	0.753	0.766	0.786	0.821
2.5	0.778	0.788	0.802	0.825	0.864
2.6	0.812	0.823	0.839	0.864	0.908
2.7	0.846	0.858	0.876	0.903	1.362
2.8	0.880	0.894	0.913	0.944	1.428
2.9	0.915	0.930	0.951	0.985	1.496
3	0.950	0.966	0.990	1.027	1.567
3.1	0.986	1.003	1.029	1.069	1.640
3.2	1.022	1.040	1.068	1.113	1.715
3.3	1.058	1.078	1.108	1.653	1.794
3.4	1.094	1.116	1.149	1.718	1.876
3.5	1.131	1.155	1.190	1.785	1.962
3.6	1.168	1.193	1.232	1.853	2.053
3.7	1.205	1.233	1.275	1.923	2.149
3.8	1.243	1.273	1.318	1.994	2.252
3.9	1.281	1.313	1.362	2.068	2.362
4	1.319	1.354	2.010	2.144	2.483
4.1	1.358	1.395	2.075	2.222	2.618
4.2	1.398	1.437	2.142	2.303	2.773
4.3	1.437	1.480	2.210	2.388	2.960
4.4	1.477	1.523	2.279	2.475	3.221
4.5	1.518	1.567	2.350	2.566	
4.6	1.559	2.302	2.422	2.662	
4.7	1.601	2.366	2.497	2.762	
4.8	1.643	2.432	2.573	2.868	
4.9	1.685	2.499	2.651	2.982	
5	1.728	2.567	2.731	3.104	
5.1	1.772	2.636	2.814	3.237	
5.2	1.816	2.706	2.900	3.385	
5.3	2.658	2.778	2.988	3.553	
5.4	2.723	2.851	3.079	3.755	
5.5	2.789	2.925	3.174	4.026	
5.6	2.856	3.002	3.274		
5.7	2.923	3.080	3.377		

Chart (2-6): Ultimate Limit Design Chart for Doubly Reinforced Sections



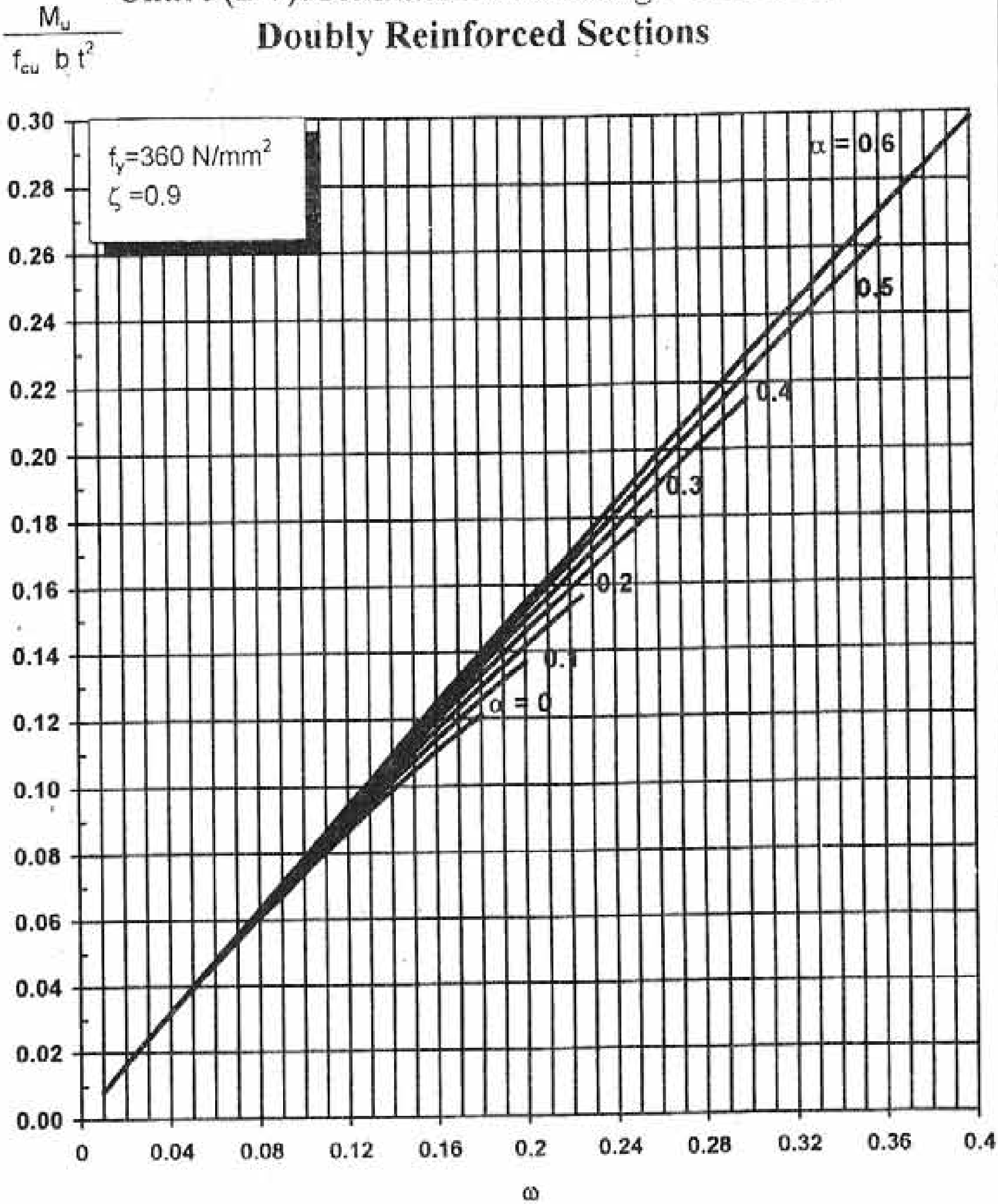


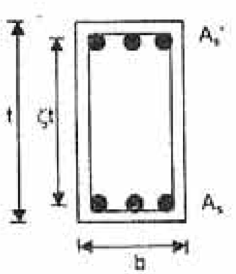
$\mu = \omega\ f_{cu} / f_y$

$A_s = \mu\ b\ t$

$A'_s = \alpha\ A_s$

Chart (2-7):Ultimate Limit Design Chart for Doubly Reinforced Sections



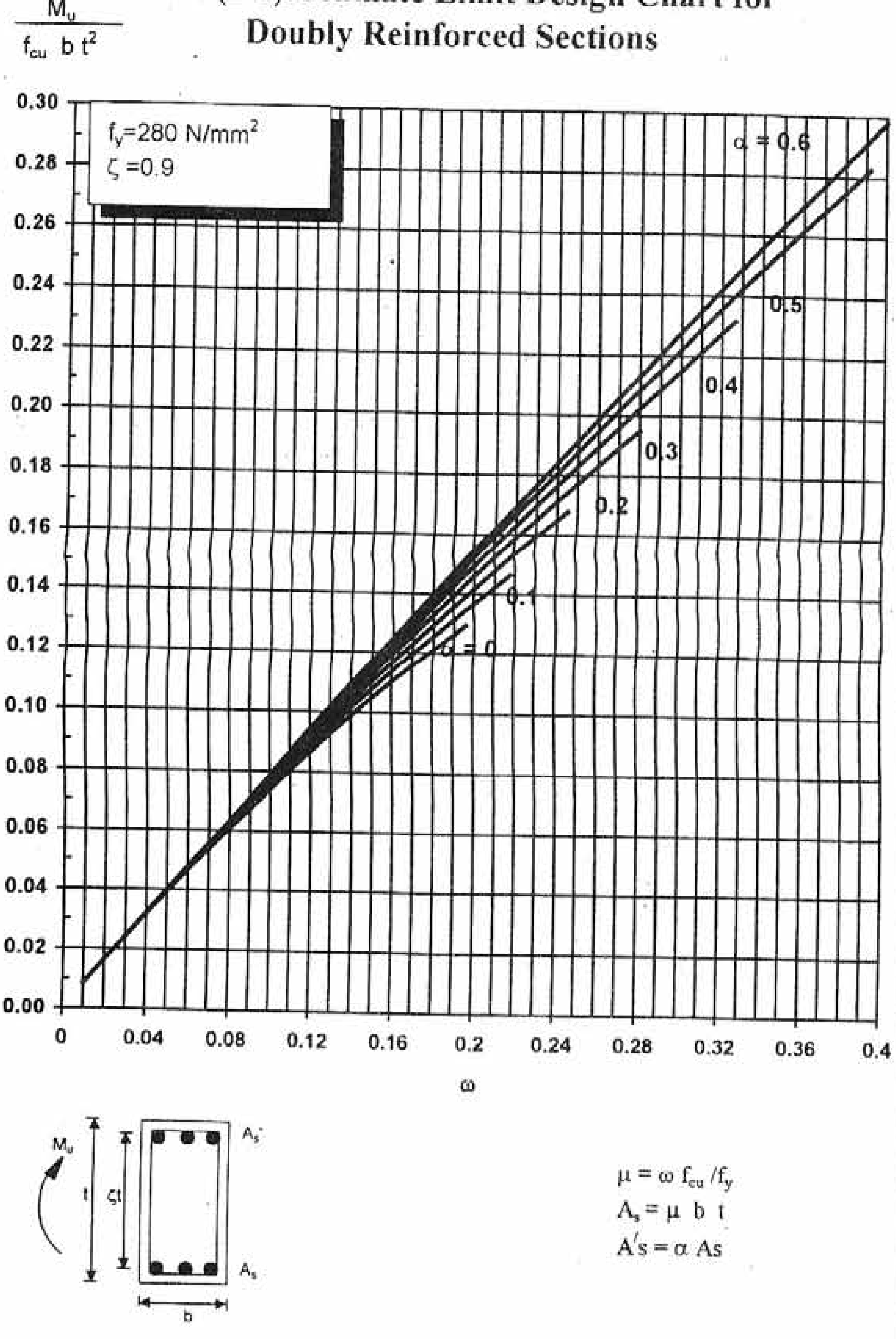


$\mu = \omega\ f_{cu} / f_y$

$A_s = \mu\ b\ t$

$A's = \alpha\ As$

Chart (2-8): Ultimate Limit Design Chart for Doubly Reinforced Sections

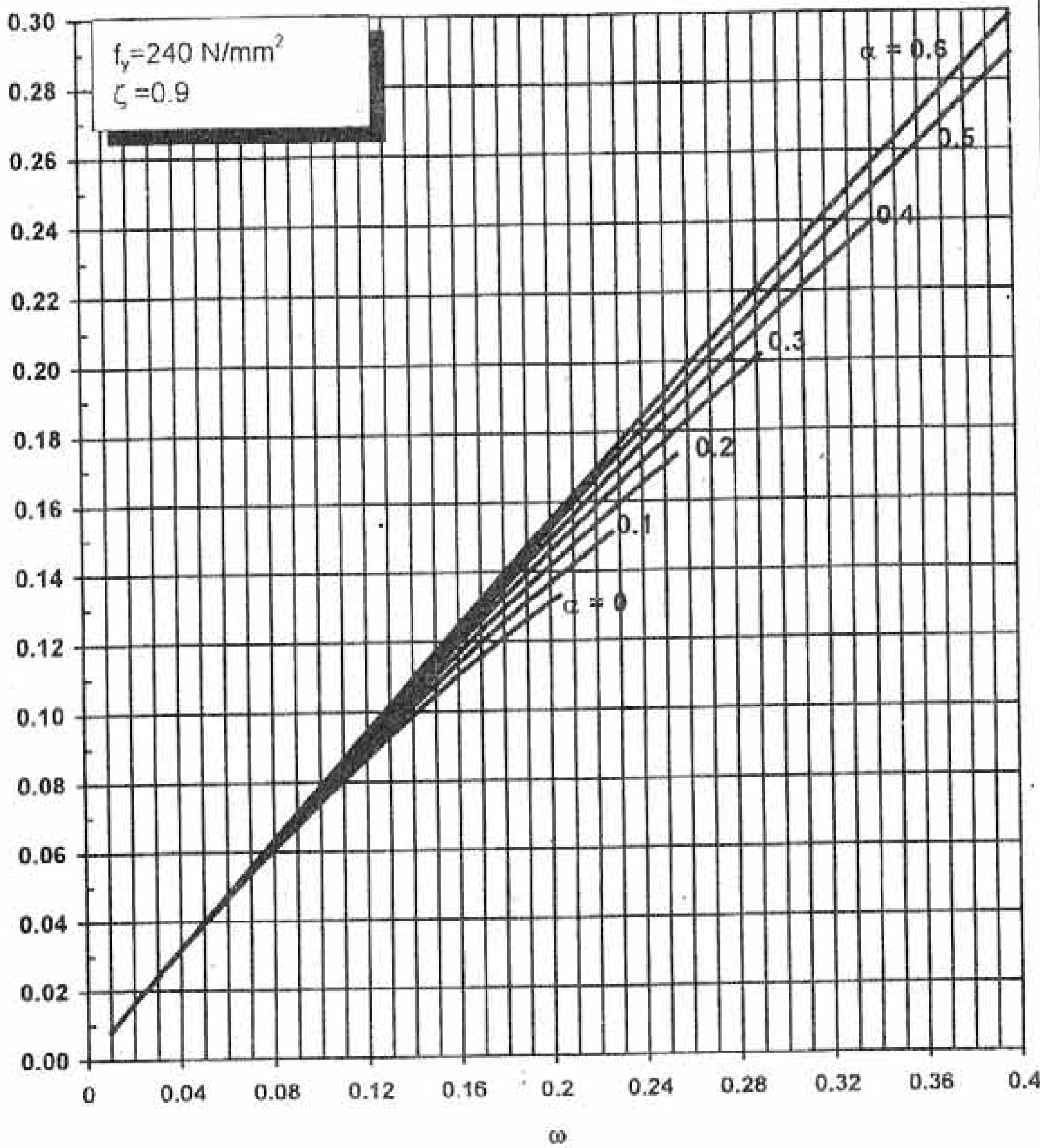


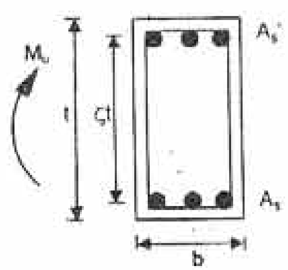
$\mu = \omega\ f_{cu} / f_y$

$A_s = \mu\ b\ t$

$A's = \alpha\ As$

Chart (2-9):Ultimate Limit Design Chart for Doubly Reinforced Sections





$\mu = \omega\ f_{cu}/f_y$

$A_s = \mu\ b\ t$

$A's = \alpha\ As$

3. AXIALLY LOADED SHORT COMPRESSION MEMBERS ($e/t \leq 0.05$)

3.1 Introduction

All compression members are to be designed for a minimum eccentricity in the two principle directions. Clause 4-2-1-3 of the code specifies the following equation for designing tied compression members

$P_u = 0.35\, f_{cu}\, A_c + 0.67\, A_{sc}\, f_y$

The above equation can be written as

$P_u = 0.35\, f_{cu}\, (A_g - \mu\, A_g) + 0.67\, f_y\, \mu\, A_g$

Dividing both sides by A_g

$$\frac{P_u}{A_g} = 0.35\, f_{cu}\, (1-\mu) + 0.67\, f_y\, \mu$$

$$\frac{P_u}{A_g} = 0.35\, f_{cu} + \mu\,(0.67\, f_y - 0.35\, f_{cu})$$

Design Charts (3-1 to 3-4) can be used for designing short columns. The following example illustrates the procedure

3.2 Design Example

Example 1:

Determine the cross section and the reinforcement required for a short braced axially loaded interior column with the following data:

$P_u = 2000$ kN

$f_{cu} = 25$ N/mm^2

$f_y = 360$ N/mm^2

Solution

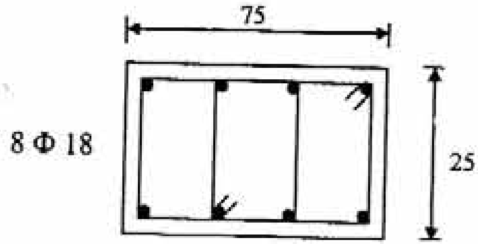
Assume μ=1% [$\mu > \mu_{min}(0.008)$ and $\mu < \mu_{max}$ (0.04)] and referring to chart with f_y = 360 N/mm^2, the required cross sectional area of the column is 1770 cm^2

A_g= 1770 cm^2

Choose 25 x 75 $\quad A_{g,chosen}$ = 1875 cm^2

$A_{s,chosen} = 0.01 \times 1770 = 17.7\ cm^2$ (8 Φ18)

$A_{s,min} = 0.006\ A_{g,chosen} = 0.006 \times 25 \times 75 = 11.25\ cm^2 < A_{s,chosen}$ . . . ok



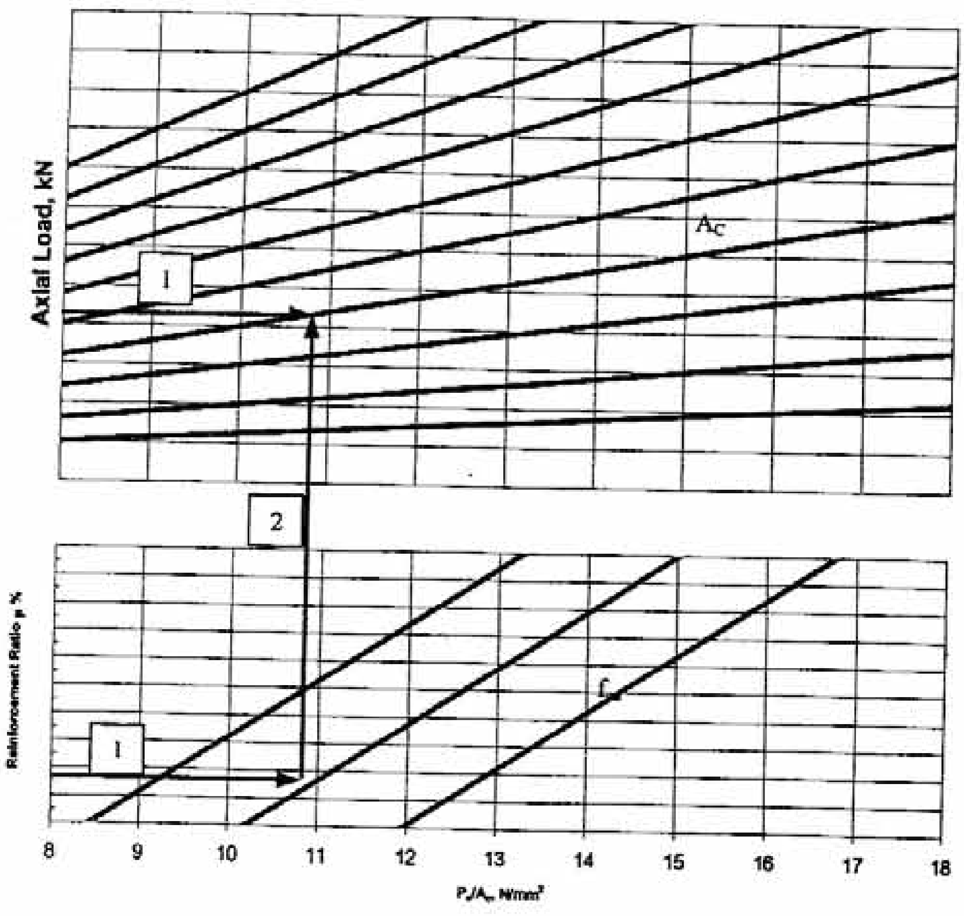
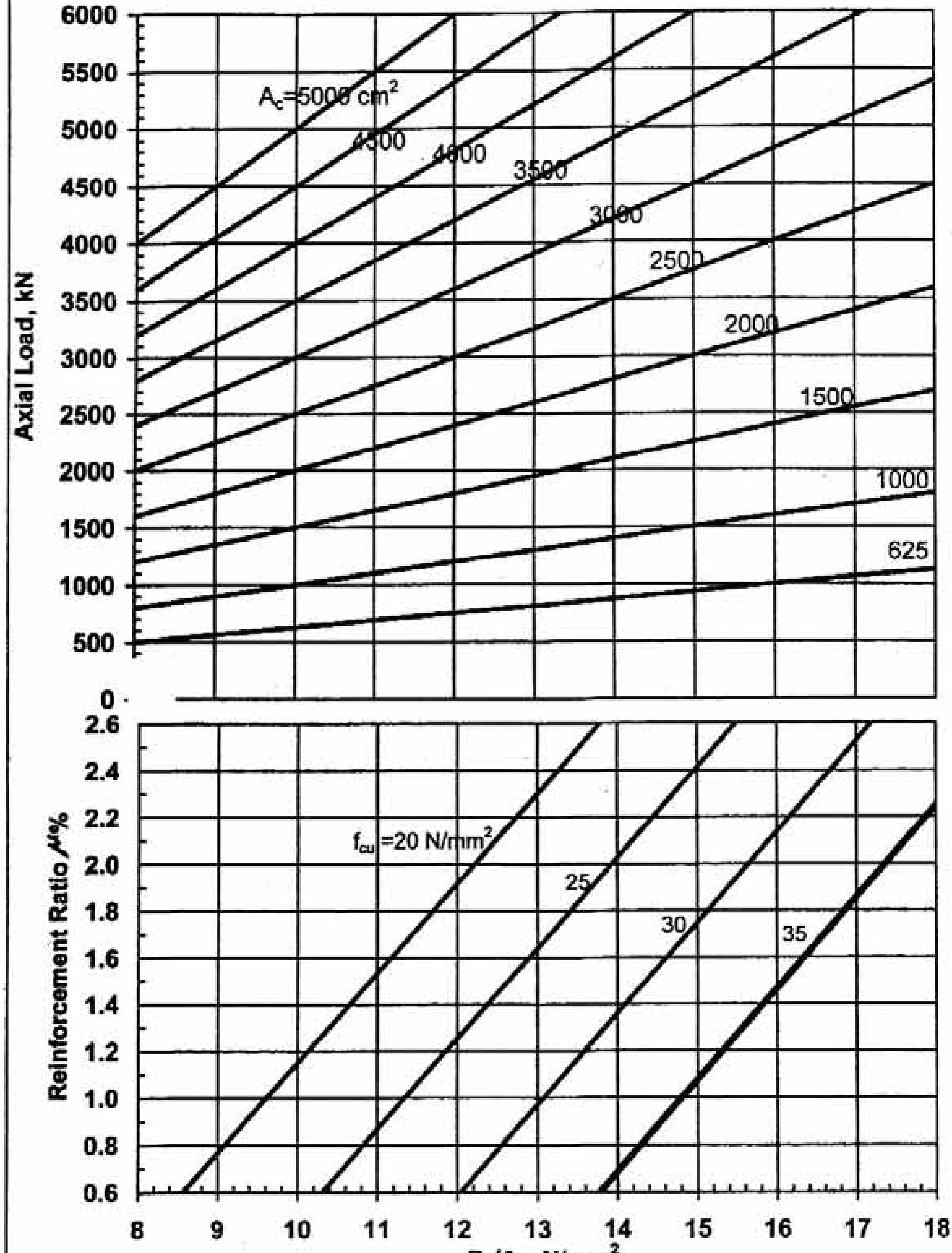


CHART (3-1): DESIGN OF AXIALLY LOADED TIED COLUMNS f_y=400 N/mm^2



ECCS 203-2001 Design Aids Compression Members

CHART (3-2):DESIGN OF AXIALLY LOADED TIED COLUMNS f_y=360 N/mm^2

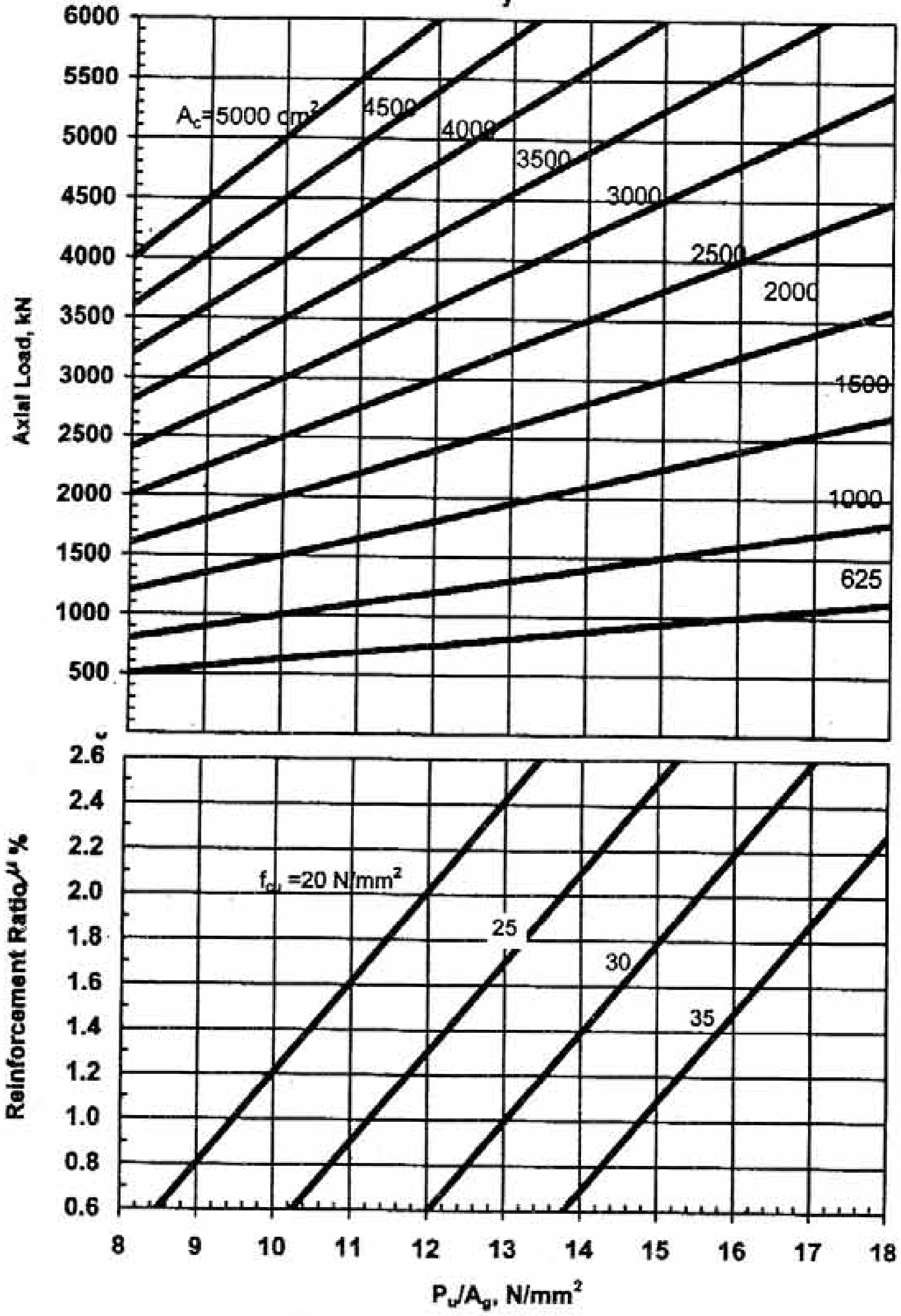


CHART (3-3):DESIGN OF AXIALLY LOADED TIED COLUMNS f_y=280 N/mm²

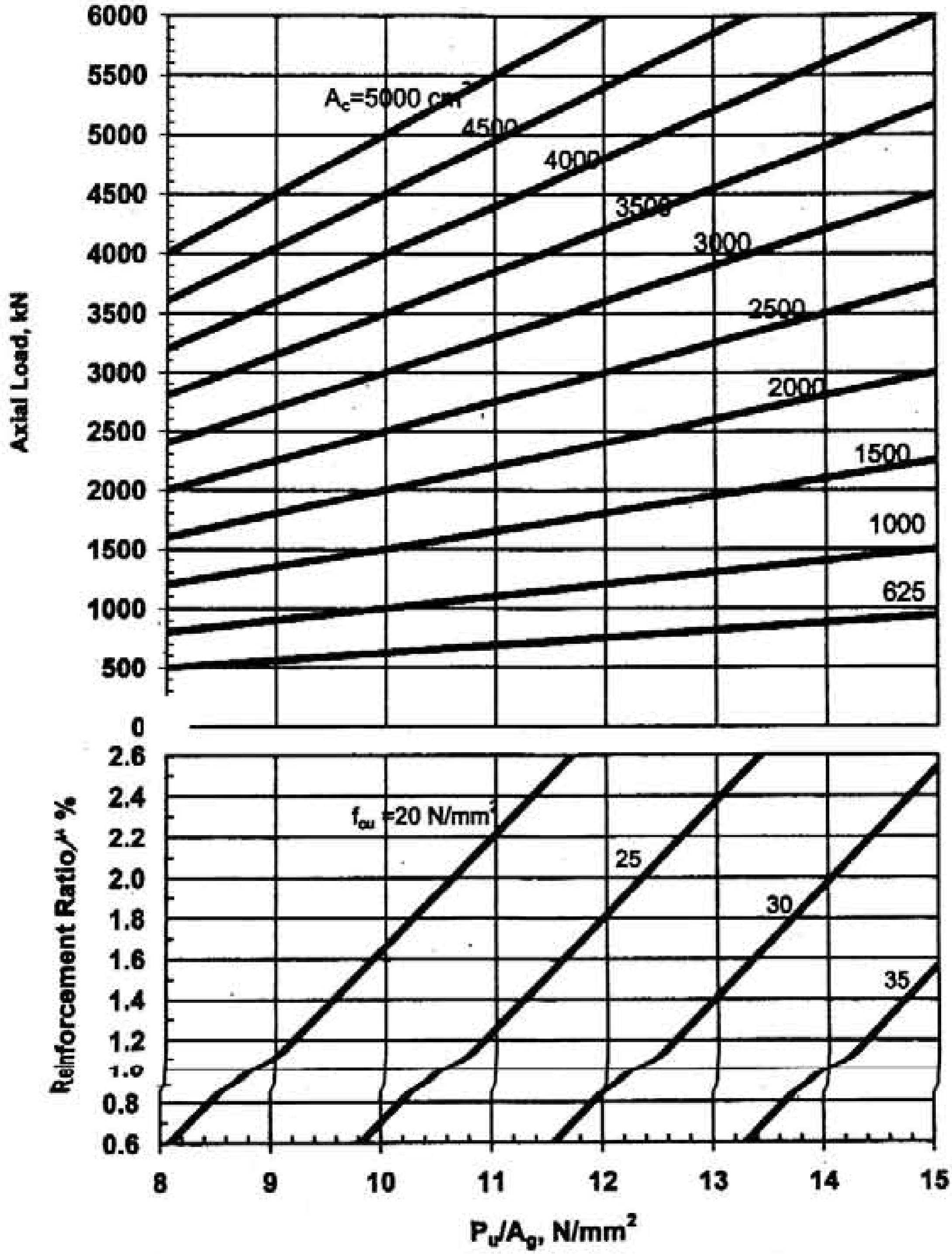
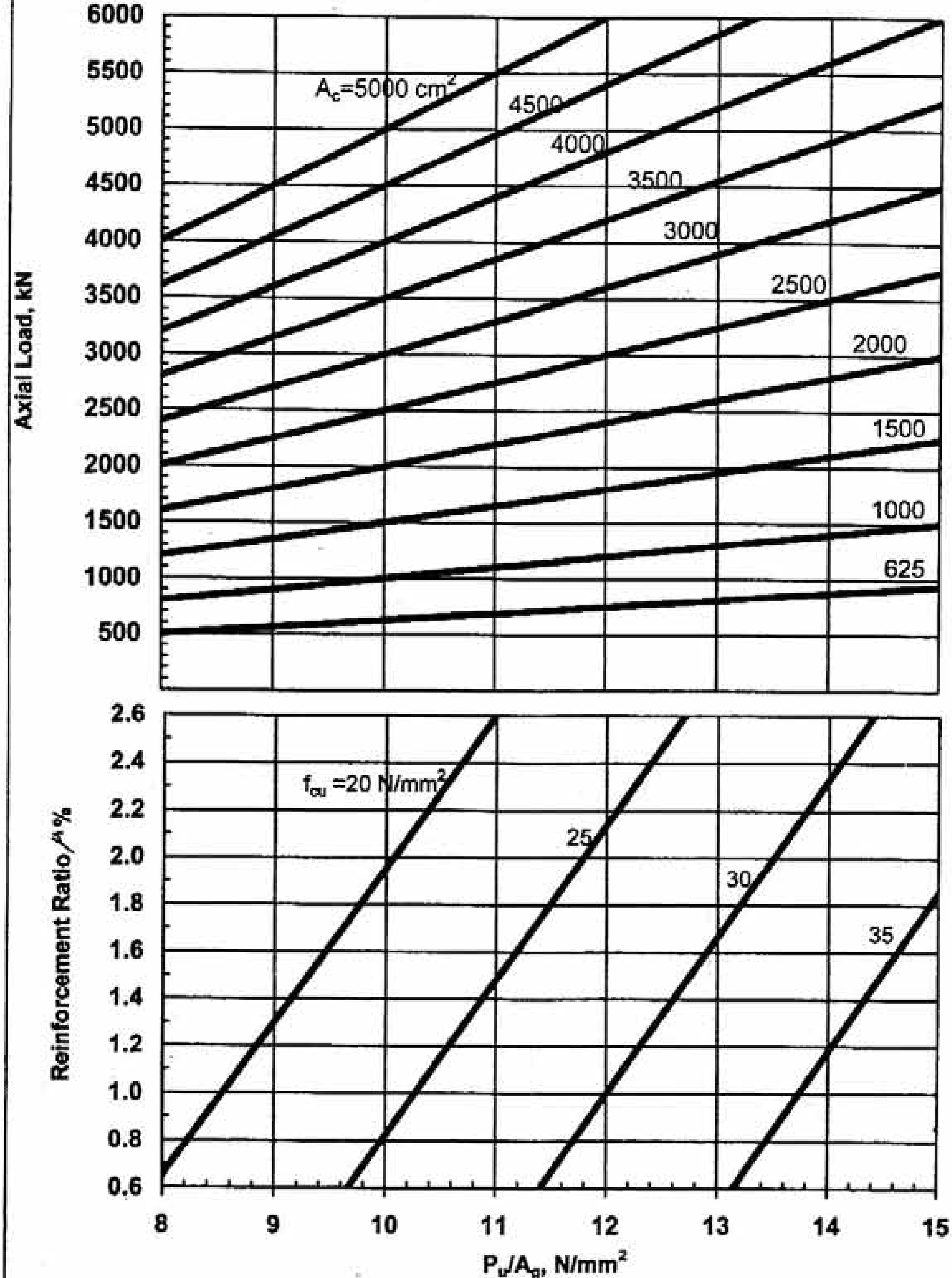


CHART (3-4):DESIGN OF AXIALLY LOADED TIED COLUMNS f_y=240 N/mm^2



4. MEMBERS SUBJECTED TO ECCENTRIC FORCES

4.1 Introduction

Columns in reinforced concrete frames are constructed to be a part of a rigid frame unless specific mechanisms in precast systems are designed to prohibit moment transfer between beams and columns at joints. Columns must be proportioned with a capacity to resist bending moment as well as axial load. Column cross sections can resist more axial load in the absence of moment than in conjunction with moment. However, ECCS 203-2001 requires that all compression members be designed for an eccentricity of at least e = 5% of the depth t or 20 mm as a minimum eccentricity. The capacity of column cross sections is described with interaction diagrams. Combinations of axial load and moment within the graphs represent safe values for the column cross section.

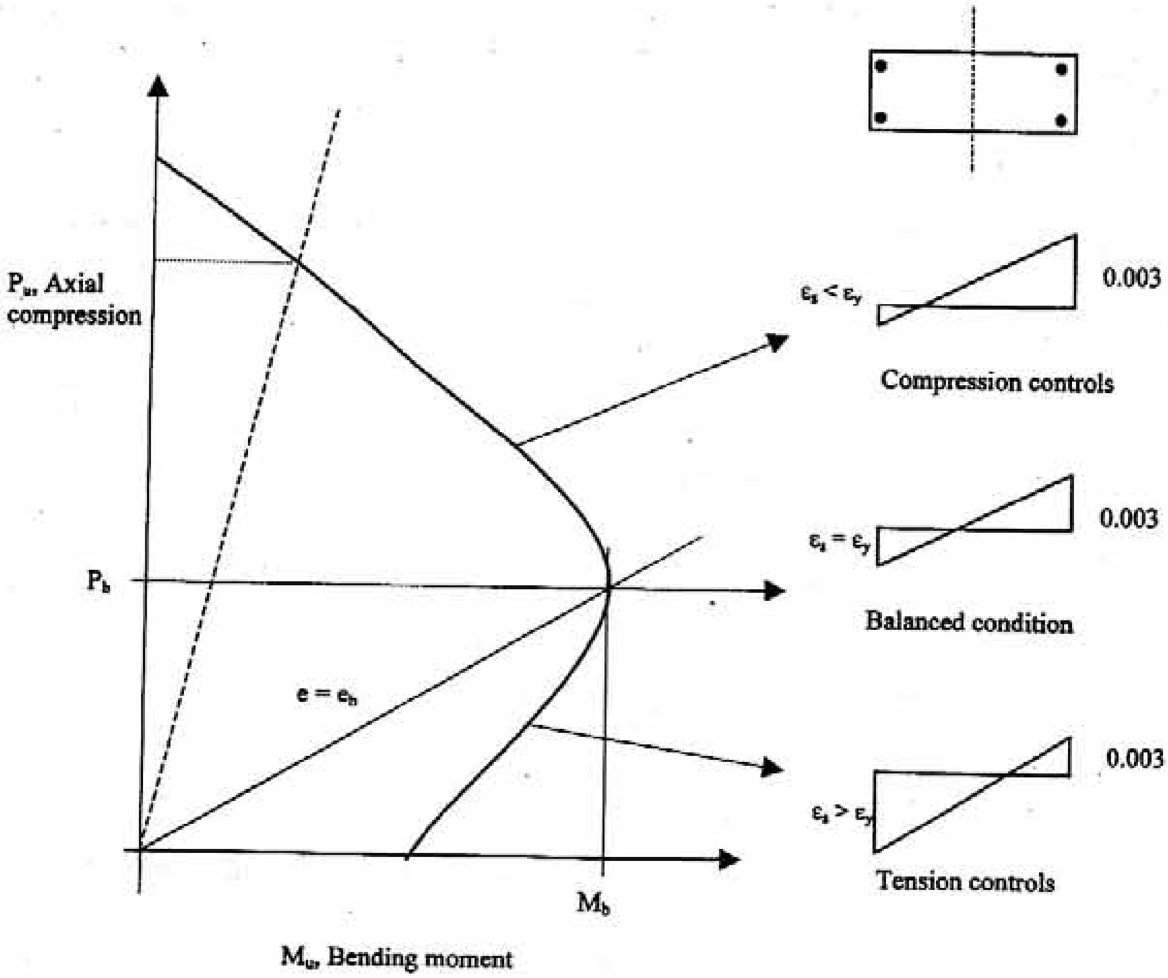
4.2 Interaction Diagram

When combined axial compression and bending act on a section having a low slenderness ratio, from the viewpoint of maximum strength of a section there may be:

(1) Compression over most or all of the section such that the compressive strain in the concrete reaches 0.003 before the tension steel yields, known as the "compression controls" region.

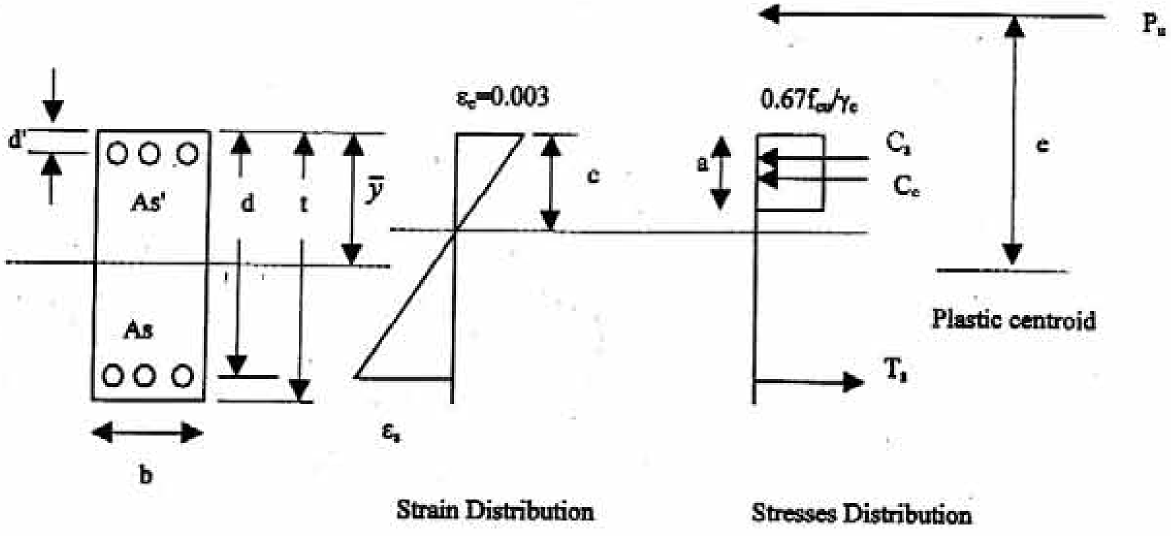
(2) Tension in a large portion of the section such that the strain in the tension reinforcement is greater than the yield point strain when the compressive strain in the concrete reaches 0.003, known as the "tension controls" region.

(3) The balanced condition is reached when the tension steel reaches its yield strain at precisely the same load level as the concrete reaches its ultimate strain, 0.003, and starts crushing.



Typical ultimate strength interaction diagram for axial compression and bending moment about one axis.

4.3 Behavior of Eccentrically Loaded Short Columns



The same principles concerning the stress distribution and the equivalent rectangular stress block applied to beams are equally applicable to columns. The figure shows a typical rectangular column cross section with strain and stress distribution diagrams. The equilibrium expressions for forces and moments can be expressed as follows:

$$P_u = 0.67\frac{f_{cu}}{\gamma_c}ba + A_s' f_s' - A_s f_s - 0.67\frac{f_{cu}}{\gamma_c}A_s' \quad \text{(a)}$$

$$M_u = P_u e = 0.67\frac{f_{cu}}{\gamma_c}ba(\bar{y}-\frac{a}{2}) + A_s' f_s'(\bar{y}-d') + A_s f_s(d-\bar{y}) - 0.67\frac{f_{cu}}{\gamma_c}A_s'(\bar{y}-d') \quad \text{(b)}$$

Where

$$f_s' = 0.003E_s\frac{c-d'}{c} \le \frac{f_y}{\gamma_s}$$

$\bar{y}$ is the distance from the extreme compression fibers to the plastic or geometric centroid.

And $\gamma_c = 1.5\left[\frac{7}{6} - \frac{e/t}{3}\right] \geq 1.5$, $\gamma_s = 1.15\left[\frac{7}{6} - \frac{e/t}{3}\right] \geq 1.15$

4.3.1 Balanced failure in rectangular column sections

From similar triangles, an expression for the depth of neutral axis c_b at balanced condition can be written as:

$$\frac{c_b}{d} = \frac{0.003}{0.003 + \frac{f_y}{E_s\,\gamma_s}}$$

$$a_b = 0.8c_b$$

The axial load corresponding to balanced condition P_{ub} and the corresponding eccentricity e_b can be determined by using equations (a) and (b) ($f_s = f_y/\gamma_s$).

$$P_{ub} = 0.67\frac{f_{cu}}{\gamma_c}ba_b + A_s^{'}f_s^{'} - A_s\frac{f_y}{\gamma_s} - 0.67\frac{f_{cu}}{\gamma_c}A_s^{'}$$

$$M_{ub} = P_{ub}e_b = 0.67\frac{f_{cu}}{\gamma_c}ba_b(\bar{y} - \frac{a}{2}) + A_s^{'}f_s^{'}(\bar{y} - d^{'}) + A_s\frac{f_y}{\gamma_s}(d - \bar{y}) - 0.67\frac{f_{cu}}{\gamma_c}A_s^{'}(\bar{y} - d')$$

4.3.2 Tension failure in rectangular sections

The initial limit state of failure in cases of large eccentricity occurs by yielding of reinforcement at the tension side. The transition from compression failure to tension failure takes place at $e=e_b$. If e is larger than e_b and $P_u < P_{ub}$, the failure will be in tension through initial yielding of the tensile reinforcement. The previous equations (a and b) are applicable in the analysis by substituting the yield strength f_y/γ_s for the stress f_s in the tension reinforcement.

$$P_u = 0.67\frac{f_{cu}}{\gamma_c}ba + A_s' f_s' - A_s\frac{f_y}{\gamma_s} - 0.67\frac{f_{cu}}{\gamma_c}A_s'$$

$$M_u = P_u e = 0.67\frac{f_{cu}}{\gamma_c}ba(\bar{y}-\frac{a}{2}) + A_s' f_s'(\bar{y}-d') + A_s\frac{f_y}{\gamma_s}(d-\bar{y}) - 0.67\frac{f_{cu}}{\gamma_c}A_s'(\bar{y}-d')$$

4.3.3 Compression failure in rectangular sections

For initial crushing of the concrete, the eccentricity e has to be less than the balanced eccentricity e_b or $P_u > P_{ub}$. The tensile reinforcement stress f_s is less than f_y/γ_s.

The previous equations (a and b) are applicable in the analysis by substituting the yield strength f_y/γ_s for the stress f_s' in the compression reinforcement The analysis process necessitates applying the basic equilibrium equations, using trial and adjustment procedure and ensuring strain compatibility checks at all stages.

$$P_u = 0.67\frac{f_{cu}}{\gamma_c}ba + A_s'\frac{f_y}{\gamma_s} - A_s f_s - 0.67\frac{f_{cu}}{\gamma_c}A_s'$$

$$M_u = P_u e = 0.67\frac{f_{cu}}{\gamma_c}ba(\bar{y}-\frac{a}{2}) + A_s'\frac{f_y}{\gamma_s}(\bar{y}-d') + A_s f_s(d-\bar{y}) - 0.67\frac{f_{cu}}{\gamma_c}A_s(\bar{y}-d')$$

Where $\quad f_s = 0.003E_s\frac{d-c}{c} \le \frac{f_y}{\gamma_s}$

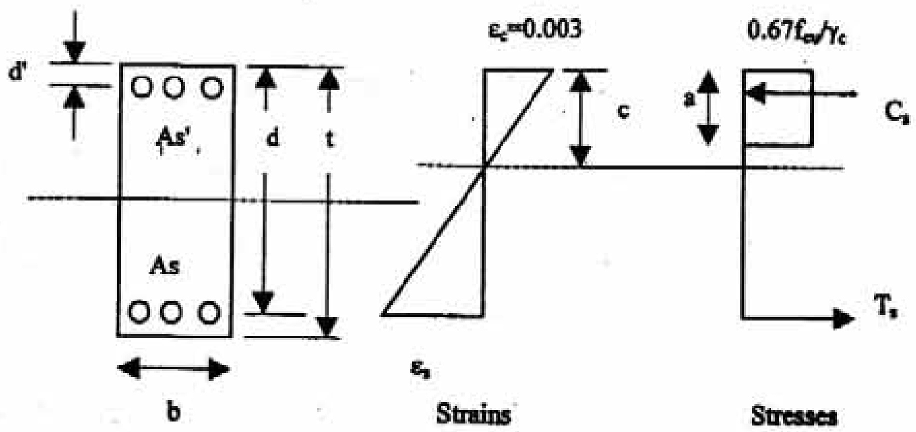
4.4 Solved Examples

Example 1: Design of a section subjected to eccentric compressive force using interaction diagrams (Case of compression failure)

Given: $P_u = 1400$ kN; $M_u = 240$ kN.m

$t = 750$ mm; $b = 250$ mm

$f_{cu} = 25$ N/mm²; $f_y = 240$ N/mm²



Design:

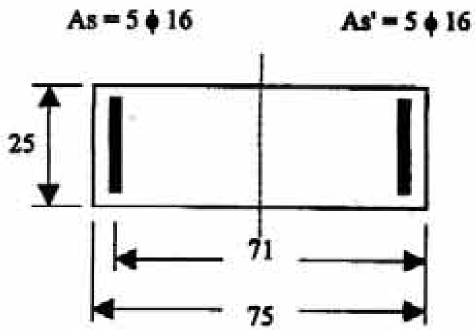
$$\frac{P_u}{bt} = \frac{1400*10^3}{250*750} = 7.5 \quad N/mm^2$$

$$\frac{M_u}{bt^2} = \frac{240*10^6}{250*750^2} = 1.7 \quad N/mm^2$$

From the interaction diagram with $f_{cu} = 25$ N/mm², $f_y = 240$ N/mm², ζ=0.9 and α=1

$\mu = 0.5\%$

$As = As' = 0.005*25*75 = 9.375$ cm²
(5 ϕ 16)



Example 2: Using the strain compatibility approach, verify the adequacy of the design obtained in example 1

$e = M_u/P_u = 240/1400 * 100 = 17.14$ cm

$e/t = 17.14/75 = 0.228$

Strength reduction factors:

$$\gamma_c = 1.5\left[\frac{7}{6}-\frac{e/t}{3}\right] = 1.636$$

$$\gamma_s = 1.15\left[\frac{7}{6}-\frac{e/t}{3}\right] = 1.26$$

Equations of equilibrium:

$$P_u = 0.67\frac{f_{cu}}{\gamma_c}ba + A_s'\frac{f_y}{\gamma_s} - A_s f_s - 0.67\frac{f_{cu}}{\gamma_c}A_s'$$

Where, $f_s = 0.003E_s\dfrac{d-c}{c}$ *and* $a = 0.8c$

$1400*10^3 = 0.67*25*250*0.8*c/1.636 + 937.5*240/1.262 - 937.5*600*(710-c)/c$

$-0.67*25*937.5/1.636$

c = 62.85 cm, a = 50.28 cm

Take moment at section plastic centroid

$$M_u = 0.67\frac{f_{cu}}{\gamma_c}ba(\frac{t}{2}-\frac{a}{2}) + A_s'\frac{f_y}{\gamma_s}(\frac{t}{2}-d') + A_s f_s(d-\frac{t}{2}) - 0.67\frac{f_{cu}}{\gamma_c}A_s'(t/2-d')$$

$= 0.67*25*250*0.8*628.5/1.636*(375-251.4) + 937.5*240/1.262*(375-40)$

$+ 937.5*600*(710-628.5)/628.5*(710-375) - 0.67*25*937.5/1.636*(375-40)$

$M_u = 240$ kN.m

The section is adequate and the accuracy of the interaction diagram has been verified.

$$Note: \varepsilon_s = 0.003 \frac{710 - 628.5}{628.5} = 0.00039 < \varepsilon_y / \gamma_s$$ (tension reinforcement did not yield)

Example 3: Design of a section subjected to eccentric compressive force using interaction diagrams (Case of tension failure)

Given: $P_u = 600$ kN; $M_u = 320$ kN.m

$t = 750$ mm; $b = 250$ mm

$f_{cu} = 25$ N/mm^2; $f_y = 240$ N/mm^2

Design:

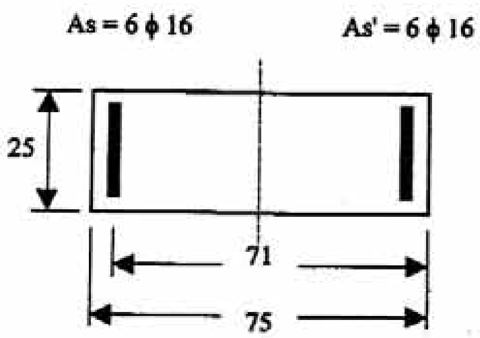
$$\frac{P_u}{bt} = \frac{600*10^3}{250*750} = \quad 3.2 \quad N/mm^2$$

$$\frac{M_u}{bt^2} = \frac{320*10^6}{250*750^2} = \quad 2.3 \quad N/mm^2$$

From the interaction diagram with $f_{cu} = 25$ N/mm^2, $f_y = 240$ N/mm^2, ζ=0.9 and α=1

$\mu = 0.64\%$

$As = As' = 0.0064*25*75 = 12$ cm^2 (6 ϕ 16)



Example 4: Using the strain compatibility approach, verify the adequacy of the design obtained in example 3

e = M_u/P_u = 320/600* 100 = 53.34 cm

e/t = 54.34/75 = 0.7112

Calculating the safety reduction factors:

$$\gamma_c = 1.5\left[\frac{7}{6} - \frac{e/t}{3}\right] = 1.39 < 1.5 \qquad \gamma_c = 1.5$$

$$\gamma_s = 1.15\left[\frac{7}{6} - \frac{e/t}{3}\right] = 1.07 < 1.15 \qquad \gamma_s = 1.15$$

Equations of equilibrium:

$$P_u = 0.67\frac{f_{cu}}{\gamma_c}ba + A_s'\frac{f_y}{\gamma_s} - A_s\frac{f_y}{\gamma_s} - 0.67\frac{f_{cu}}{\gamma_c}A_s'$$

$600*10^3$ = 0.67*25*250*a/1.5 + 1200*240/1.15 -1200*240/1.15-0.67*25*1200/1.5

a = 21.0 cm , c=26.25 cm

$$M_u = 0.67\frac{f_{cu}}{\gamma_c}ba(\frac{t}{2} - \frac{a}{2}) + A_s'\frac{f_y}{\gamma_s}(\frac{t}{2} - d') + A_s\frac{f_y}{\gamma_s}(d - \frac{t}{2}) - 0.67\frac{f_{cu}}{\gamma_c}A_s'(t/2 - d')$$

= 0.67*25*250*210/1.5*(375-210/2) + 1200*240/1.15*(375-40)

+ 1200*240/1.15*(710-375) - 0.67*25*1200/1.5*(375-40)

M_u = 320 kN.m

The section is adequate and the accuracy of the interaction diagram has been verified.

$$Note: \varepsilon_s = 0.003\frac{710 - 262.5}{262.5} = 0.0051 > \varepsilon_y/\gamma_s$$ (tension reinforcement is yielded)

Example 5: Design of unsymmetrical rectangular section subjected to eccentric compressive force using design aids for flexure (M_{su} approach) for tension failure cases

Given: $P_u = 600$ kN; $M_u = 320$ kN.m

$t = 750$ mm; $b = 250$ mm

$f_{cu} = 25$ N/mm^2; $f_y = 240$ N/mm^2

$As' = 6$ cm^2 (3 ϕ 16)

Design:

$e = M_u/P_u = 320 * 100 / 600 = 53.33$ cm

$e_s = e + t/2 -$ cover $= 53.33 + 37.5 - 4 = 86.83$ cm

$M_{us} = P_u\, e_s = 600 * 0.868 = 520$ kN.m

Since,

$$M_{us} = \overline{A}_{s1}\, f_y(d - a/2) + A_s' f_y(d - d')/\gamma_s$$

Where

$$\overline{A}_{s1} = A_{s1} \pm \frac{P_u}{f_y/\gamma_s} \qquad \& \qquad A_{s1} = A_s - A_s'$$

and take the +ve sign for compressive loads and -ve sign for tensile loads

therefore,

$$\frac{M_{us}}{bd^2} = \frac{520*10^6}{250*710^2} = 4.13$$

$$\mu_s' f_y(1 - d'/d)/\gamma_s = \frac{3*2*100}{250*710}*240*(1-40/710)/1.15 = 0.666$$

$\overline{M}_{us}/(bd^2) = \overline{\mu}_{s1}' f_y(1 - a/2d)/\gamma_s = 4.13 - 0.666 = 3.464 \quad N/mm^2$

$\overline{\mu}_{s1} = 0.0205$

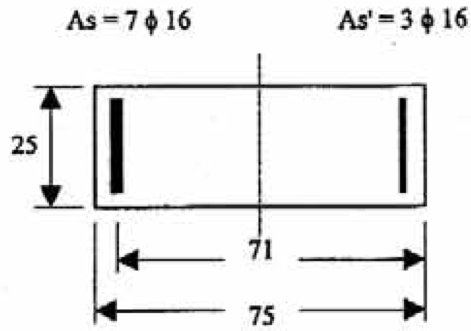
$\overline{A}_{s1} = \overline{\mu}_{s1} bd = 0.0205 * 25 * 71 = 36.4 \ cm^2$

$A_{s1} = \overline{A}_{s1} - P_u\gamma_s / f_y = 36.4 - 600 * 10^3 * 1.15/240/100 = 7.65 \qquad cm^2$

Total area of tensile reinforcement, A_s

$A_s = A_{s1} + A_s' = 7.65 + 6 = 13.65 \ cm^2$ (7 ϕ 16)

With A_s' = (3 ϕ 16)



Example 6: Using the strain compatibility approach, verify the adequacy of the design obtained in example 5

$e = M_u/P_u = 320/600 * 100 = 53.34$ cm

$e/t = 54.34/75 = 0.7112$

Strain reduction factors:

$$\gamma_c = 1.5\left[\frac{7}{6} - \frac{e/t}{3}\right] = 1.39 < 1.5 \qquad \gamma_c = 1.5$$

$\gamma_s = 1.15\left[\frac{7}{6} - \frac{e/t}{3}\right] = 1.07 < 1.15 \qquad \gamma_s = 1.15$

Equations of equilibrium:

$$P_u = 0.67\frac{f_{cu}}{\gamma_c}ba + A_s^{'}\frac{f_y}{\gamma_s} - A_s\frac{f_y}{\gamma_s} - 0.67\frac{f_{cu}}{\gamma_c}A_s^{'}$$

$600*10^3$ = 0.67*25*250*a/1.5 + 600*240/1.15 -1400*240/1.15-0.67*25*600/1.5

a = 27.7 cm

$$M_u = 0.67\frac{f_{cu}}{\gamma_c}ba(\frac{t}{2} - \frac{a}{2}) + A_s^{'}\frac{f_y}{\gamma_s}(\frac{t}{2} - d^{'}) + A_s\frac{f_y}{\gamma_s}(d - \frac{t}{2}) - 0.67\frac{f_{cu}}{\gamma_c}A_s^{'}(t/2 - d')$$

= 0.67*25*250*a/1.5*(375-a/2) + 600*240/1.15*(375-40)

+ 1400*240/1.15*(710-375) - 0.67*25*600/1.5*(375-40)

M_u = 320 kN.m

The section is adequate. Also, the accuracy of the interaction diagram has been verified.

Example 7: Design of unsymmetrical rectangular sections subjected to eccentric tensile force using design aids for flexure (M_{su} approach)

Given: P_u = 220 kN (tension); M_u = 216 kN.m

t = 750 mm; b = 250 mm

f_{cu} = 25 N/mm^2; f_y = 240 N/mm^2

As' = 0

Design:

e = M_u/P_u = 216 * 100 / 220 = 98.18 cm

$e_s = e - t/2 + \text{cover} = 98.18 - 37.5 + 4 = 64.68$ cm

$M_{us} = P_u\, e_s = 220 * 0.6468 = 142.3$ kN.m

Since,

$$M_{us} = \overline{A}_{s1} f_y (d - a/2)/\gamma_s + A_s' f_y (d - d')/\gamma_s$$

Where $\overline{A}_{s1} = A_{s1} \pm P_u \gamma_s / f_y$

$$\frac{M_{us}}{bd^2} = \frac{142.2*10^6}{250*710^2} = 1.128$$

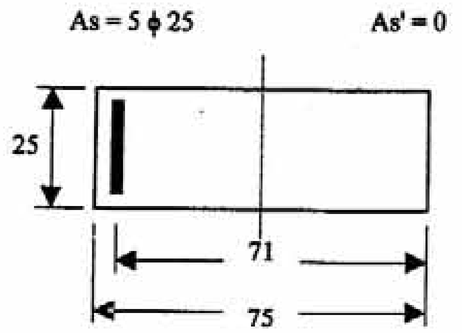
$$M_{us}/(bd^2) = \overline{\mu}_s f_y (1 - a/2d)/\gamma_s = 1.128$$

$$\overline{\mu}_s = 0.0056$$

$$\overline{A}_s = \overline{\mu}_s bd = 0.0057*25*71 = 10.12 \quad cm^2$$

$$A_s = \overline{A}_s + P_u \gamma_s / f_y = 10.12 + 220*10^3*1.15/240/100 = 20.66\, cm^2$$

Use A_s (5 ϕ 25)



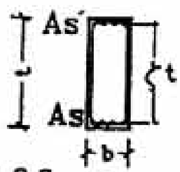
DESIGN AIDS

INTERACTION DIAGRAMS FOR ECCENTRIC FORCES

RECTANGULAR SECTION

i–Top & Bott. R.F.T Only

$\alpha = \frac{A_s'}{A_s}$

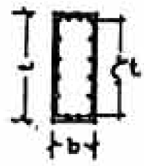


$f_y =$ 240, 280, 360, 400 — α = 1.00, 0.80 — ζ = 0.9, 0.8, 0.7

ii–R.F.T Along The Perimeter

$f_y =$ 240, 280, 360, 400 — ζ = 0.9, 0.8

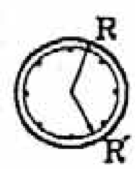
As



CIRCULAR SECTION

$f_y =$ 240, 280, 360, 400 — ζ = 0.9, 0.8, 0.7

$\zeta = \frac{R}{R'}$



How To Use Interaction Diagrams

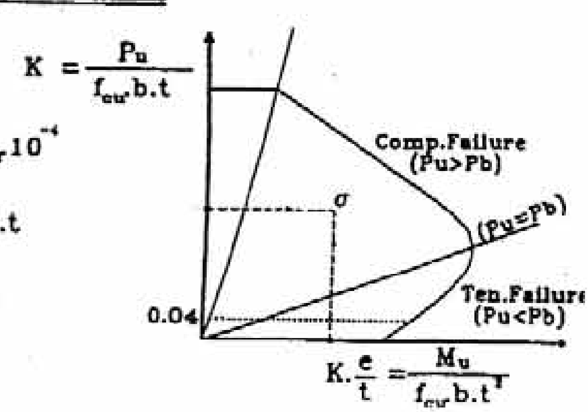
$K = \frac{P_u}{f_{cu}\,b.t}$ (unitless)

$K.\frac{e}{t} = \frac{M_u}{f_{cu}\,b.t^2}$ (unitless)

$\mu = \sigma\, f_{cu}\, 10^{-4}$

$A_s = \mu.b.t$

$A_s' = \alpha\, A_s$



5. INTERACTION DIAGRAM

4.5 Design Of Section Subjected To Eccentric Compression Force:

a) Balanced conditions : Is the limit between compression and tension failure.

Balanced eccentric force $P_b = 0.67\frac{f_{cu}}{\gamma_c} a_b\, b - \frac{A_s f_y}{\gamma_s} + \frac{A`s f_y}{\gamma_s}$

$$P_b = 0.67\frac{f_{cu}}{\gamma_c} 0.8\left(\frac{600}{600+\frac{f_y}{\gamma_s}}\right) b\,d - \frac{A_s f_y}{\gamma_s} + \frac{A`s f_y}{\gamma_s}$$

$$P_b = 214.4\left(\frac{f_{cu}}{600+\frac{f_y}{\gamma_s}}\right) b\,d - \frac{A_s f_y}{\gamma_s} + \frac{A`s f_y}{\gamma_s}$$

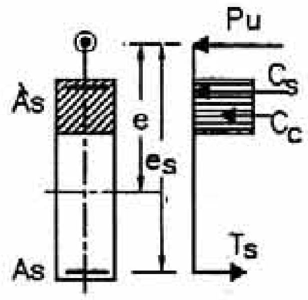
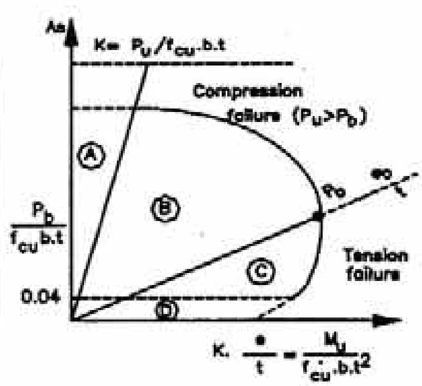


Table of K_b (balanced) $= \dfrac{P_b}{f_{cu}\, b\, t}$

(case of $A`_s = A_s$)

type of steel	240/350	280/450	360/520	400/600
ζ=0.7	0.228	0.219	0.200	0.191
ζ=0.8	0.241	0.232	0.212	0.203
ζ=0.9	0.255	0.244	0.224	0.214



Use of Interaction Diagrams

(Design charts for sections subjected to eccentric compression force P_u)

For different values of f_y, $\mu = A_s / b\,t$, $\alpha = A`_s / A_s$, $\zeta = \dfrac{d-d`}{t}$

Procedure of design of section:

i- For a given sec. calculate $k = \dfrac{P_u}{f_{cu}\, b\, t}$

ii- Calculate $k\dfrac{e}{t} = \dfrac{M_u}{f_{cu}\, b\, t^2}$

iii- Locate the point on the interaction diagram.

If the point is at the zone A (Eccentric force with min eccentricity $e \le 0.05\,t$) : Design as short column.

iv- For column with separate stirrups : $P_u = 0.35\, f_{cu}\, A_c + 0.67\, A_s\, f_y$

If the point is at zone B (case of compression failure)

iv- Design using the interaction diagram and get ρ :

then $\mu = \rho\, f_{cu} \times 10^{-4}$

$A_s = \mu\, b\, t$, $A'_s = \alpha\, A_s$

If the point is at the zone C (case of tension failure)

Section can be designed as if subjected to simple bending using design charts (2-1, 2-2, 2-3, 2-4)for cases where the contribution of the compression reinforcement can be neglected as follows:

iv- Calculate $e = M_u / P_u$, $e_s = e + t/2$ - cover , $M_{us} = P_u\, e_s$

v- Use chart 2-3 , Calculate C_1 from $d = C_1 \sqrt{M_{us} / f_{cu}\, b}$

vi- From chart get j , thus $A_s = \dfrac{M_{us}}{j\, d\, f_y} - \dfrac{P_u}{f_y / \gamma_s}$ where $\gamma_s = 1.15$

OR:

v- Use curves 2-1 or 2-2 , select suitable curve for (f_y , f_{cu})

vi- Calculate $R_u = M_u/bd^2$

vii- From chart get μ , thus $A_s = \mu\, b\, d - \dfrac{P_u}{f_y / \gamma_s}$ where $\gamma_s = 1.15$

If the point is at the zone D $\left(\dfrac{P_u}{f_{cu}\, b\, t} \le 0.04 \right)$

Case of small eccentric force P_u which can be neglected and section is to be designed for simple bending only .

The interaction diagrams are given for values of :

$f_y = 240$, 280 , 360 , 400
$\alpha = A'_s / A_s = 1$, 0.8
$\zeta = \dfrac{d - d'}{t} = 0.9$, 0.8, 0.7

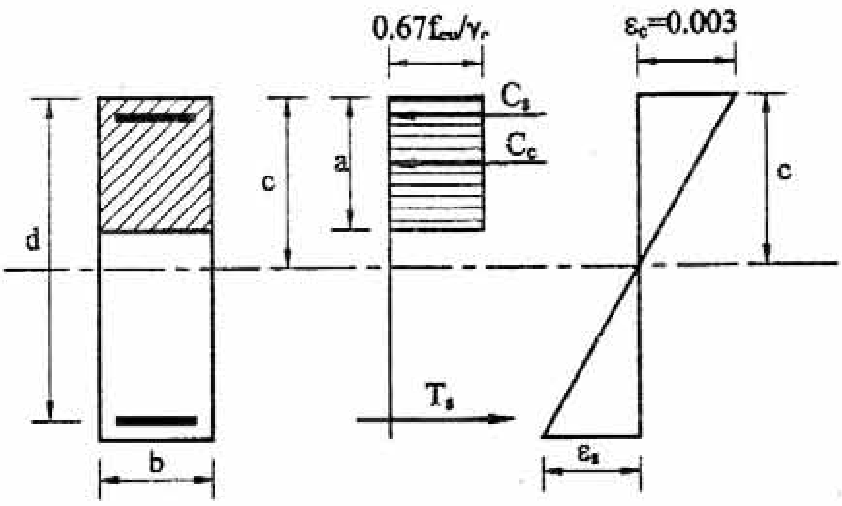
Note : f_{cu} is not a parameter and is included in the charts.

4.6 Solved Examples for Sections Subjected to Eccentric Force

Example 1:

Design a section subjected to eccentric compressive force using interaction diagrams.

Given: $P_u = 1400$ kN, $t = 750$ mm, $f_{cu} = 25$ N/mm^2, $M_u = 240$ kN.m, $b = 250$ mm, $f_y = 240$ N/mm^2



Design:

$$\frac{P_u}{f_{cu}.b.t} = \frac{1400\times10^3}{25\times250\times750} = 0.30$$

$$\frac{M_u}{f_{cu}.b.t^2} = \frac{240\times10^6}{25\times250\times750^2} = 0.068$$

From the interaction diagram for $f_y = 240$ N/mm^2, ξ=0.9, α=1.0

get $\rho = 2$

$$e = \frac{Mu}{Pu} = \frac{240}{1400} = 0.17m = 170mm \quad \Rightarrow\Rightarrow \quad \frac{e}{t} = \frac{170}{750} = 0.227 \; (comp.\ failure)$$

$\mu = \rho \cdot f_{cu} \cdot 10^{-4} = 2 \times 25 \times 10^{-4} = 0.005$

$A_s = \mu \cdot b \cdot t = 0.005 \times 250 \times 750 = 937.5$ mm^2

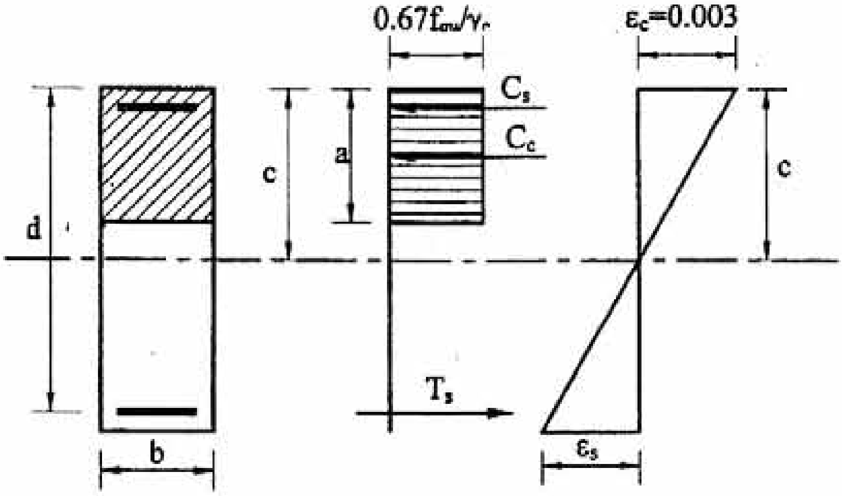
$A_s' = \alpha \cdot A_s = 1.0 \times 937.5 = 937.5$ mm^2

Take $A_s = 5 \varnothing 16$ & $A_s' = 5 \varnothing 16$

Example 2:

Design of a section subjected to eccentric compressive force using interaction diagrams.

Given: $P_u = 600$ kN, $t = 750$ mm, $f_{cu} = 25$ N/mm^2, $M_u = 320$ kN.m, $b = 250$ mm, $f_y = 240$ N/mm^2



Design:

$$\frac{P_u}{f_{cu}.b.t} = \frac{600\times10^3}{25\times250\times750} = 0.128$$

$$\frac{M_u}{f_{cu}.b.t^2} = \frac{320\times10^6}{25\times250\times750^2} = 0.091$$

From the interaction diagram for $f_y = 240$ N/mm^2, $\xi=0.9$, $\alpha=1.0$
get $\rho = 2.5$

$$e = \frac{Mu}{Pu} = \frac{320}{600} = 0.533\text{m} = 533\text{mm} \quad \Rightarrow\Rightarrow \quad \frac{e}{t} = \frac{533}{750} = 0.71 \text{ (tension failure)}$$

$\mu = \rho \cdot f_{cu} \cdot 10^{-4} = 2.5 \times 25 \times 10^{-4} = 0.00625$
$A_s = \mu \cdot b \cdot t = 0.00625 \times 250 \times 750 = 1172$ mm^2
$A_s` = \alpha \cdot A_s = 1.0 \times 1172 = 1172$ mm^2
Take $A_s = 6 \varnothing 16$ & $A_s` = 6 \varnothing 16$

Example 3:

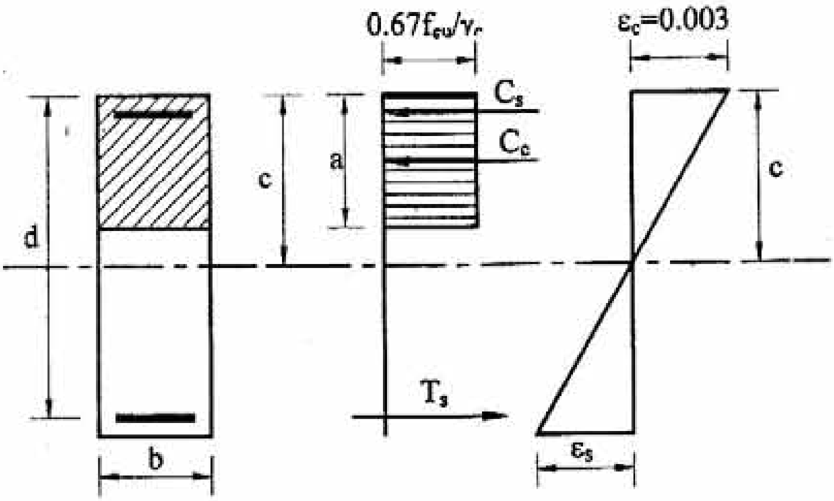
Design of unsymmetrical rectangular section subjected to eccentric compressive force using interaction diagrams.

Given: $P_u = 600$ kN, $M_u = 320$ kN.m

$t = 750$ mm, $b = 250$ mm

$f_{cu} = 25$ N/mm^2, $f_y = 240$ N/mm^2

$\alpha = 0.80$



Design:

$$\frac{P_u}{f_{cu}.b.t} = \frac{600\times10^3}{25\times250\times750} = 0.128$$

$$\frac{M_u}{f_{cu}.b.t^2} = \frac{230\times10^6}{25\times250\times750^2} = 0.091$$

From the interaction diagram for $f_y = 240$ N/mm^2, $\xi=0.9$, $\alpha=0.80$

get $\rho = 2.6$

$$e = \frac{Mu}{Pu} = \frac{320}{600} = 0.533\text{m} = 533\text{mm} \quad \Rightarrow\Rightarrow \quad \frac{e}{t} = \frac{533}{750} = 0.71 \text{ (tension failure)}$$

$\mu = \rho \, . \, f_{cu} \, . \, 10^{-4} = 2.6 \times 25 \times 10^{-4} = 0.0065$

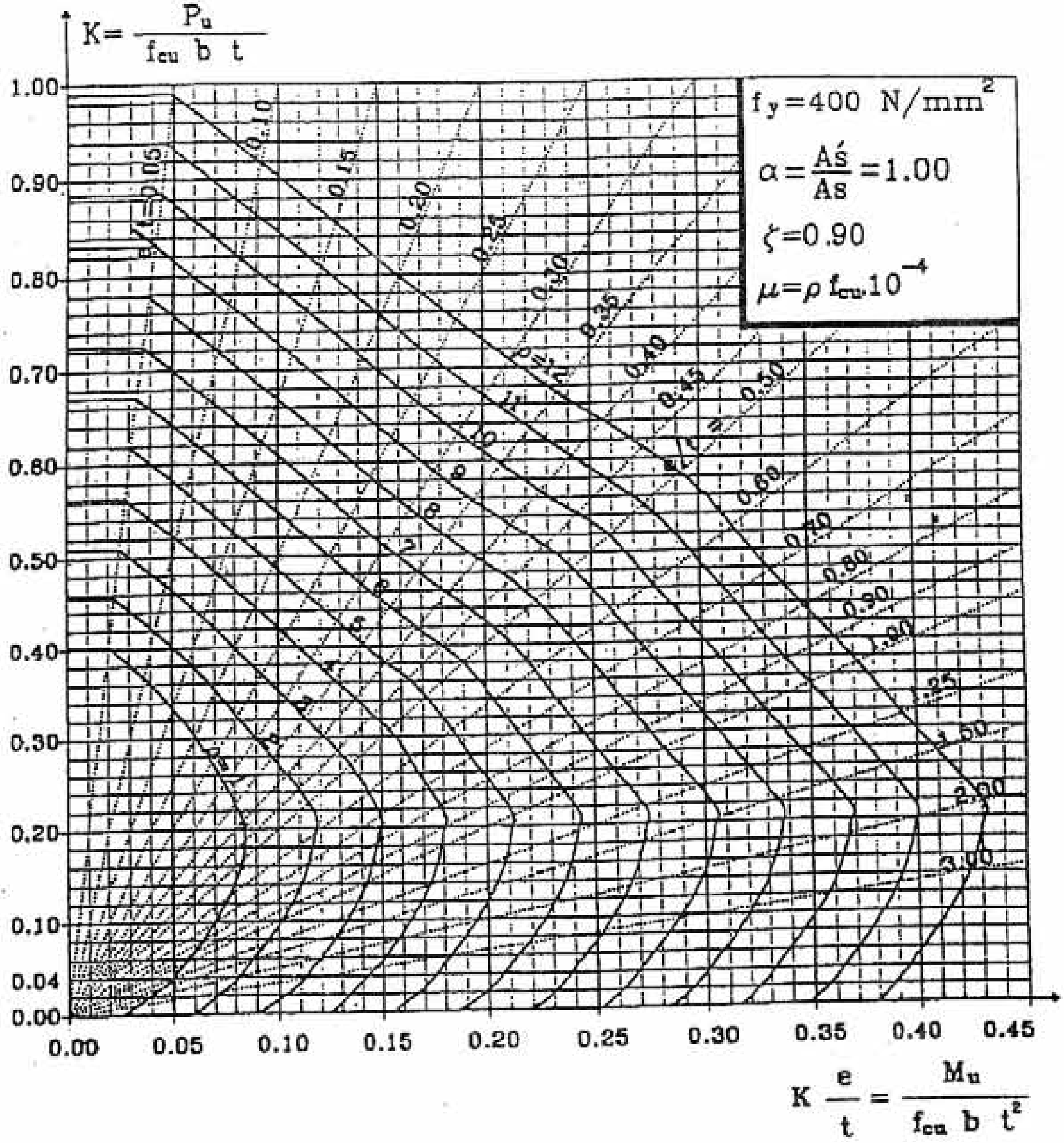
$A_s = \mu \, . \, b \, . \, t = 0.0065 \times 250 \times 750 = 1219$ mm^2

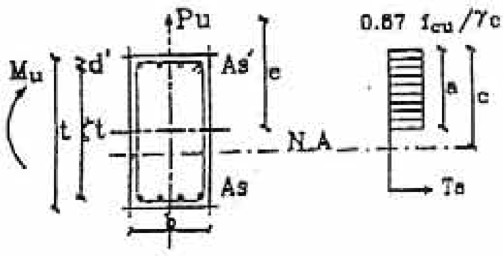
$A_s' = \alpha \, . \, A_s = 0.8 \times 1219 = 975$ mm^2

Take $A_s = 4 \varnothing 16 + 3 \varnothing 14$ & $A_s' = 5 \varnothing 16$

Chart (4-1) : INTERACTION DIAGRAMS

FOR DESIGN OF SECTIONS SUBJECTED TO ECCENTRIC COMP. FORCES





$\mu = \rho\ f_{cu}.10^{-4}$

$As = \mu\ b\ t$

$A\acute{s} = \alpha\ As$

$\zeta = \frac{d-d'}{t}$

R–Sec.

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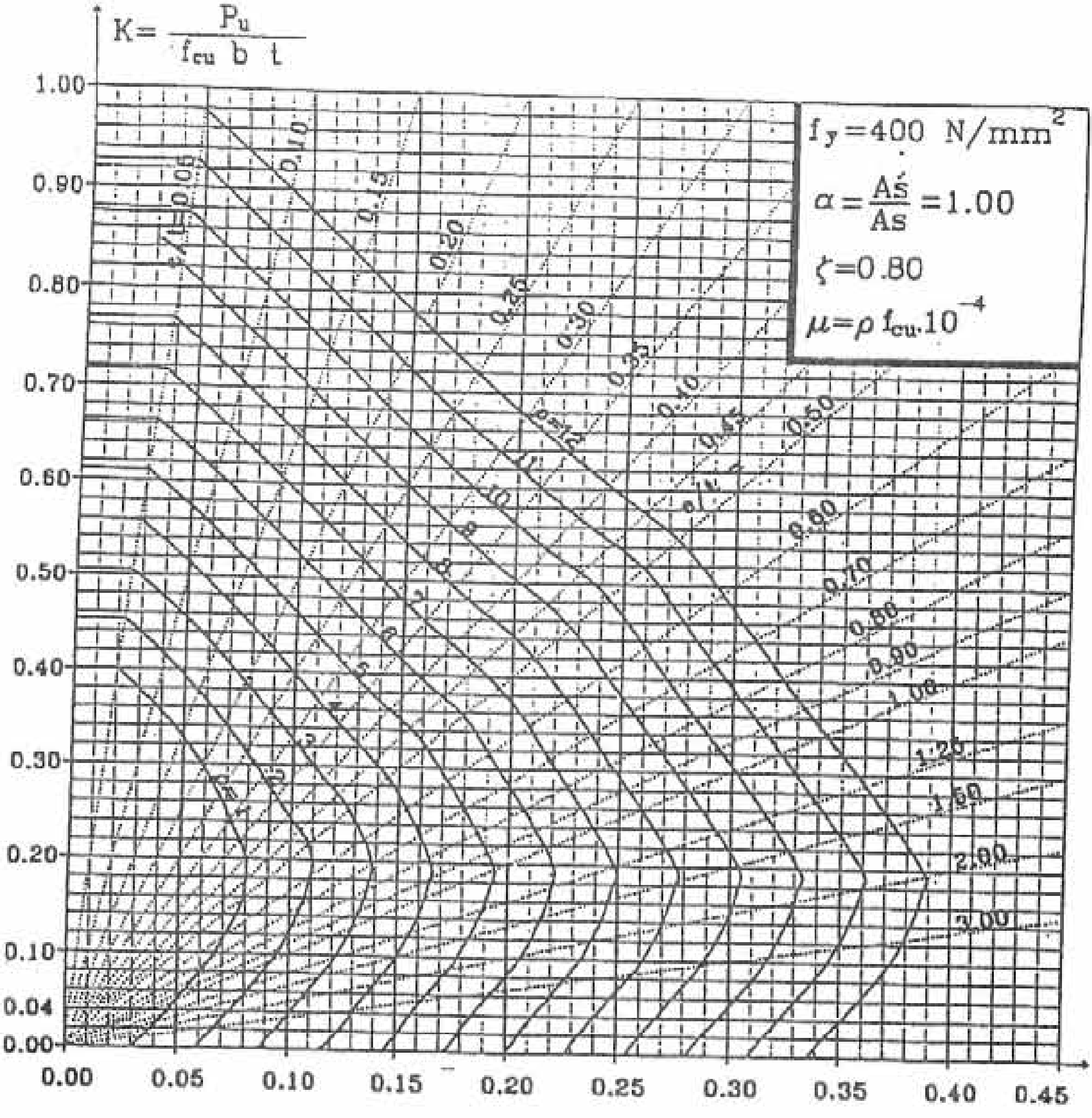
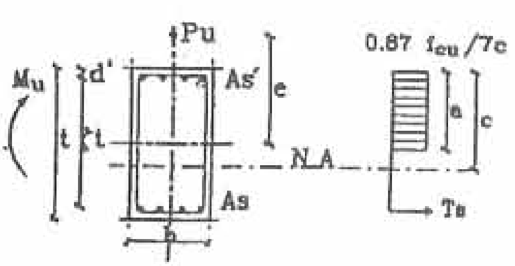
Chart (4-2) : INTERACTION DIAGRAMS

FOR DESIGN OF SECTIONS SUBJECTED TO ECCENTRIC COMP. FORCES





$\mu = \rho\ f_{cu}.10^{-4}$

$As = \mu\ b\ t$

$A\acute{s} = \alpha\ As$

$\zeta = \frac{d - d'}{t}$

R-Sec.

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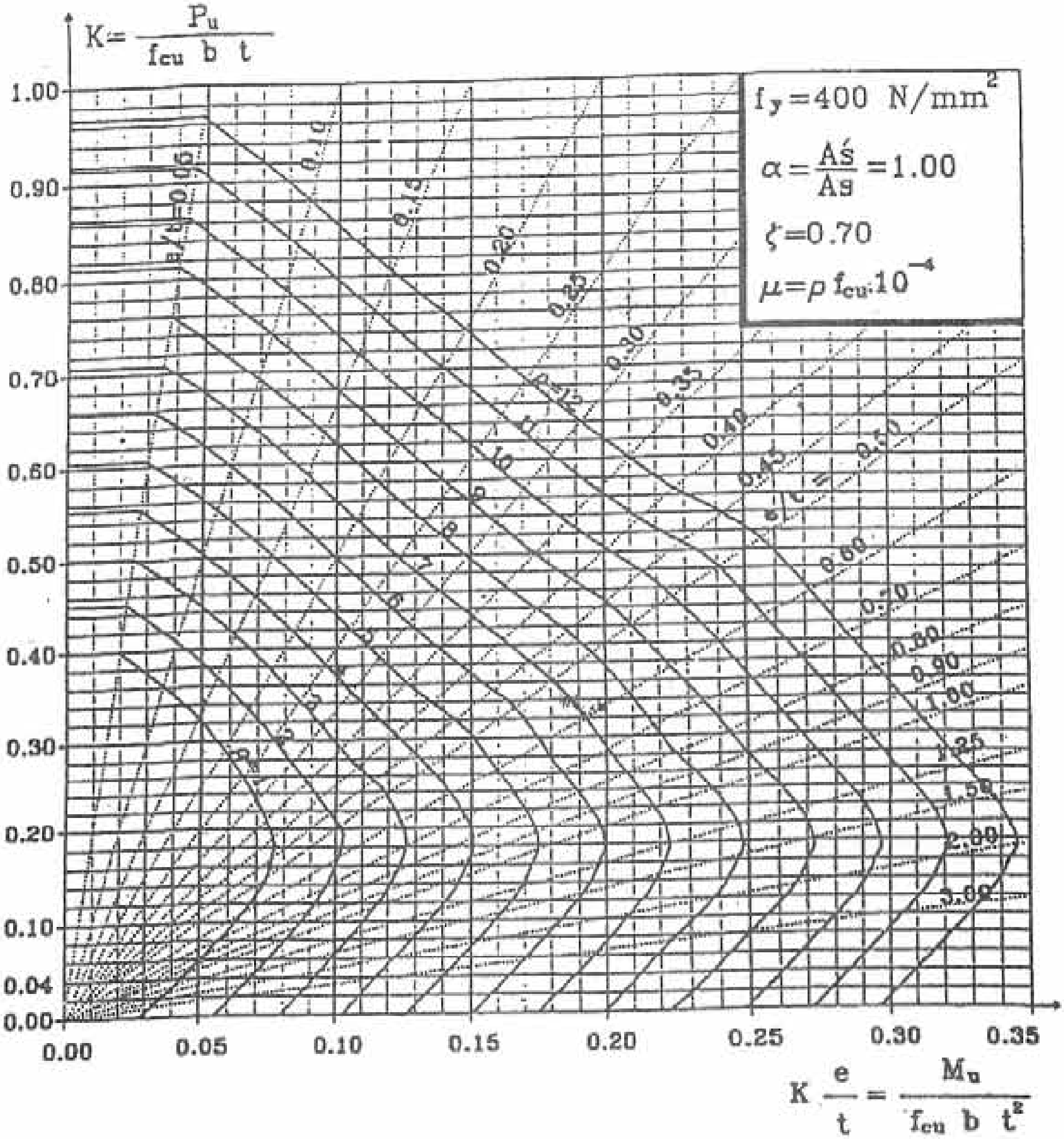
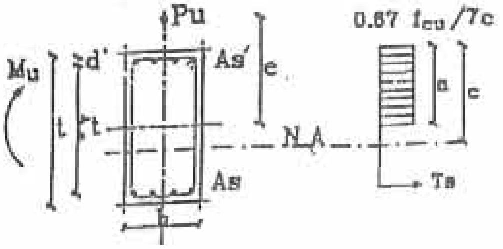
Chart (4-3) : INTERACTION DIAGRAMS

FOR DESIGN OF SECTIONS SUBJECTED TO ECCENTRIC COMP. FORCES





$\mu = \rho\ f_{cu}.10^{-4}$

$As = \mu\ b\ t$

$A\acute{s} = \alpha\ As$

$\zeta = \frac{d - d'}{t}$

R-Sec.

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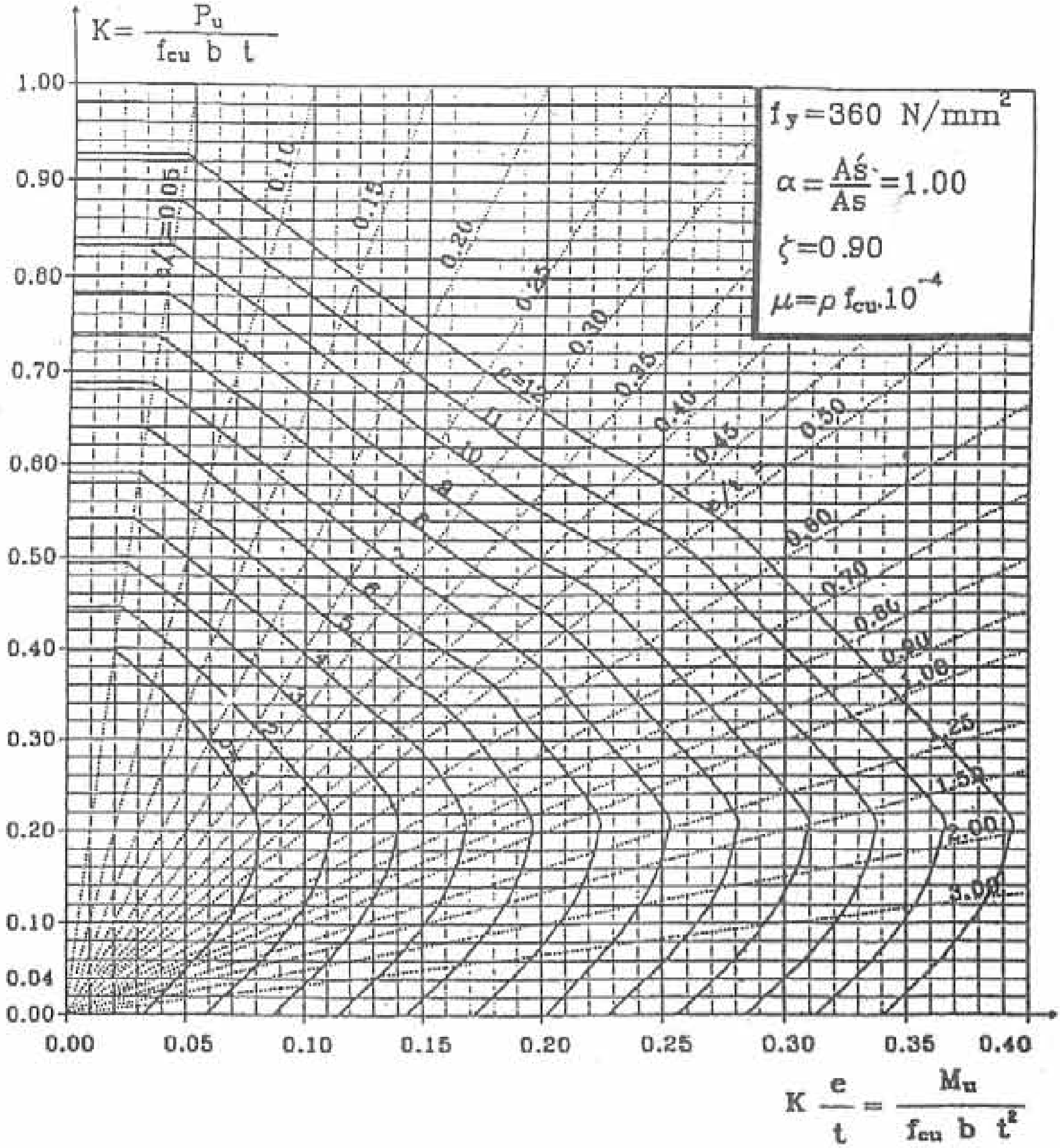
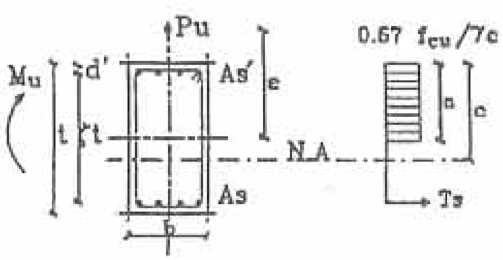
Chart (4-4) : INTERACTION DIAGRAMS

FOR DESIGN OF SECTIONS SUBJECTED TO ECCENTRIC COMP. FORCES



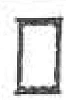


$\mu = \rho\ f_{cu}.10^{-4}$

$As = \mu\ b\ t$

$A\acute{s} = \alpha\ As$

$\zeta = \frac{d - d'}{t}$



R–Sec.

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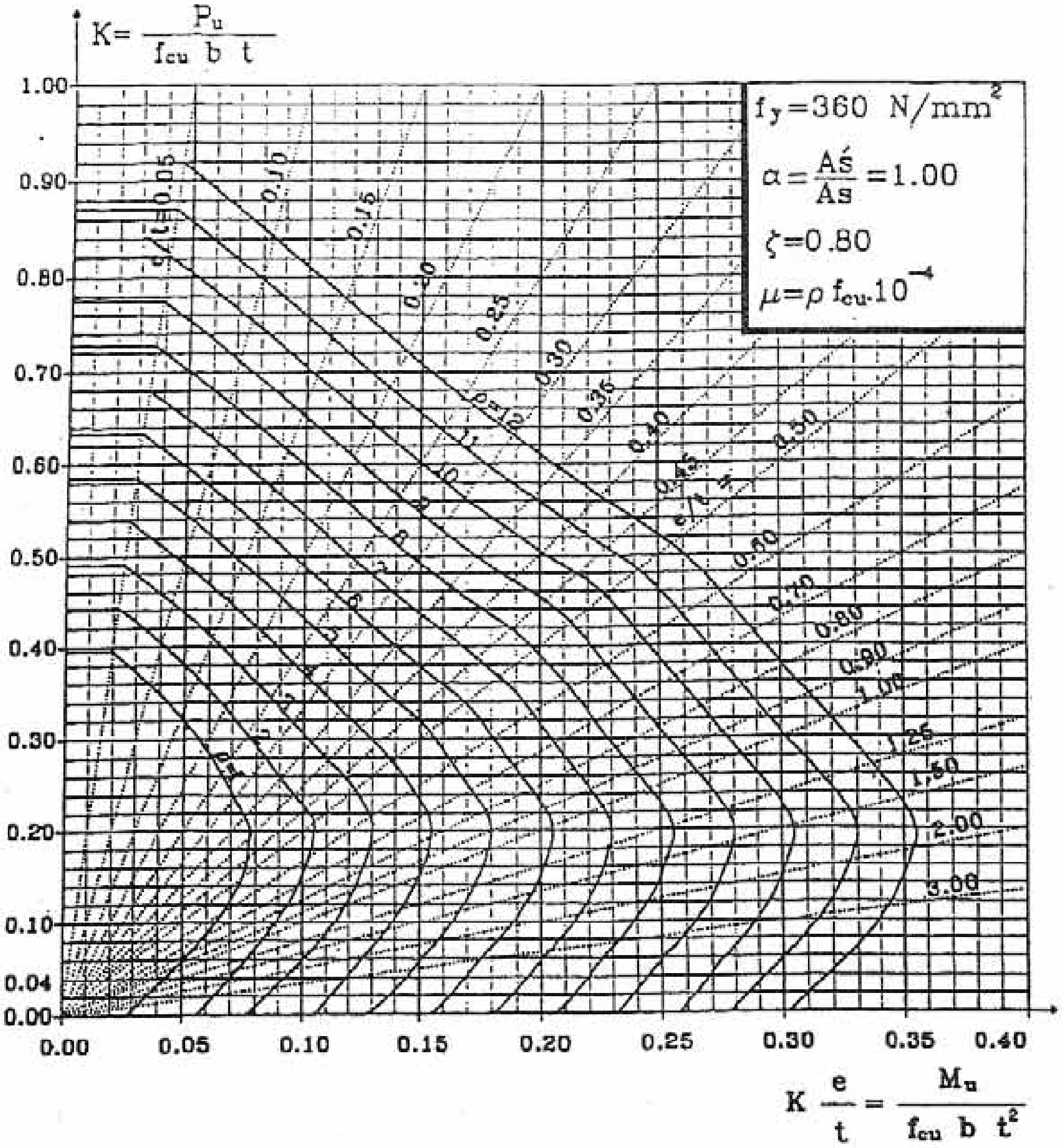
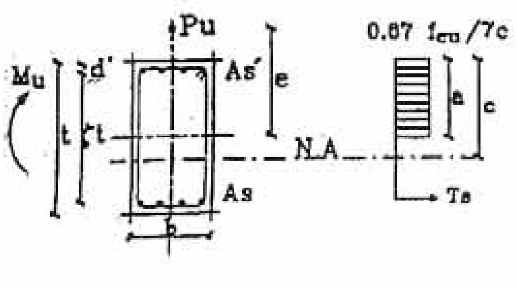
Chart (4-5) : INTERACTION DIAGRAMS

FOR DESIGN OF SECTIONS SUBJECTED TO ECCENTRIC COMP. FORCES





$\mu = \rho\ f_{cu} . 10^{-4}$

$As = \mu\ b\ t$

$A\acute{s} = \alpha\ As$

$\zeta = \frac{d - d'}{t}$

R–Sec.

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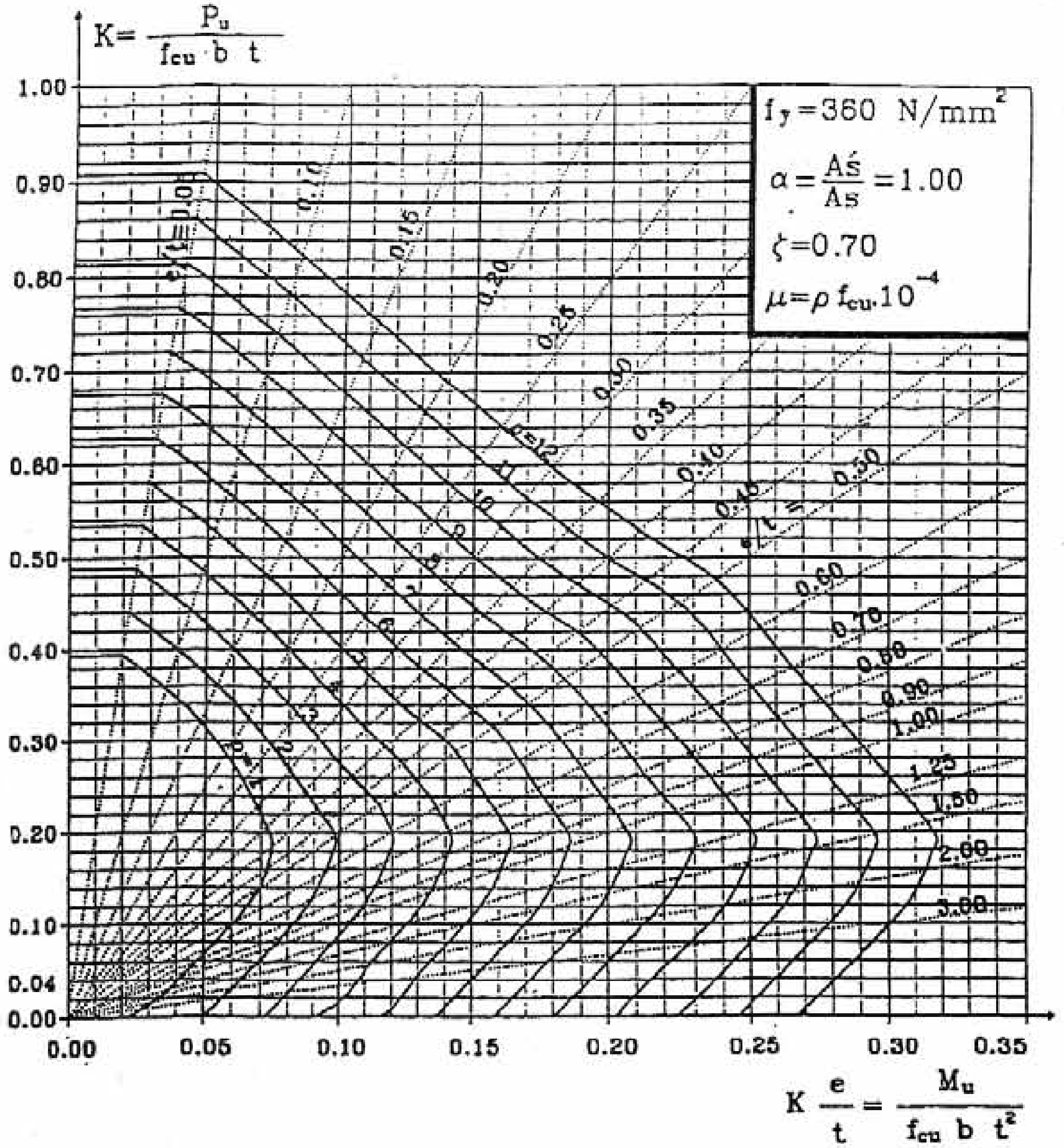
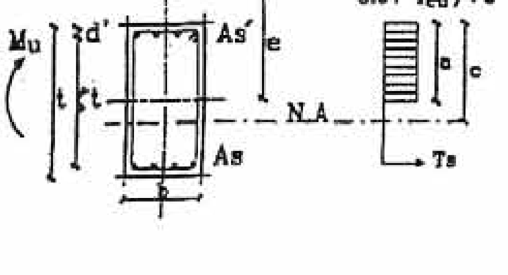
Chart (4-6) : INTERACTION DIAGRAMS

FOR DESIGN OF SECTIONS SUBJECTED TO ECCENTRIC COMP. FORCES





$\mu = \rho \, f_{cu} . 10^{-4}$

$As = \mu \ b \ t$

$A\acute{s} = \alpha \ As$

$\zeta = \frac{d - d'}{t}$

R–Sec.

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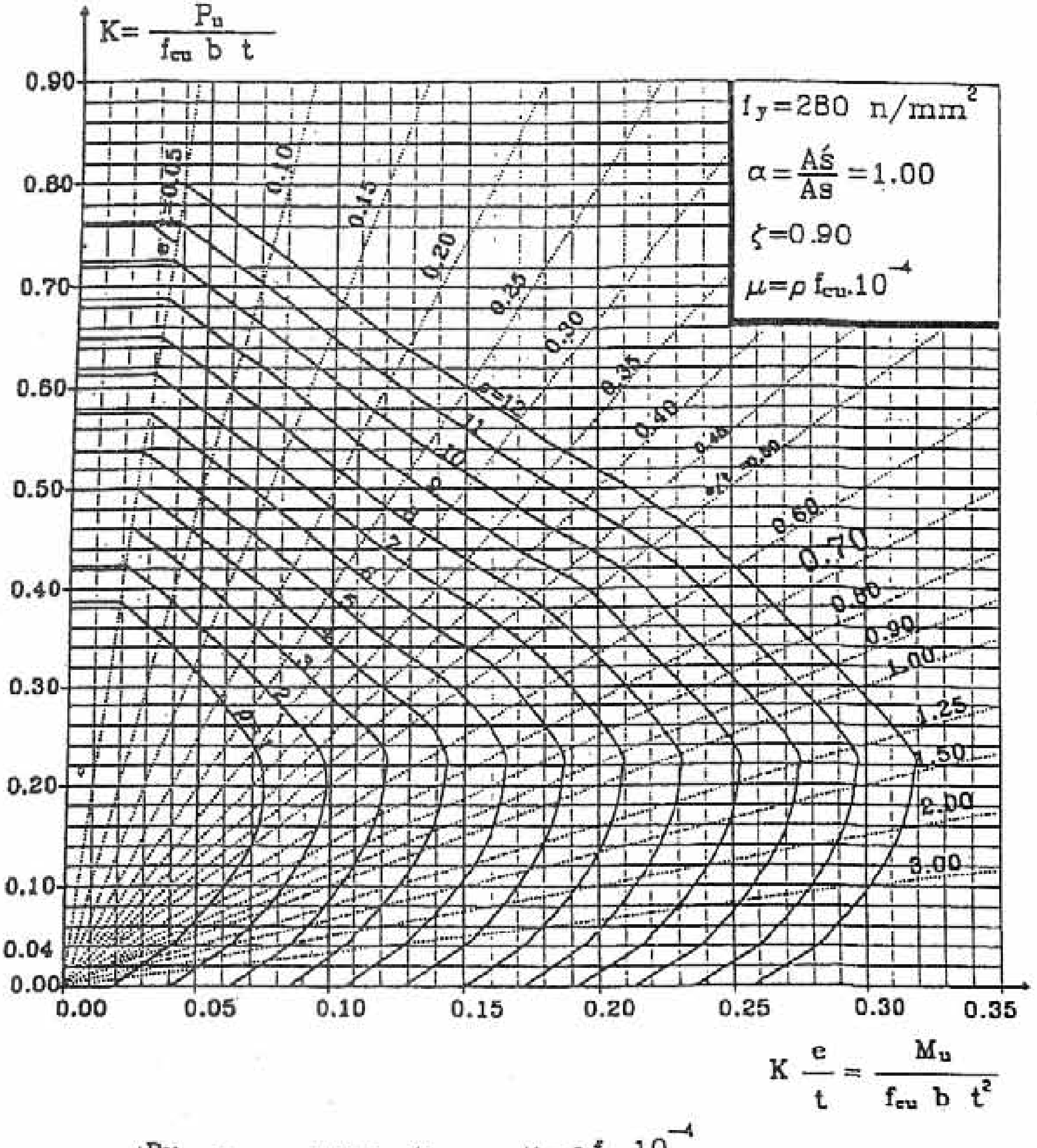
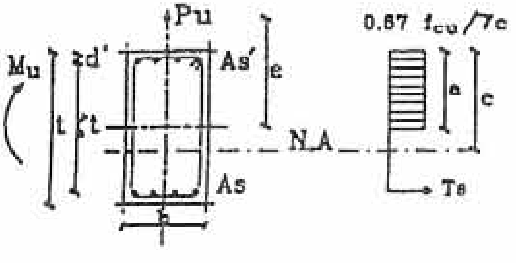
Chart (4-7) : INTERACTION DIAGRAMS

FOR DESIGN OF SECTIONS SUBJECTED TO ECCENTRIC COMP. FORCES





$\mu = \rho\ f_{cu}.10^{-4}$

$As = \mu\ b\ t$

$A\acute{s} = \alpha\ As$

$\zeta = \frac{d - d'}{t}$

R–Sec.

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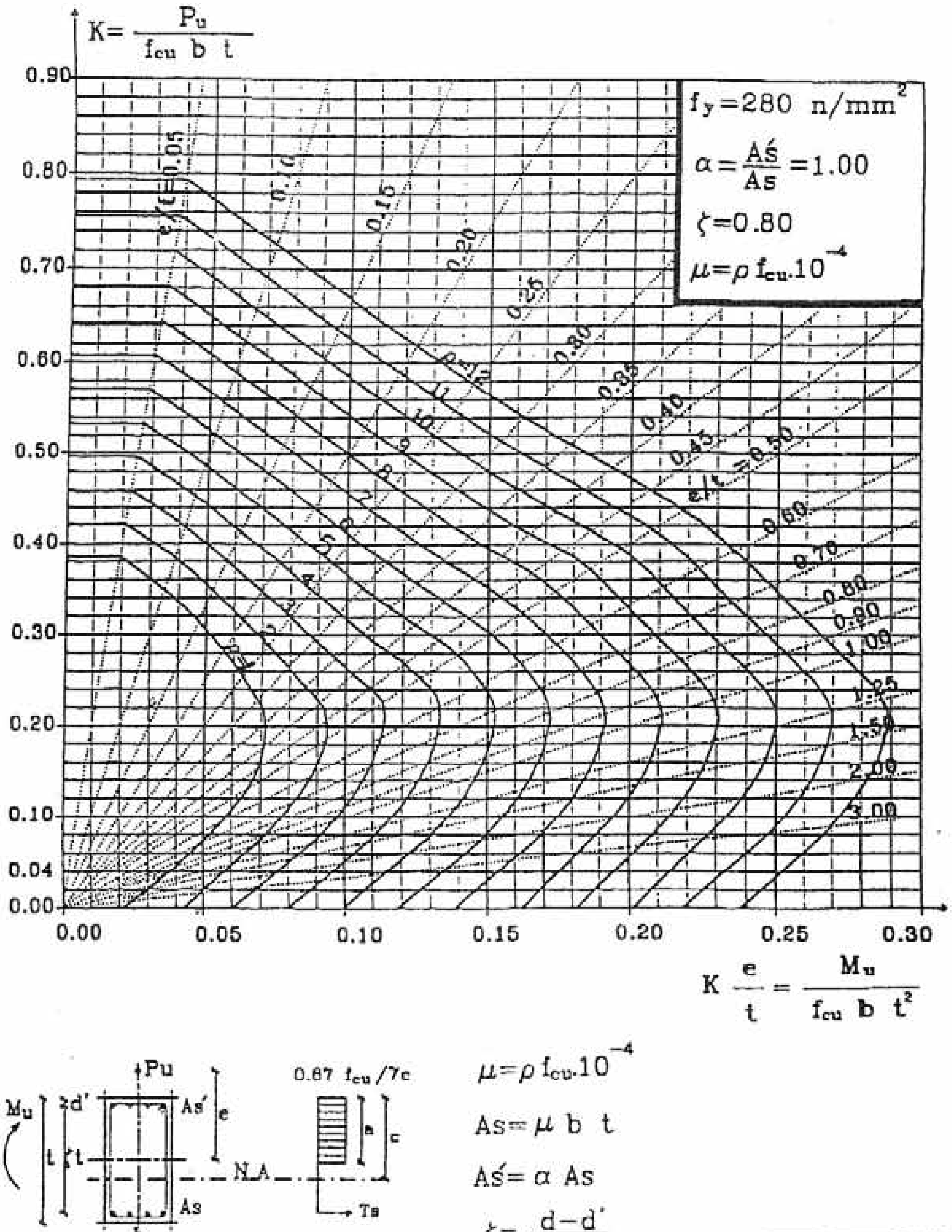
Chart (4-8) : INTERACTION DIAGRAMS

FOR DESIGN OF SECTIONS SUBJECTED TO ECCENTRIC COMP. FORCES



$\mu = \rho\, f_{cu} . 10^{-4}$

$A_s = \mu\ b\ t$

$A_s' = \alpha\ A_s$

$\zeta = \frac{d - d'}{t}$

R-Sec.

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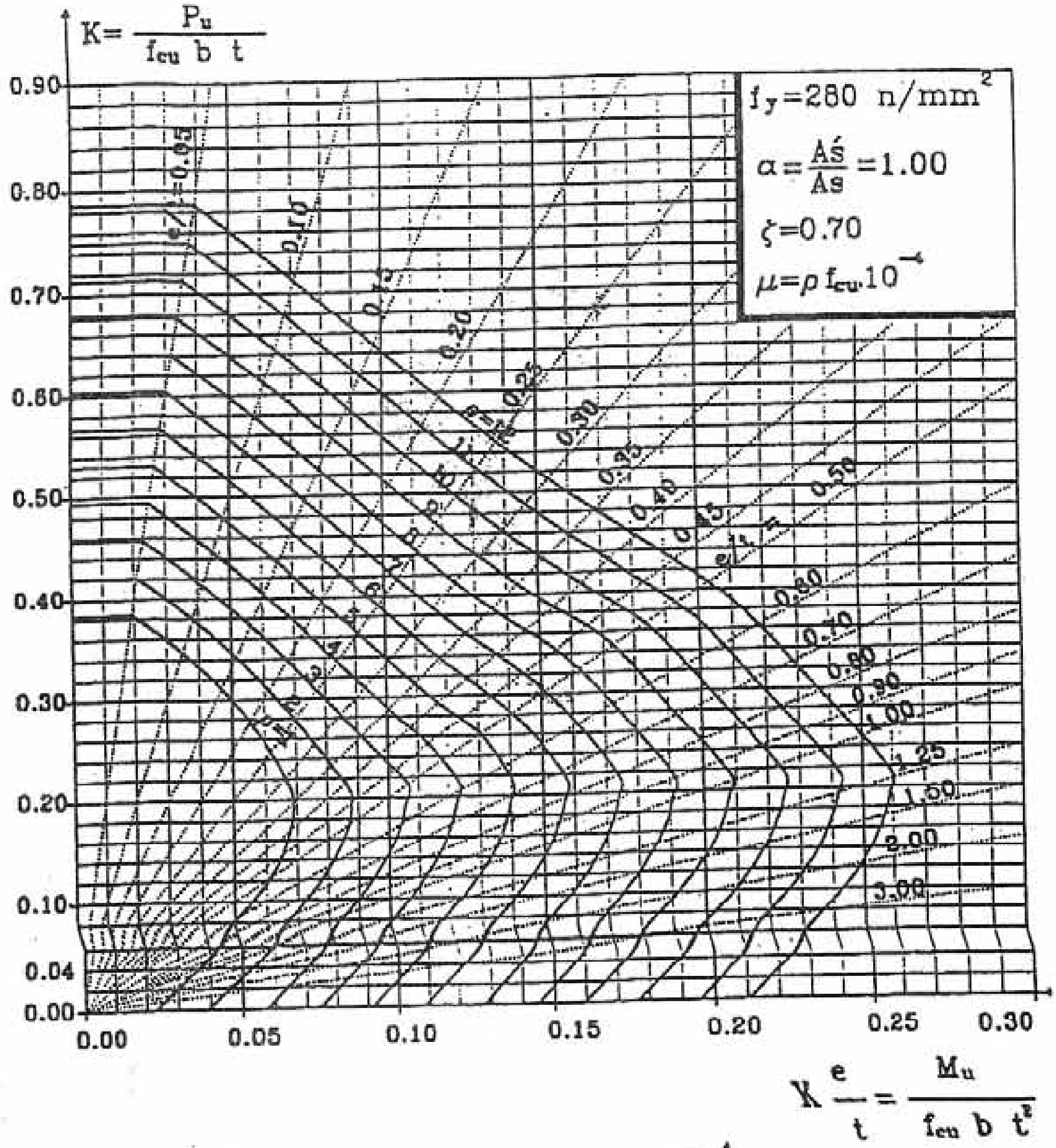
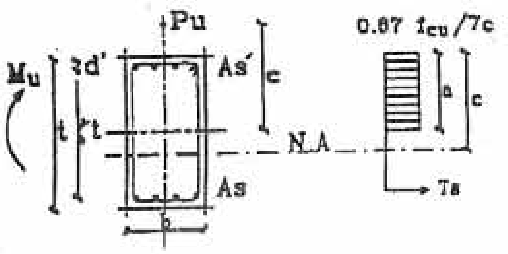
Chart (4-9) : INTERACTION DIAGRAMS

FOR DESIGN OF SECTIONS SUBJECTED TO ECCENTRIC COMP. FORCES





$\mu = \rho\ f_{cu} \cdot 10^{-4}$

$A_s = \mu\ b\ t$

$A_s' = \alpha\ A_s$

$\zeta = \frac{d - d'}{t}$

R-Sec.

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Chart (4-10) : INTERACTION DIAGRAMS

FOR DESIGN OF SECTIONS SUBJECTED TO ECCENTRIC COMP. FORCES

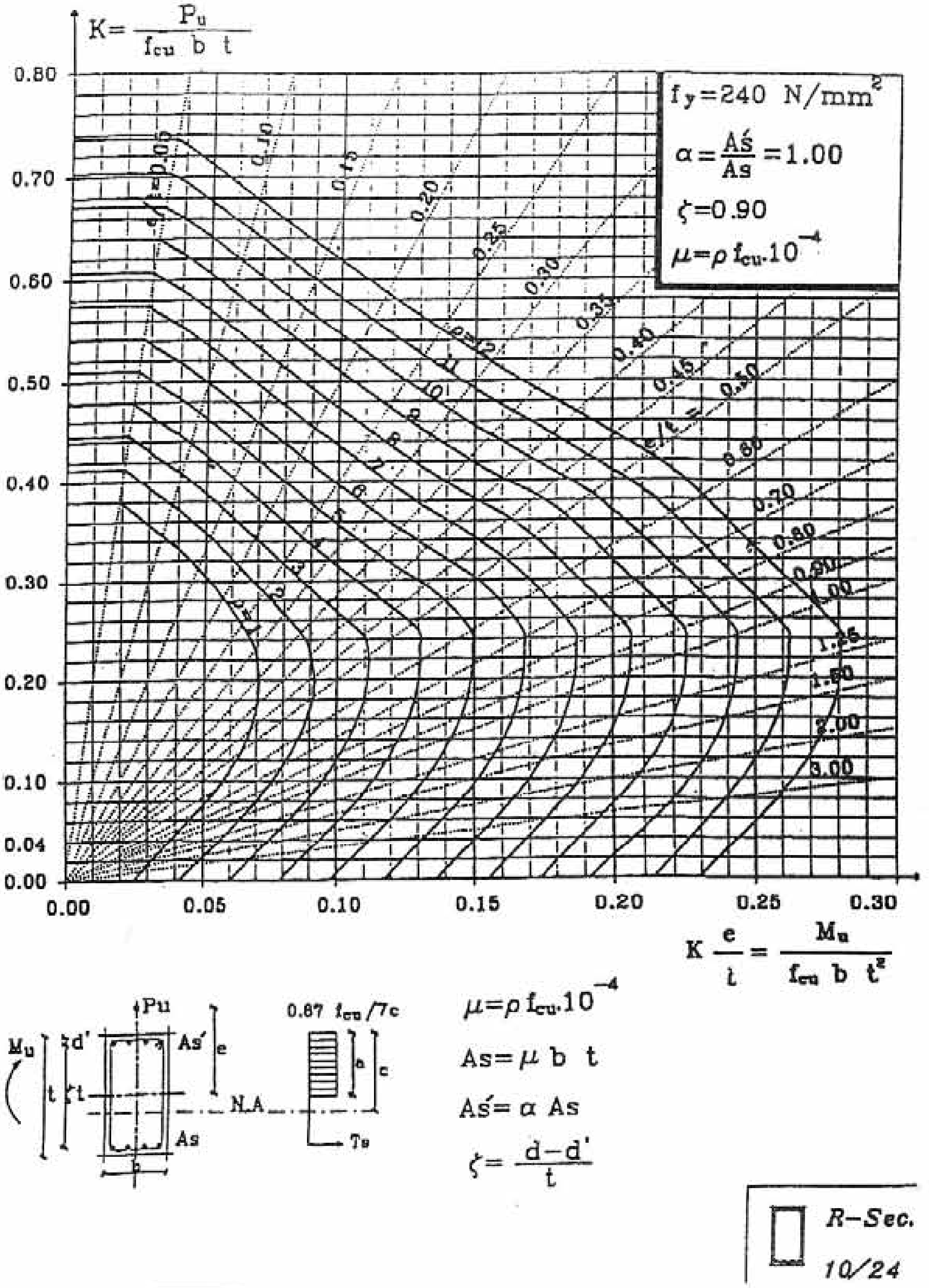
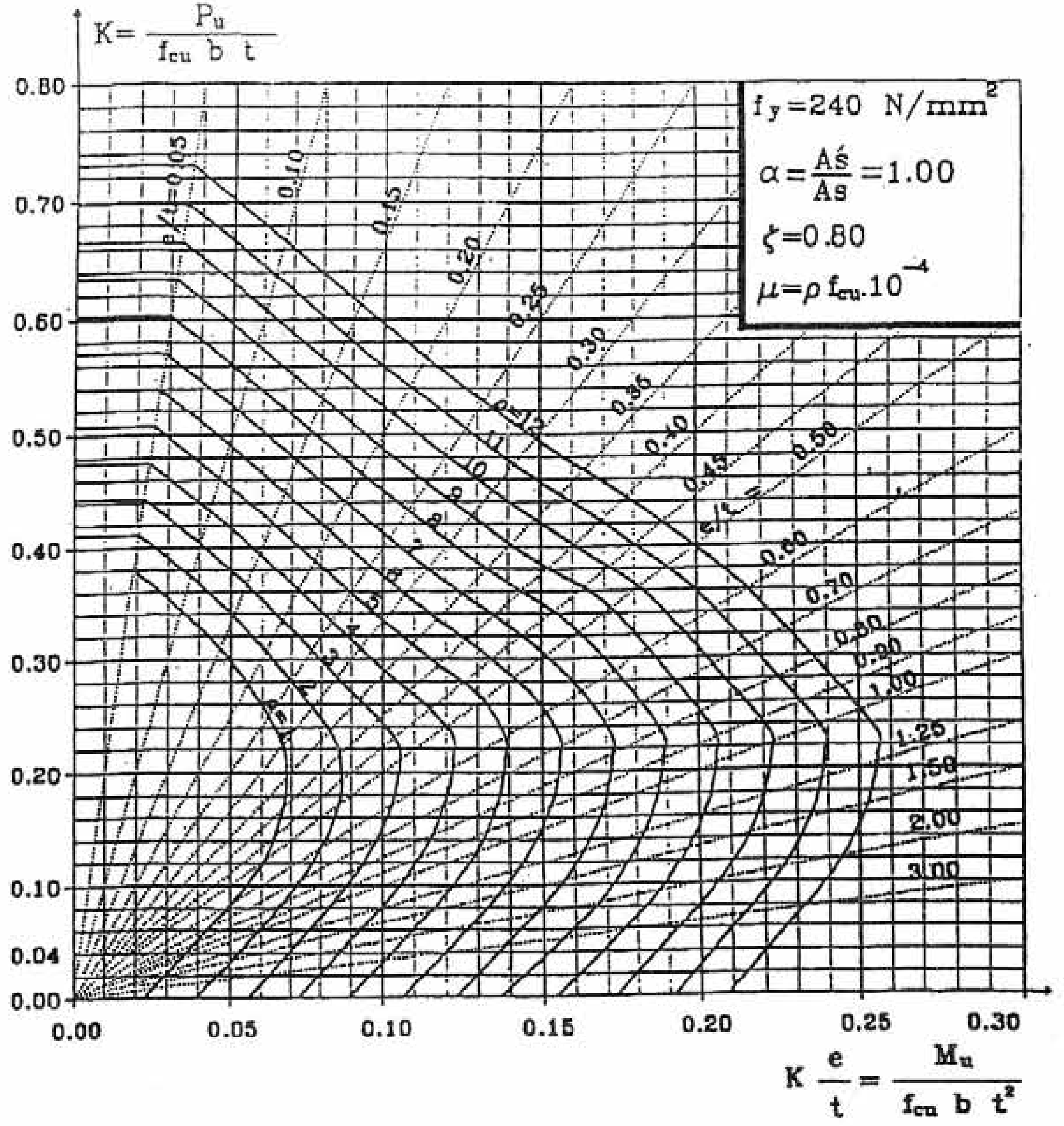
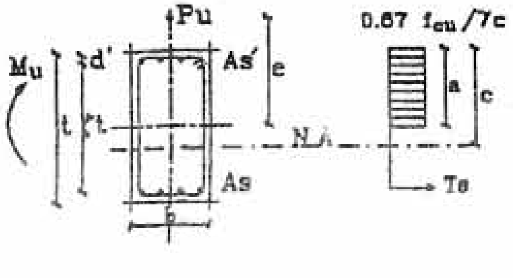


Chart (4-11) : INTERACTION DIAGRAMS

FOR DESIGN OF SECTIONS SUBJECTED TO ECCENTRIC COMP. FORCES





$\mu = \rho\ f_{cu}.10^{-4}$

$As = \mu\ b\ t$

$A\acute{s} = \alpha\ As$

$\zeta = \frac{d - d'}{t}$

R–Sec.

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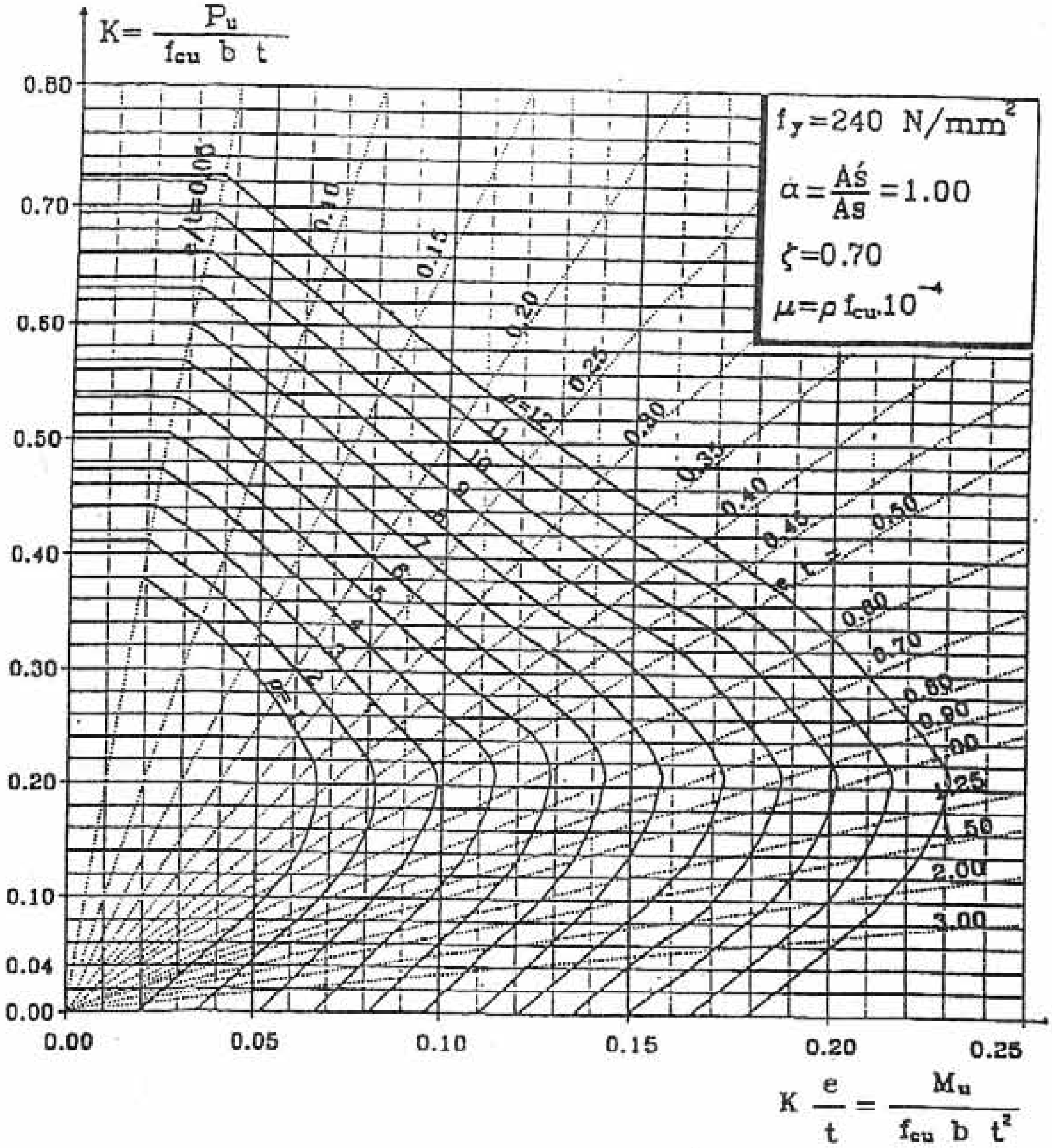
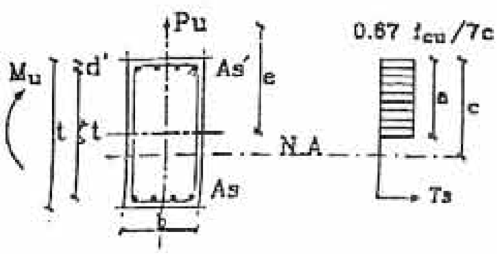
Chart (4-12) : INTERACTION DIAGRAMS

FOR DESIGN OF SECTIONS SUBJECTED TO ECCENTRIC COMP. FORCES





$\mu = \rho\, f_{cu} \cdot 10^{-4}$

$As = \mu\ b\ \ t$

$As' = \alpha\ As$

$\zeta = \dfrac{d - d'}{t}$

R–Sec.

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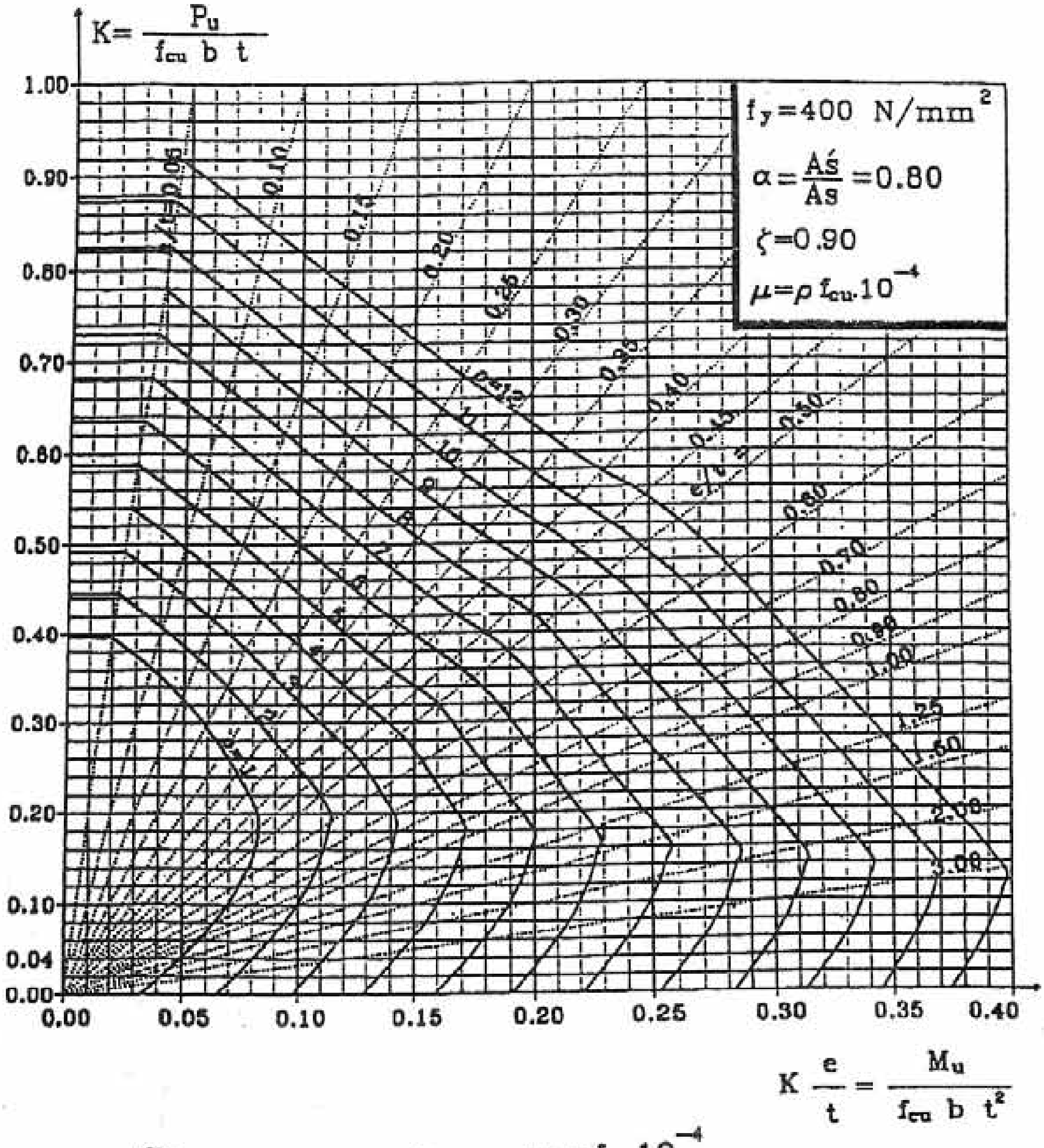
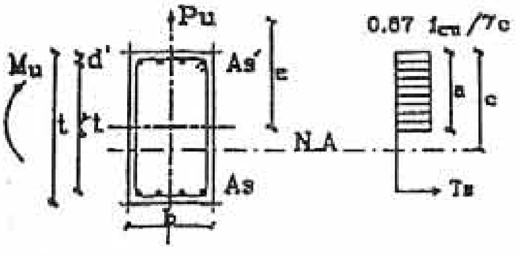
Chart (4-13) : INTERACTION DIAGRAMS

FOR DESIGN OF SECTIONS SUBJECTED TO ECCENTRIC COMP. FORCES





$\mu = \rho\ f_{cu} \cdot 10^{-4}$

$As = \mu\ b\ t$

$A's = \alpha\ As$

$\zeta = \frac{d - d'}{t}$

R-Sec.

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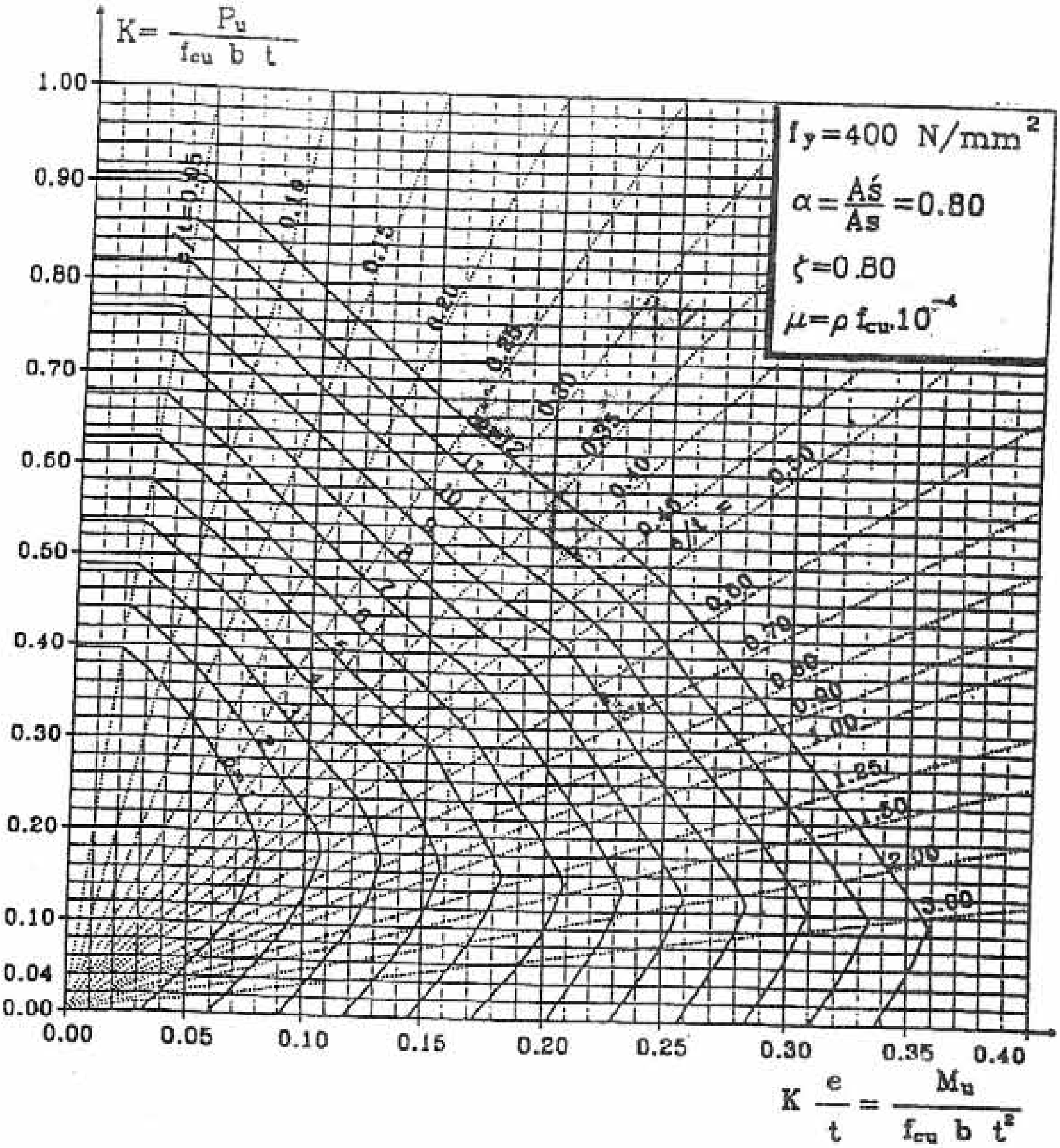
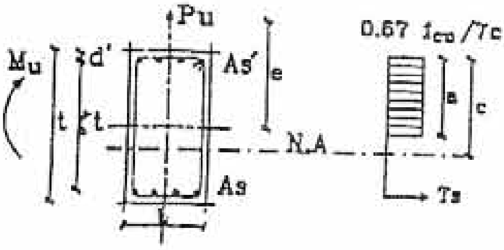
Chart (4-14) : INTERACTION DIAGRAMS

FOR DESIGN OF SECTIONS SUBJECTED TO ECCENTRIC COMP. FORCES





$\mu = \rho \, f_{cu} . 10^{-4}$

$As = \mu \; b \; t$

$As' = \alpha \; As$

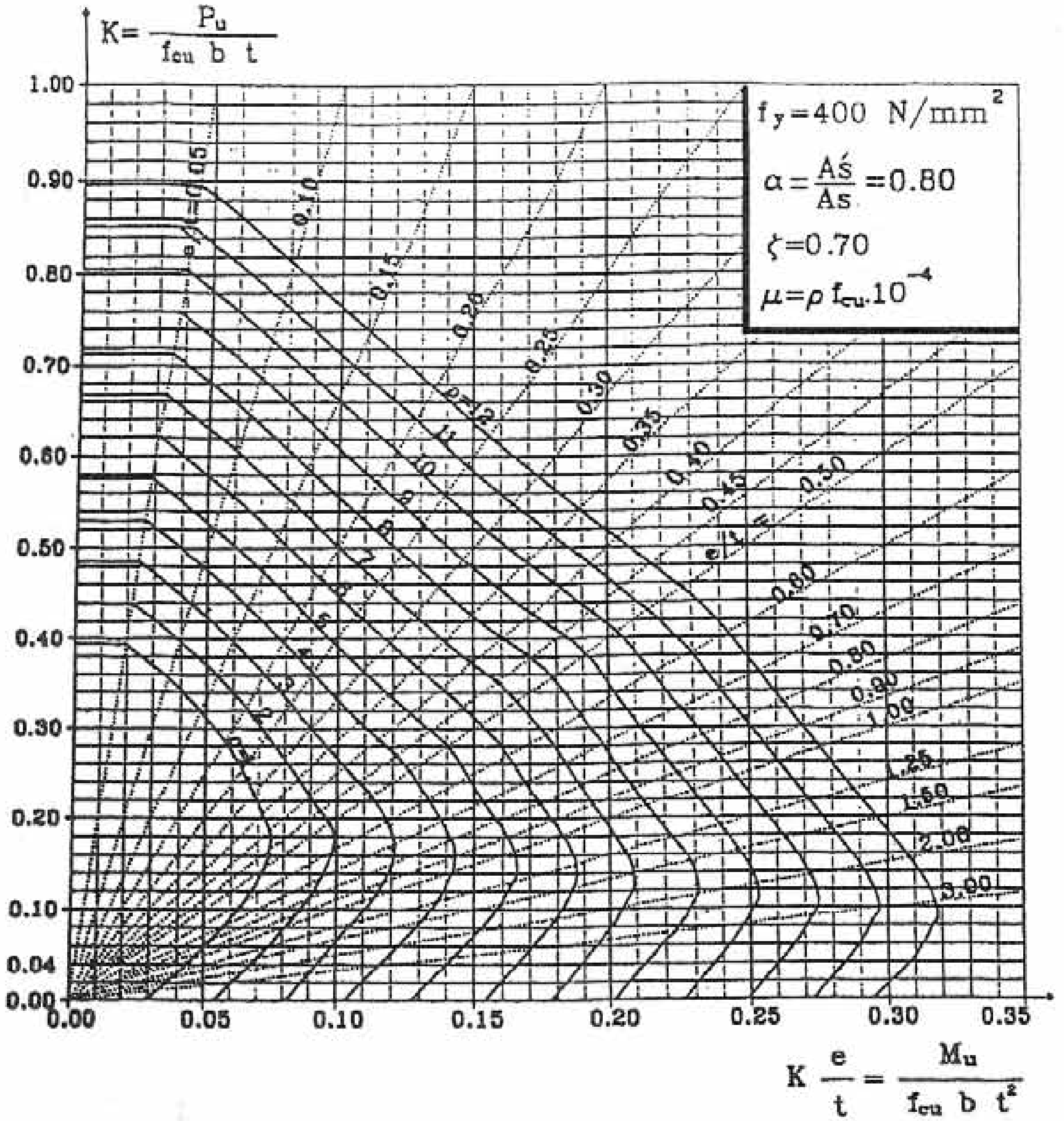
$\zeta = \frac{d-d'}{t}$

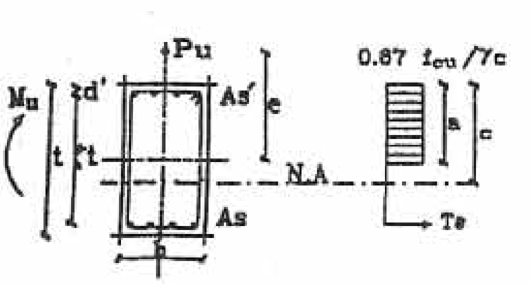
R-Sec.

14/24

Chart (4-15) : INTERACTION DIAGRAMS

FOR DESIGN OF SECTIONS SUBJECTED TO ECCENTRIC COMP. FORCES





$\mu = \rho\ f_{cu} . 10^{-4}$

$A_s = \mu\ b\ t$

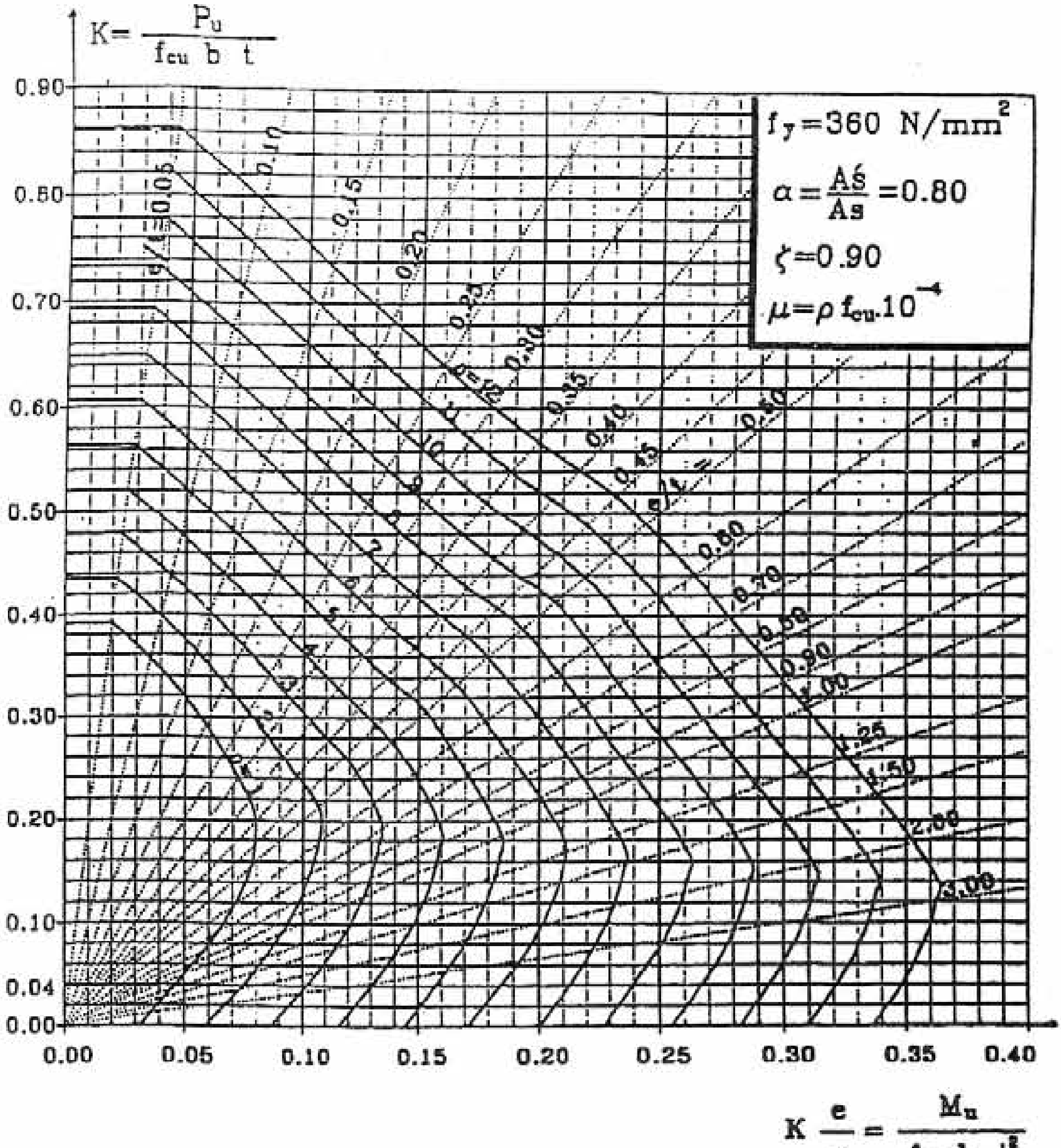
$A_s' = \alpha\ A_s$

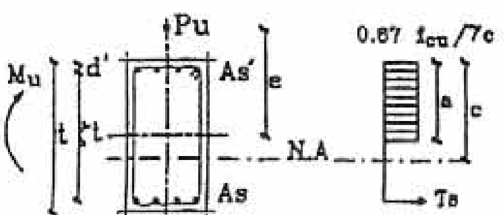
$\zeta = \frac{d - d'}{t}$

R–Sec. 15/24

Chart (4-16) : INTERACTION DIAGRAMS

FOR DESIGN OF SECTIONS SUBJECTED TO ECCENTRIC COMP. FORCES



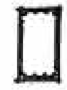


$\mu = \rho\ f_{cu} . 10^{-4}$

$A_s = \mu\ b\ t$

$A_s' = \alpha\ A_s$

$\zeta = \frac{d - d'}{t}$



R–Sec.

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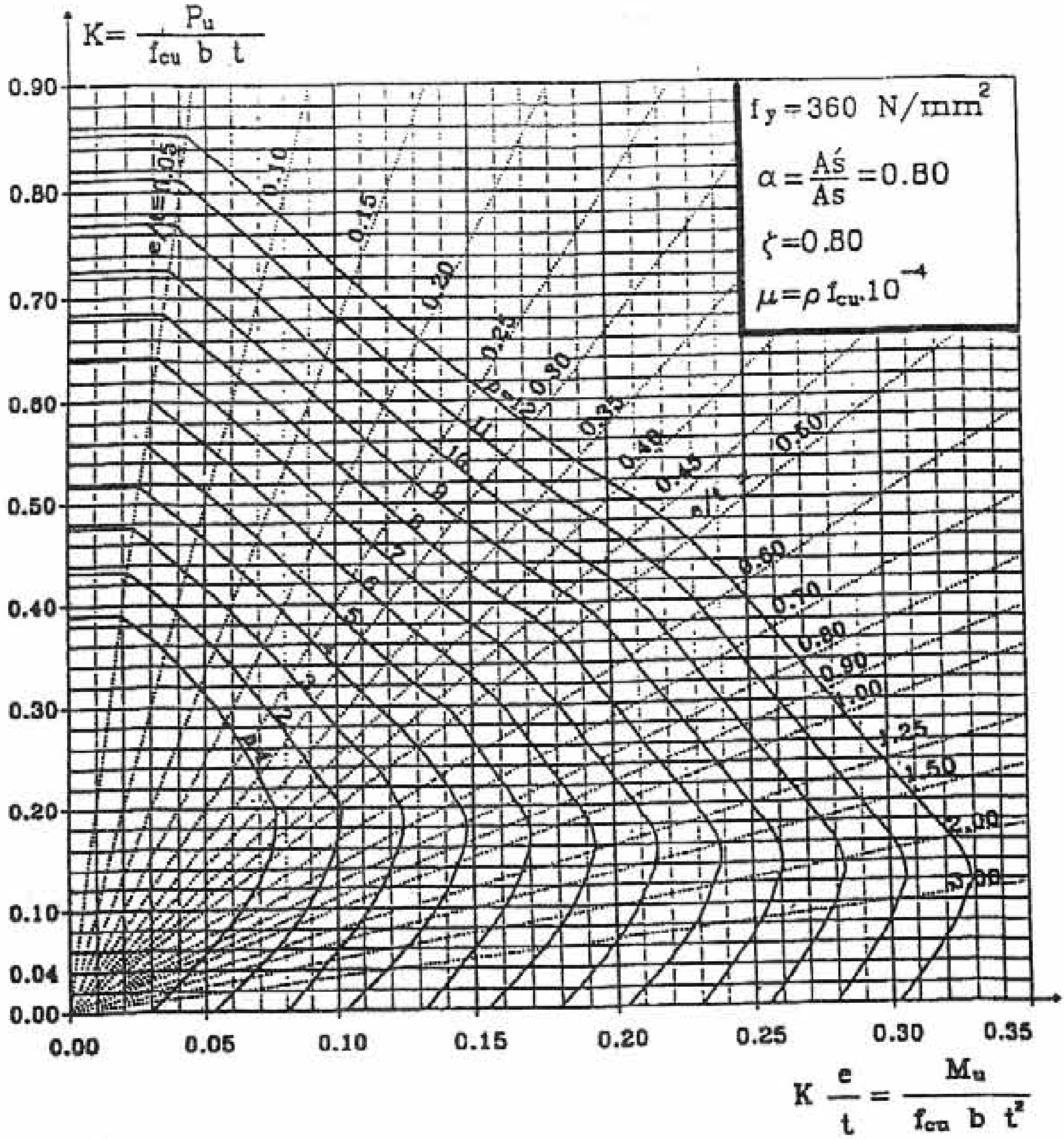
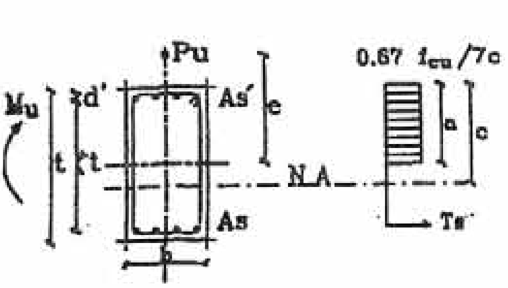
Chart (4-17) : INTERACTION DIAGRAMS

FOR DESIGN OF SECTIONS SUBJECTED TO ECCENTRIC COMP. FORCES





$\mu = \rho\, f_{cu} . 10^{-4}$

$As = \mu\ b\ t$

$As' = \alpha\ As$

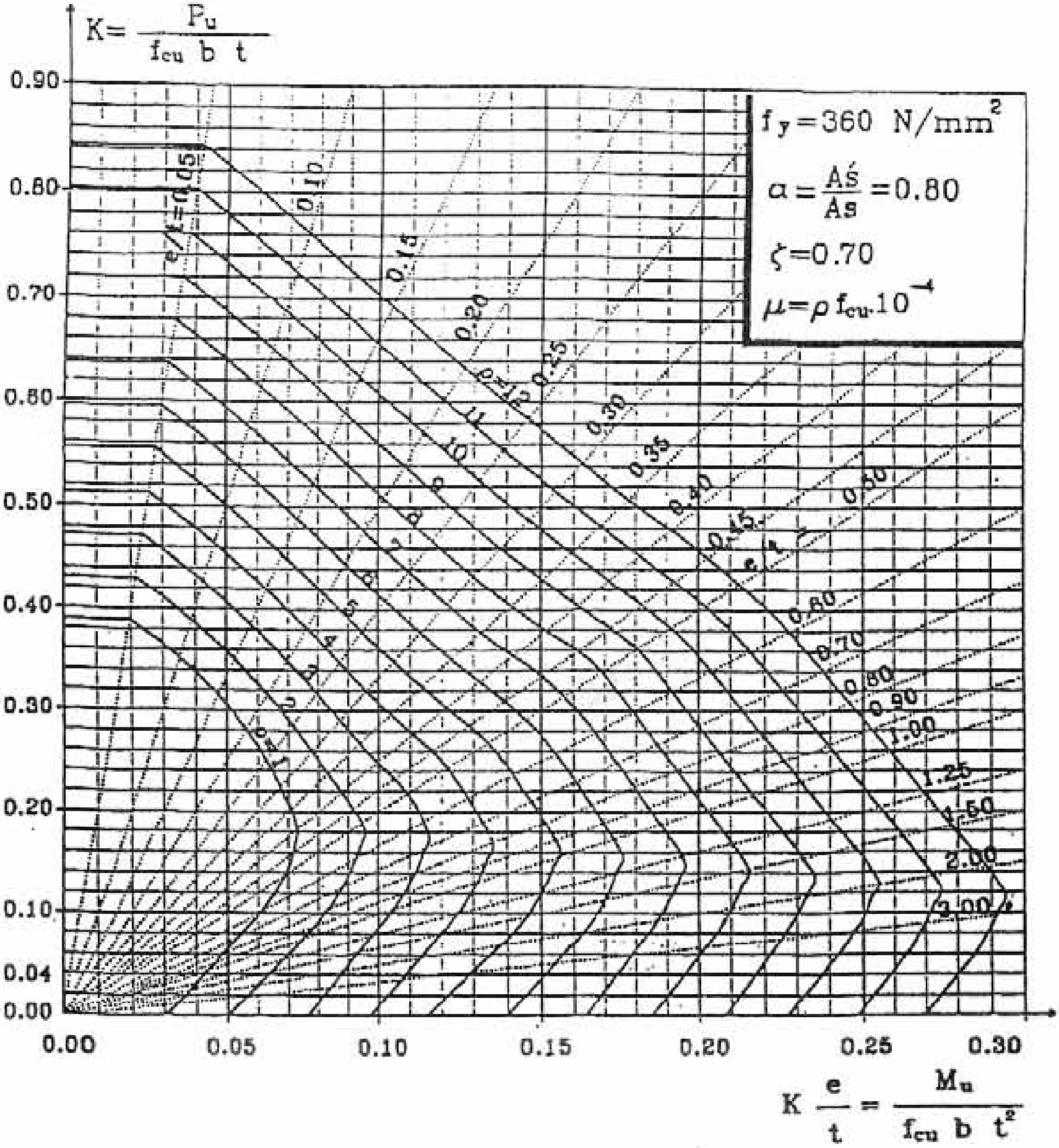
$\zeta = \dfrac{d - d'}{t}$

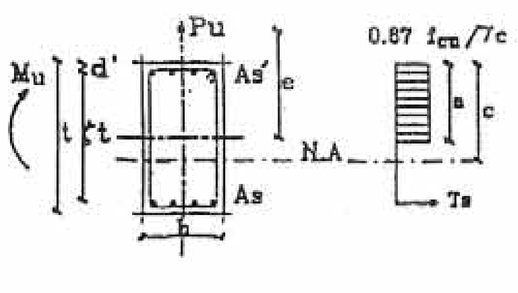
R–Sec.

17/24

Chart (4-18) : INTERACTION DIAGRAMS

FOR DESIGN OF SECTIONS SUBJECTED TO ECCENTRIC COMP. FORCES





$\mu = \rho\ f_{cu} . 10^{-4}$

$A_s = \mu\ b\ t$

$A_s' = \alpha\ A_s$

$\zeta = \frac{d - d'}{t}$

R–Sec.

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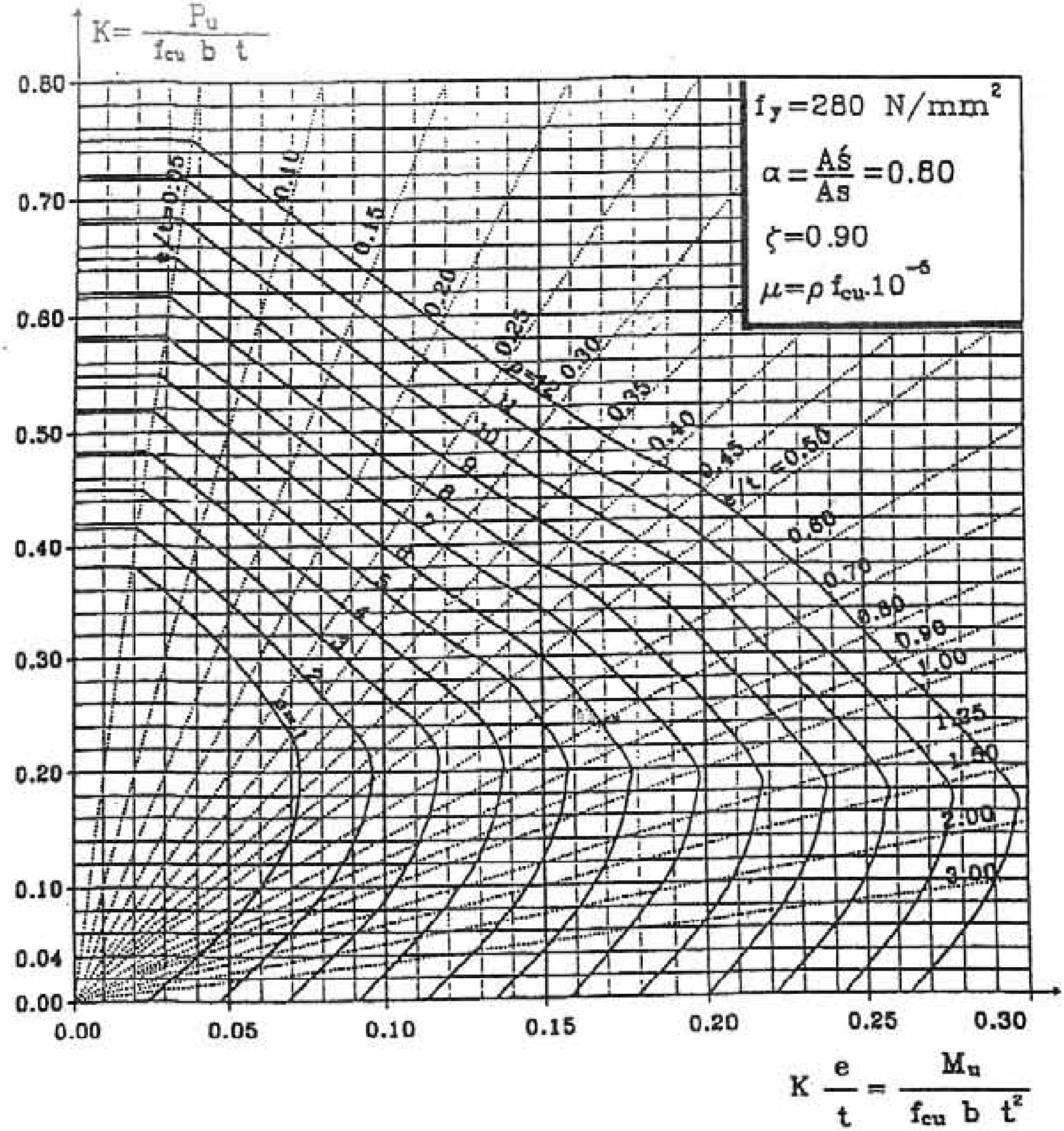
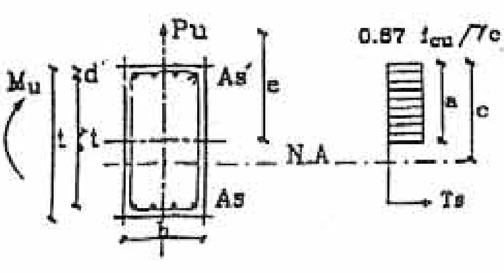
Chart (4-19) : INTERACTION DIAGRAMS

FOR DESIGN OF SECTIONS SUBJECTED TO ECCENTRIC COMP. FORCES





$\mu=\rho\ f_{cu}.10^{-4}$

$As=\mu\ b\ t$

$As'=\alpha\ As$

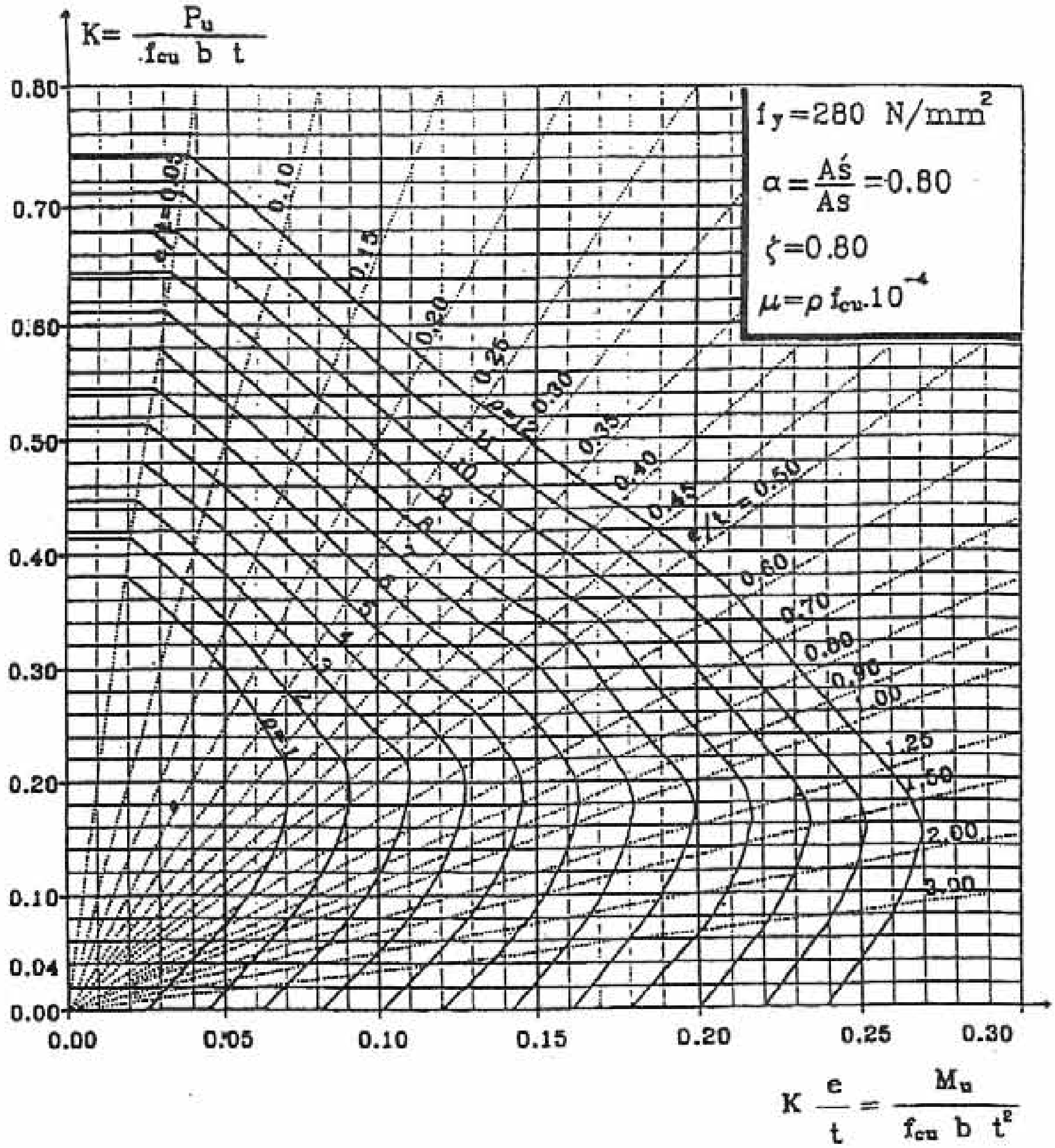
$\zeta=\frac{d-d'}{t}$

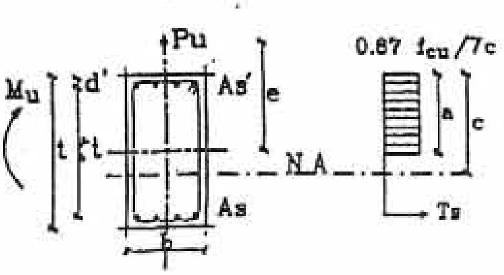
R–Sec.

19/24

Chart (4-20) : INTERACTION DIAGRAMS

FOR DESIGN OF SECTIONS SUBJECTED TO ECCENTRIC COMP. FORCES





$\mu = \rho\ f_{cu} \cdot 10^{-4}$

$A_s = \mu\ b\ .\ t$

$A_s' = \alpha\ A_s$

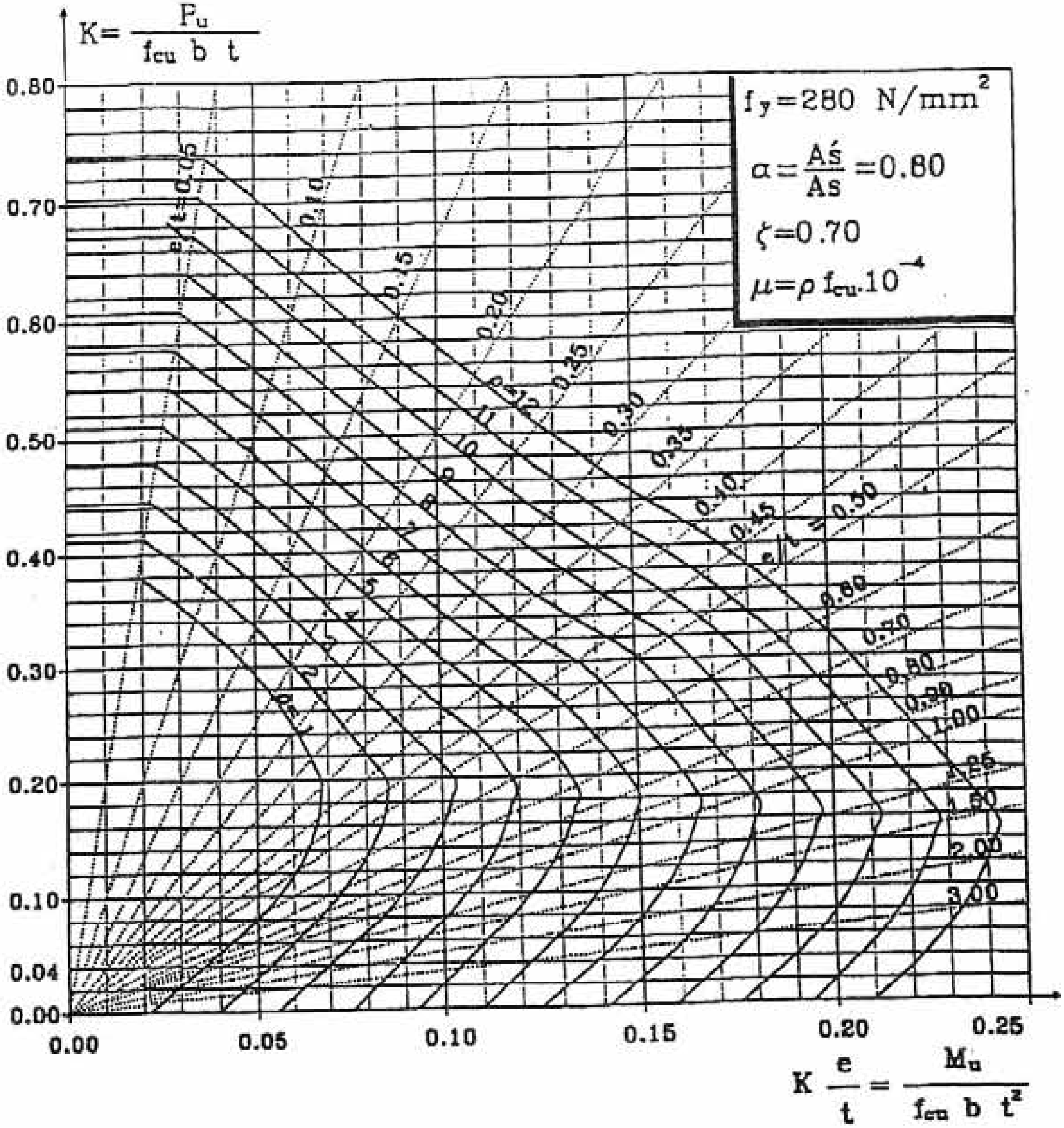
$\zeta = \frac{d-d'}{t}$

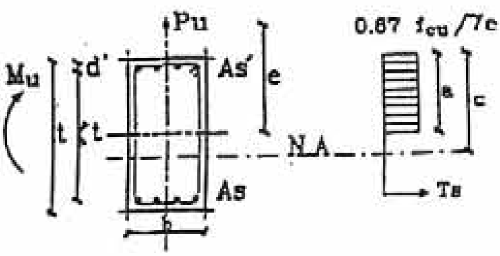
R-Sec.

20/24

Chart (4-21) : INTERACTION DIAGRAMS

FOR DESIGN OF SECTIONS SUBJECTED TO ECCENTRIC COMP. FORCES





$\mu=\rho\ f_{cu}.10^{-4}$

$As=\mu\ b\ t$

$A\acute{s}=\alpha\ As$

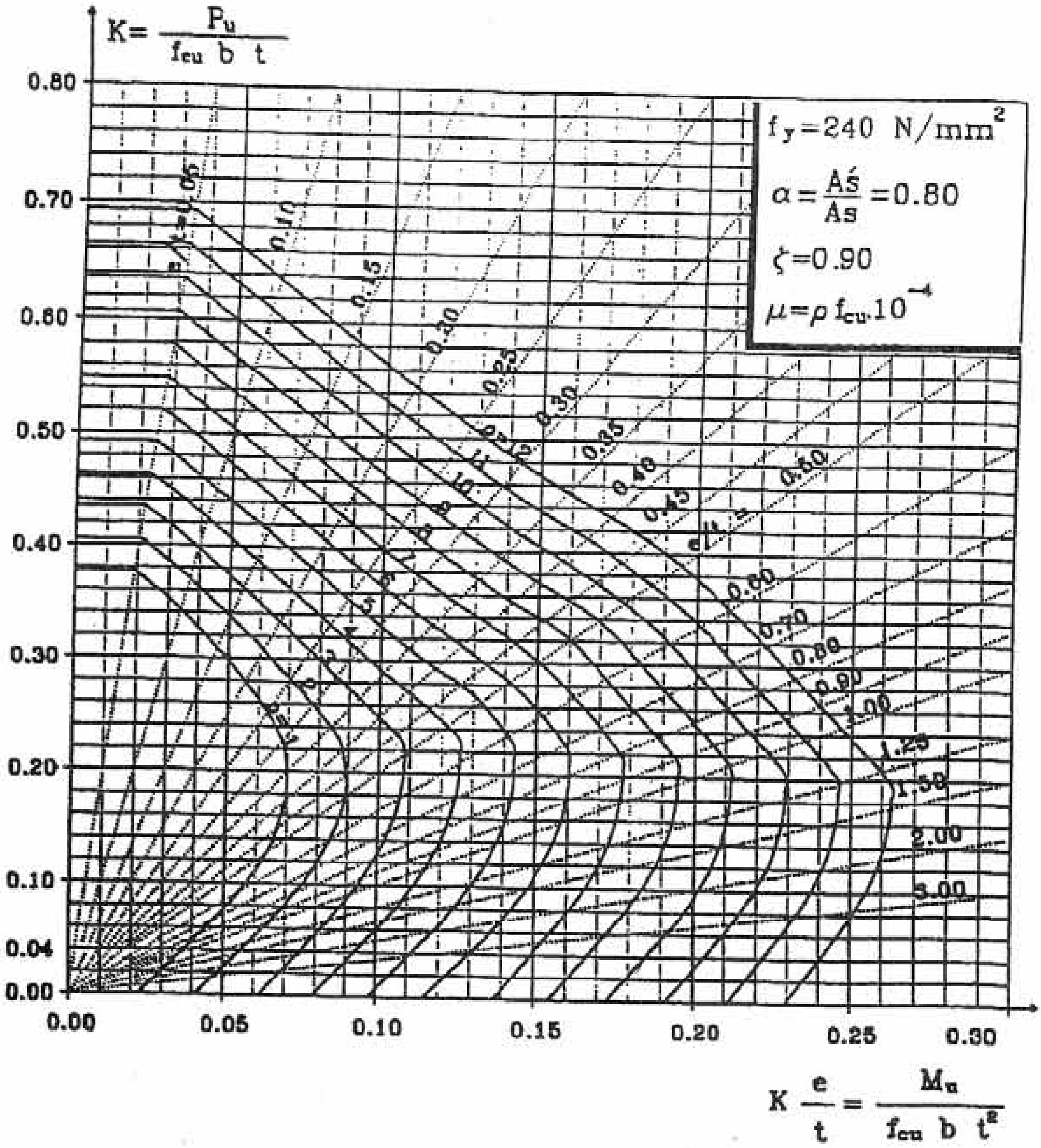
$\zeta=\frac{d-d'}{t}$

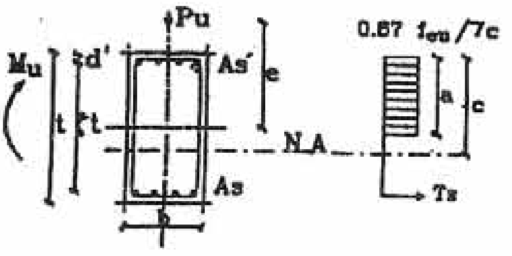
R–Sec.

21/24

Chart (4-22) : INTERACTION DIAGRAMS

FOR DESIGN OF SECTIONS SUBJECTED TO ECCENTRIC. COMP. FORCES





$\mu = \rho\, f_{cu}.10^{-4}$

$As = \mu\ b\ t$

$As' = \alpha\ As$

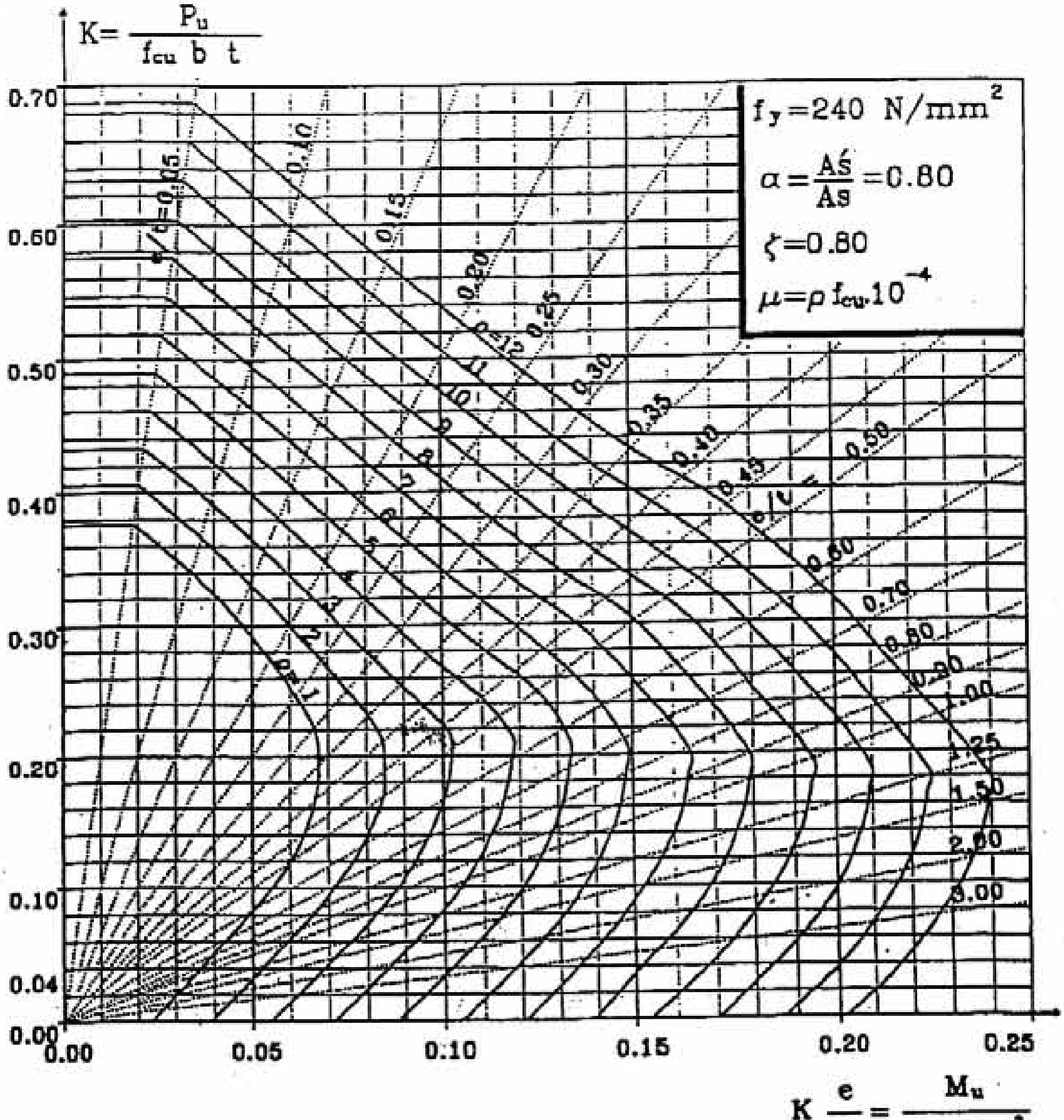
$\zeta = \frac{d - d'}{t}$

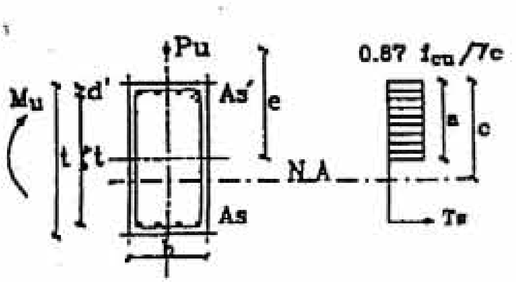
R-Sec.

22/24

Chart (4-23) : INTERACTION DIAGRAMS

FOR DESIGN OF SECTIONS SUBJECTED TO ECCENTRIC COMP. FORCES





$\mu = \rho\ f_{cu} \cdot 10^{-4}$

$As = \mu\ b\ t$

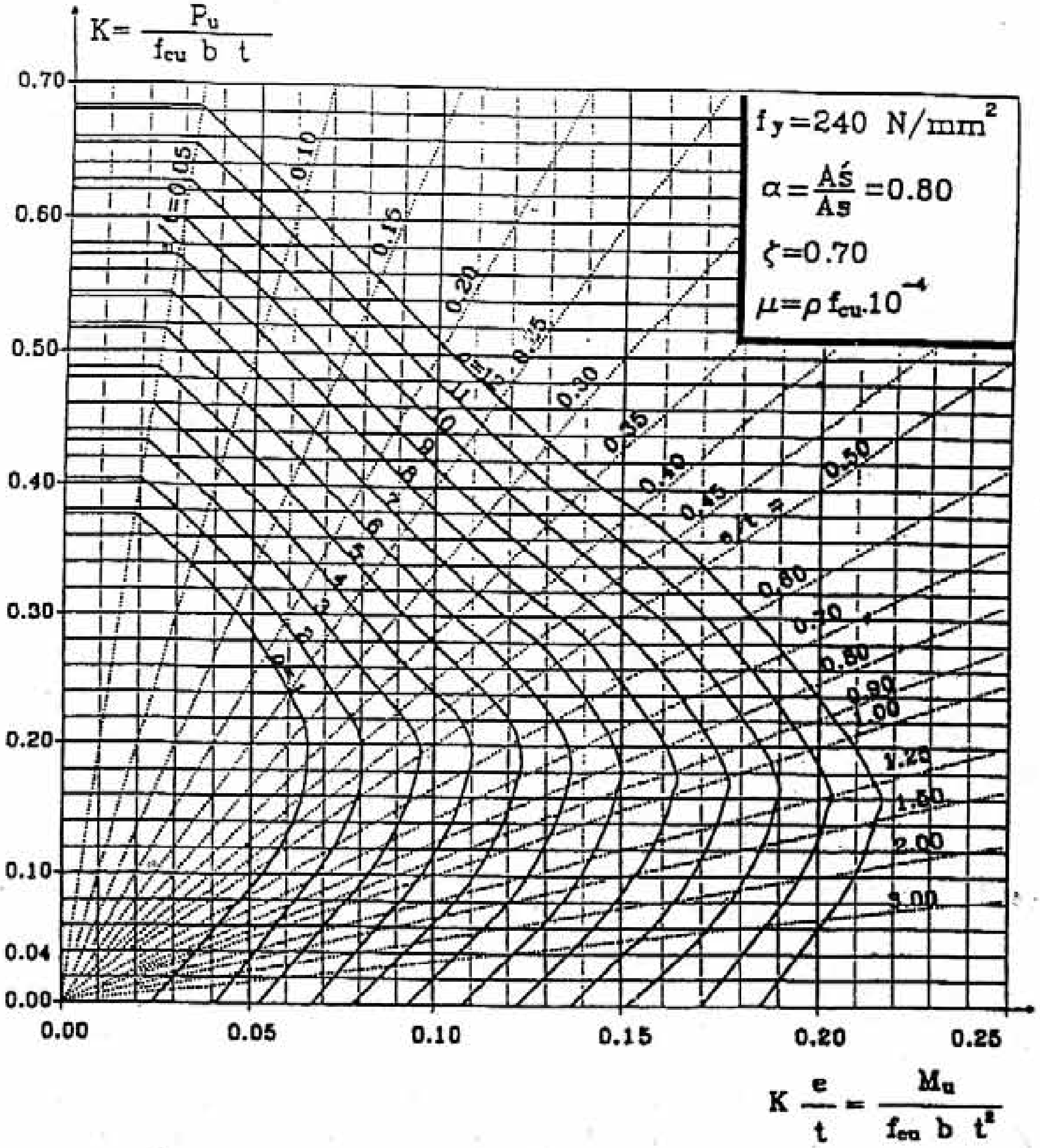
$A\acute{s} = \alpha\ As$

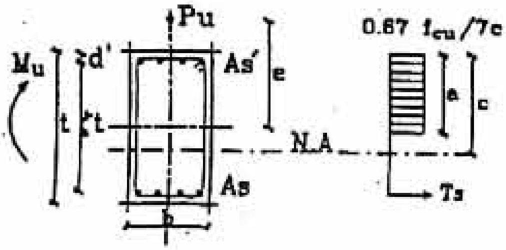
$\zeta = \frac{d - d'}{t}$

R-Sec. 23/24

Chart (4-24) : INTERACTION DIAGRAMS

FOR DESIGN OF SECTIONS SUBJECTED TO ECCENTRIC COMP. FORCES





$\mu = \rho \, f_{cu} . 10^{-4}$

$As = \mu \; b \; t$

$As' = \alpha \; As$

$\zeta = \dfrac{d - d'}{t}$

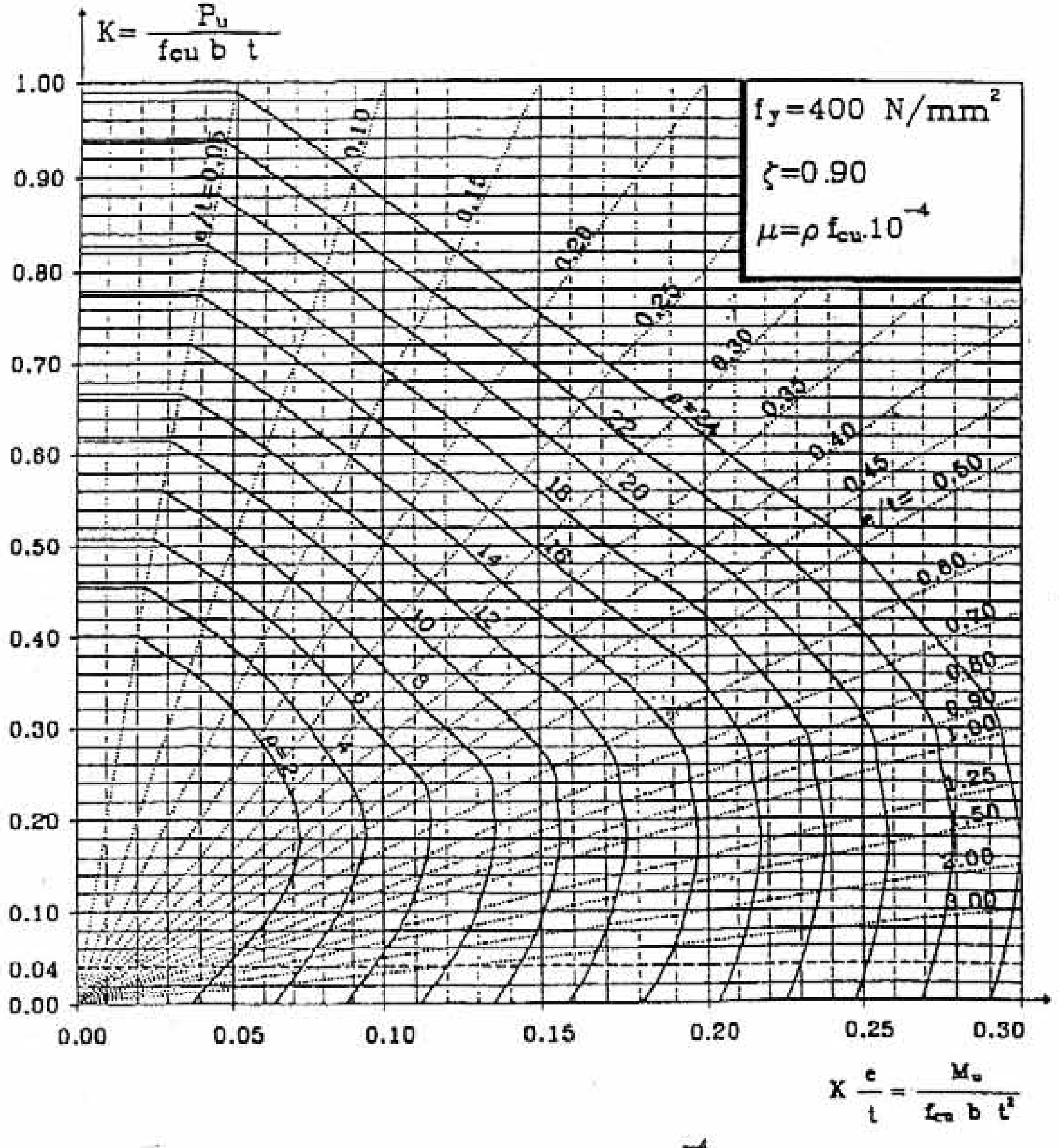
R–Sec.

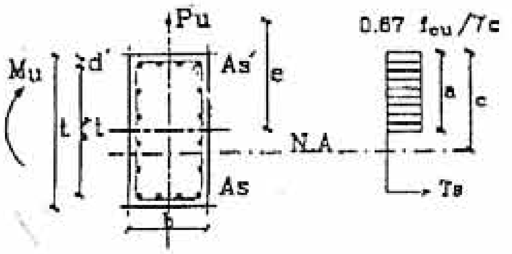
24/24

Chart (4-25) : INTERACTION DIAGRAMS

FOR DESIGN OF SECTIONS SUBJECTED TO ECCENTRIC COMP. FORCES

(Uniformly Distributed Steel)





$\mu=\rho\ f_{cu}.10^{-4}$

$A_{s\,(total)}=\mu\ b\ t$

$\zeta=\frac{d-d'}{t}$

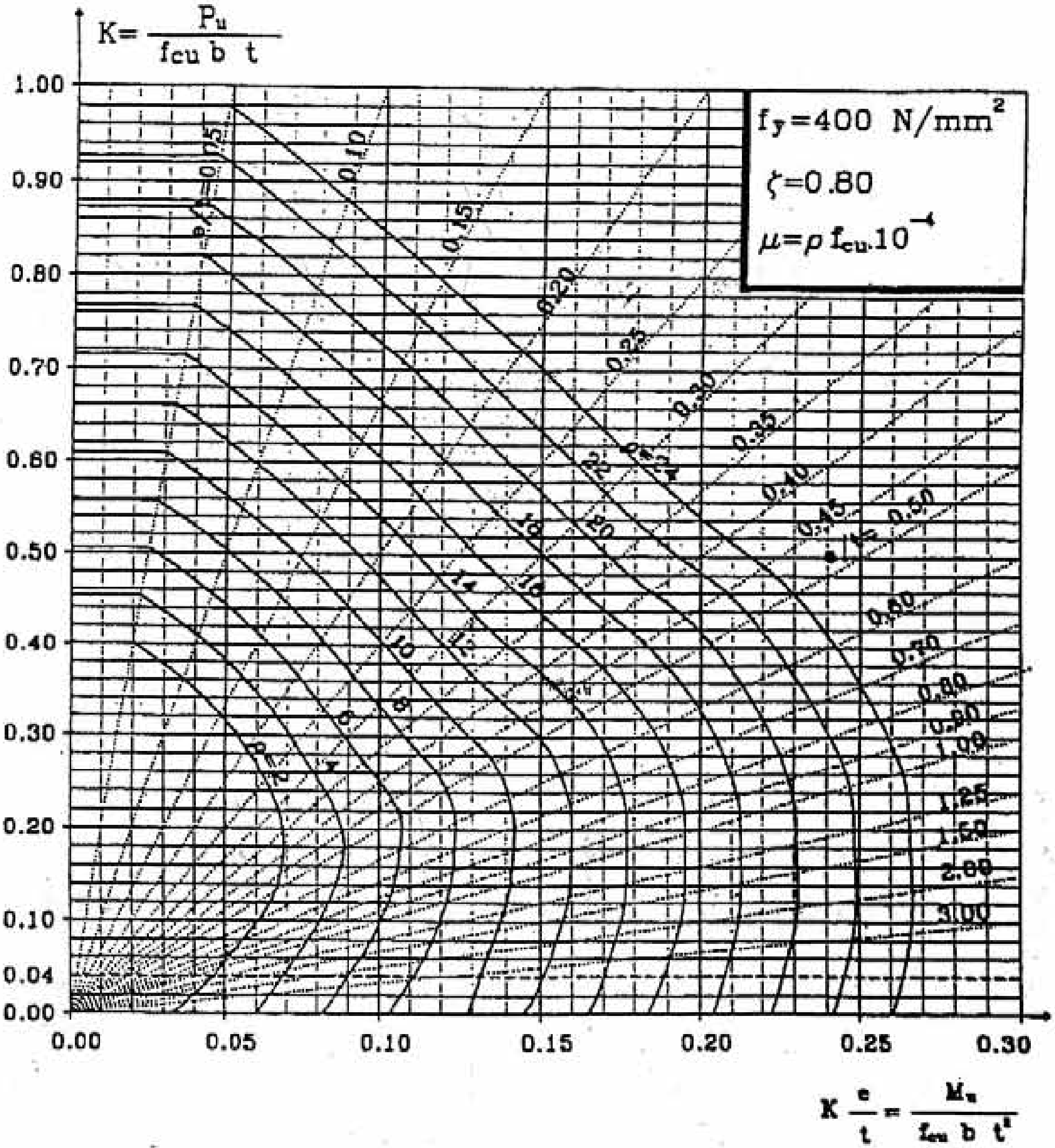
R–Sec.

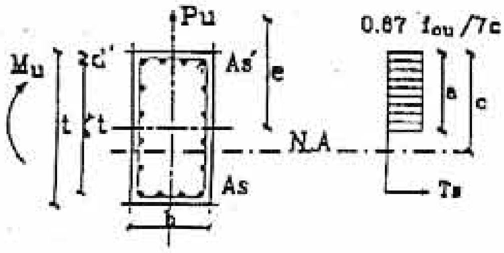
1/8

Chart (4-26) : INTERACTION DIAGRAMS

FOR DESIGN OF SECTIONS SUBJECTED TO ECCENTRIC COMP. FORCES

(Uniformly Distributed Steel)

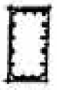




$\mu = \rho\, f_{cu} . 10^{-4}$

$A_s \ (\text{total}) = \mu\, b\, t$

$\zeta = \dfrac{d - d'}{t}$



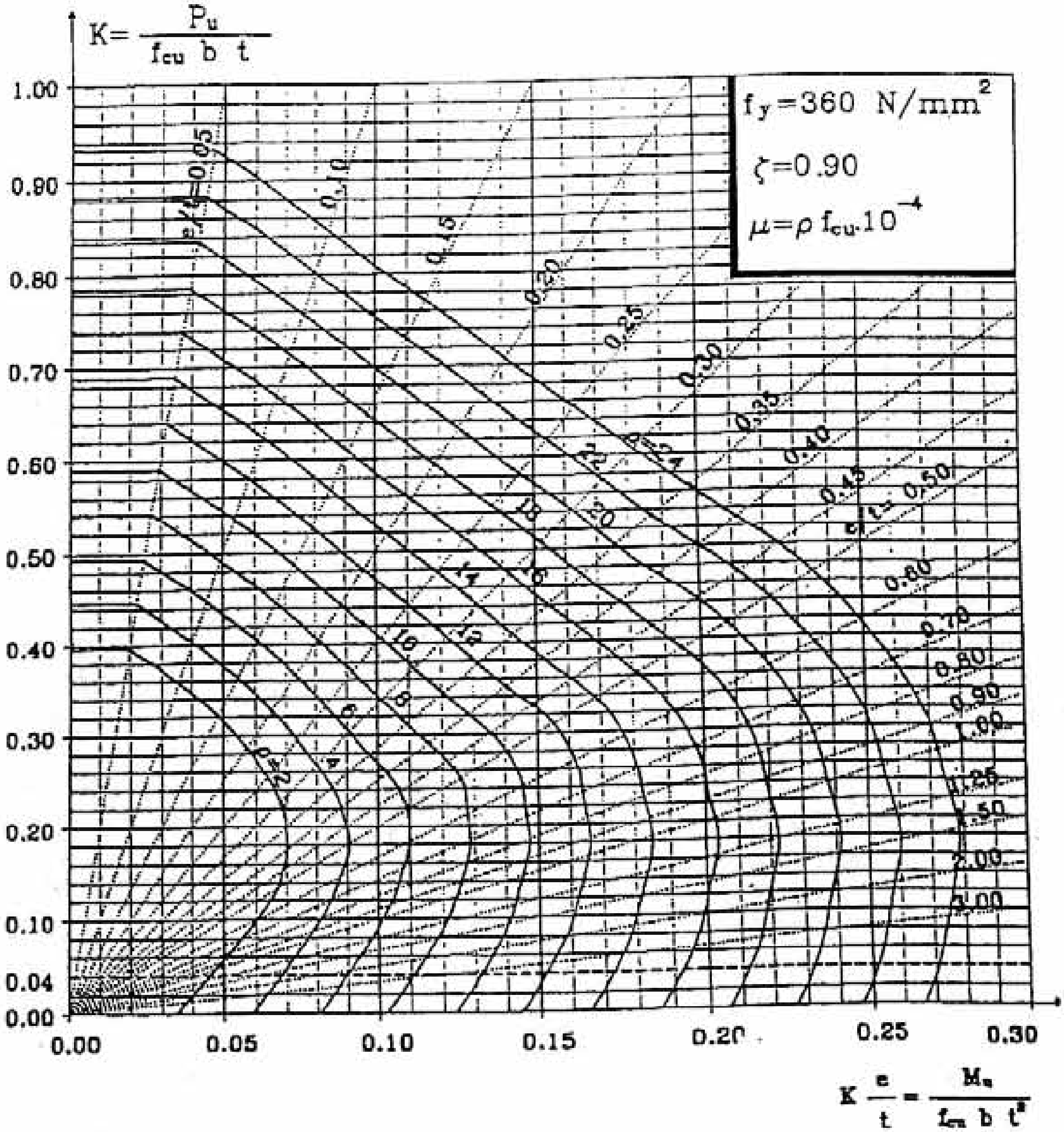
R–Sec.

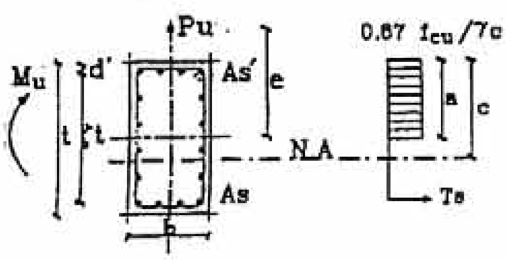
2/8

Chart (4-27) : INTERACTION DIAGRAMS

FOR DESIGN OF SECTIONS SUBJECTED TO ECCENTRIC COMP. FORCES

(Uniformly Distributed Steel)





$\mu = \rho\ f_{cu} . 10^{-4}$

$As_{(total)} = \mu\ b\ t$

$\zeta = \frac{d - d'}{t}$

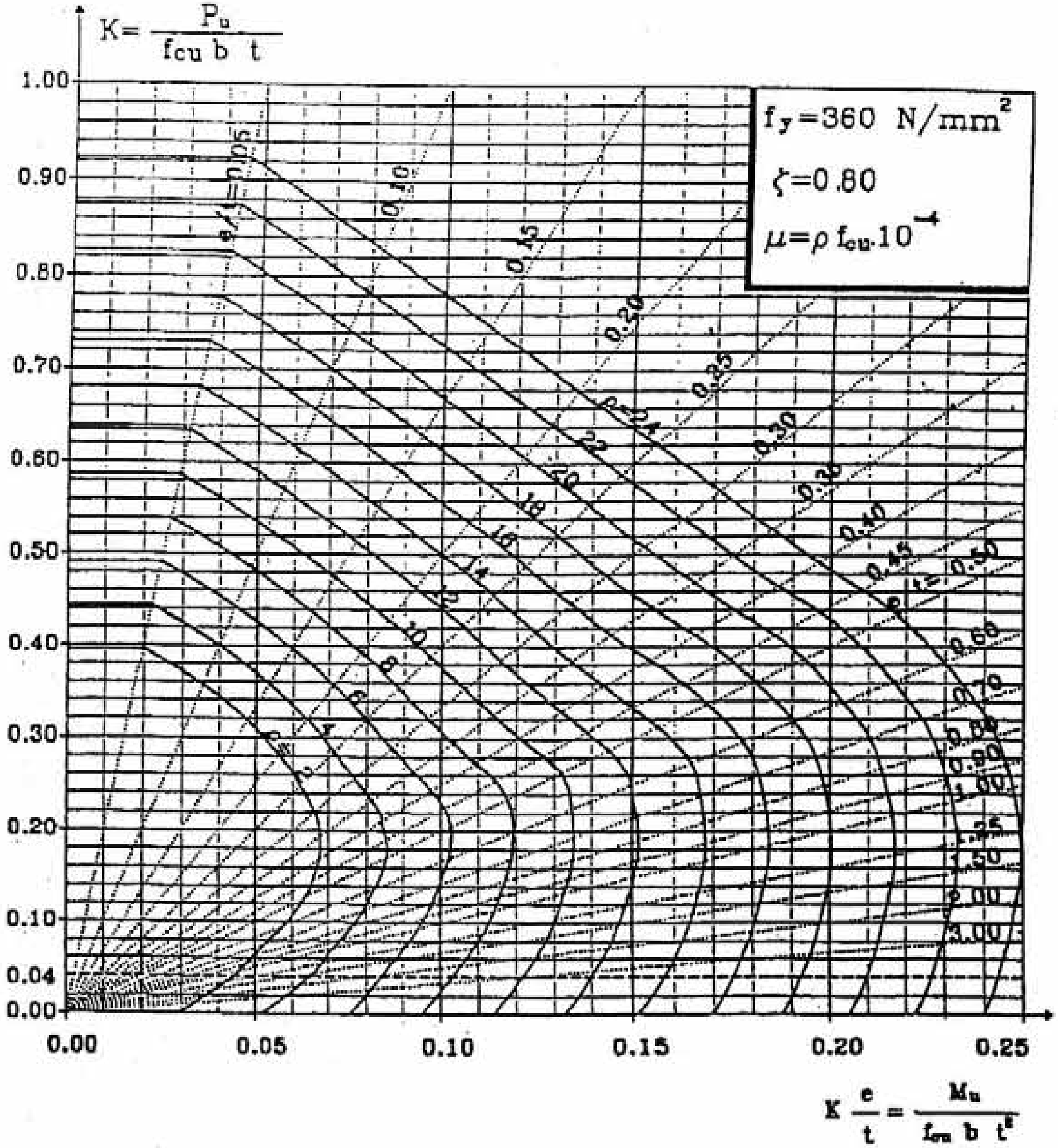
R-Sec.

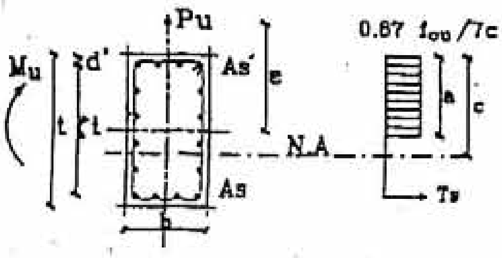
3/8

Chart (4-28) : INTERACTION DIAGRAMS

FOR DESIGN OF SECTIONS SUBJECTED TO ECCENTRIC COMP. FORCES

(Uniformly Distributed Steel)





$$\mu = \rho\, f_{cu} \cdot 10^{-4}$$

$$A_{s\,(total)} = \mu\, b\, t$$

$$\zeta = \frac{d - d'}{t}$$

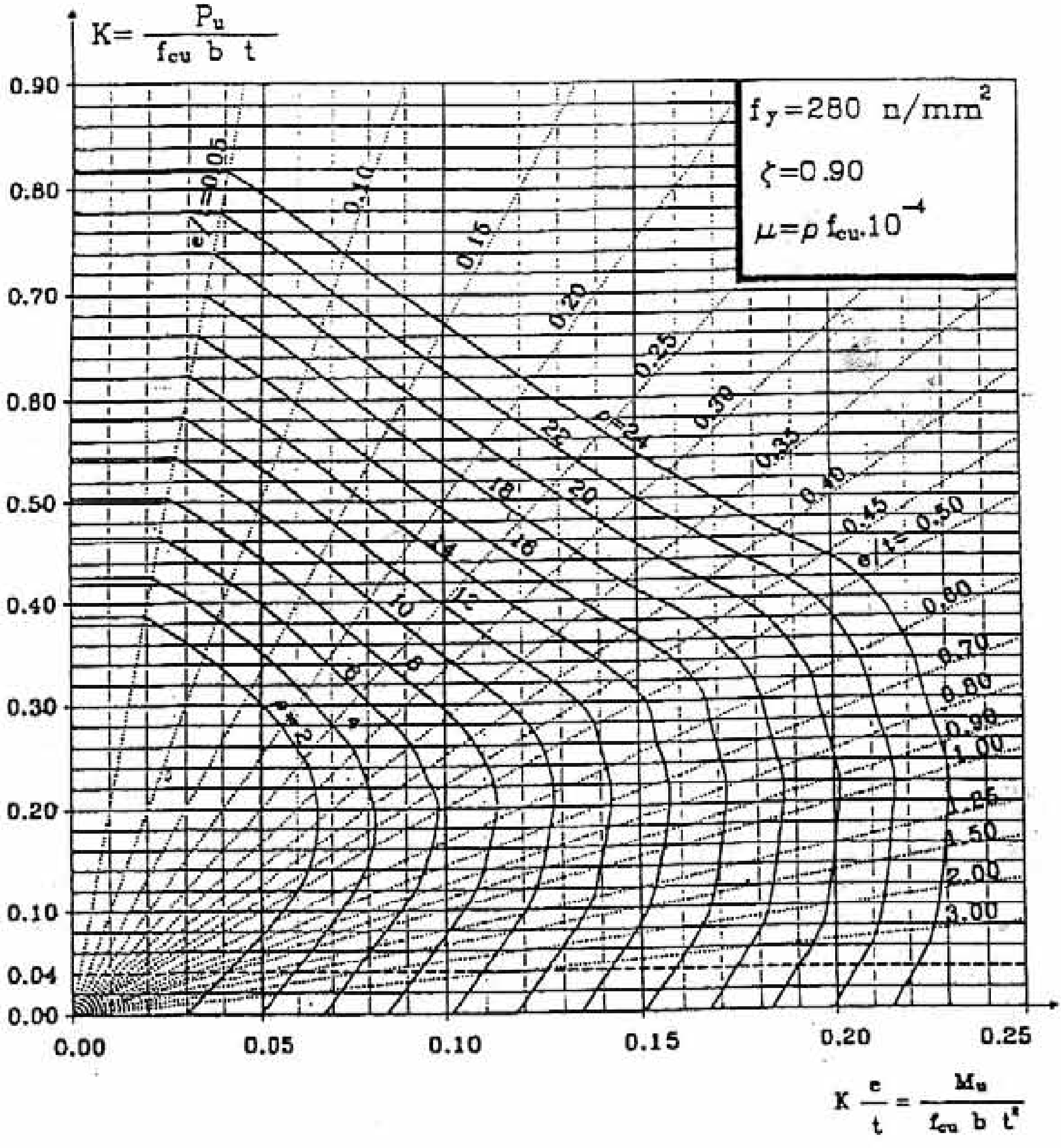
R–Sec.

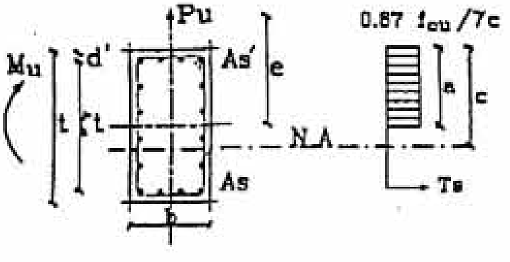
4/8

Chart (4-29) : INTERACTION DIAGRAMS

FOR DESIGN OF SECTIONS SUBJECTED TO ECCENTRIC COMP. FORCES

(Uniformly Distributed Steel)





$\mu=\rho\ f_{cu}.10^{-4}$

$As_{(total)}=\mu\ b\ t$

$\zeta=\frac{d-d'}{t}$

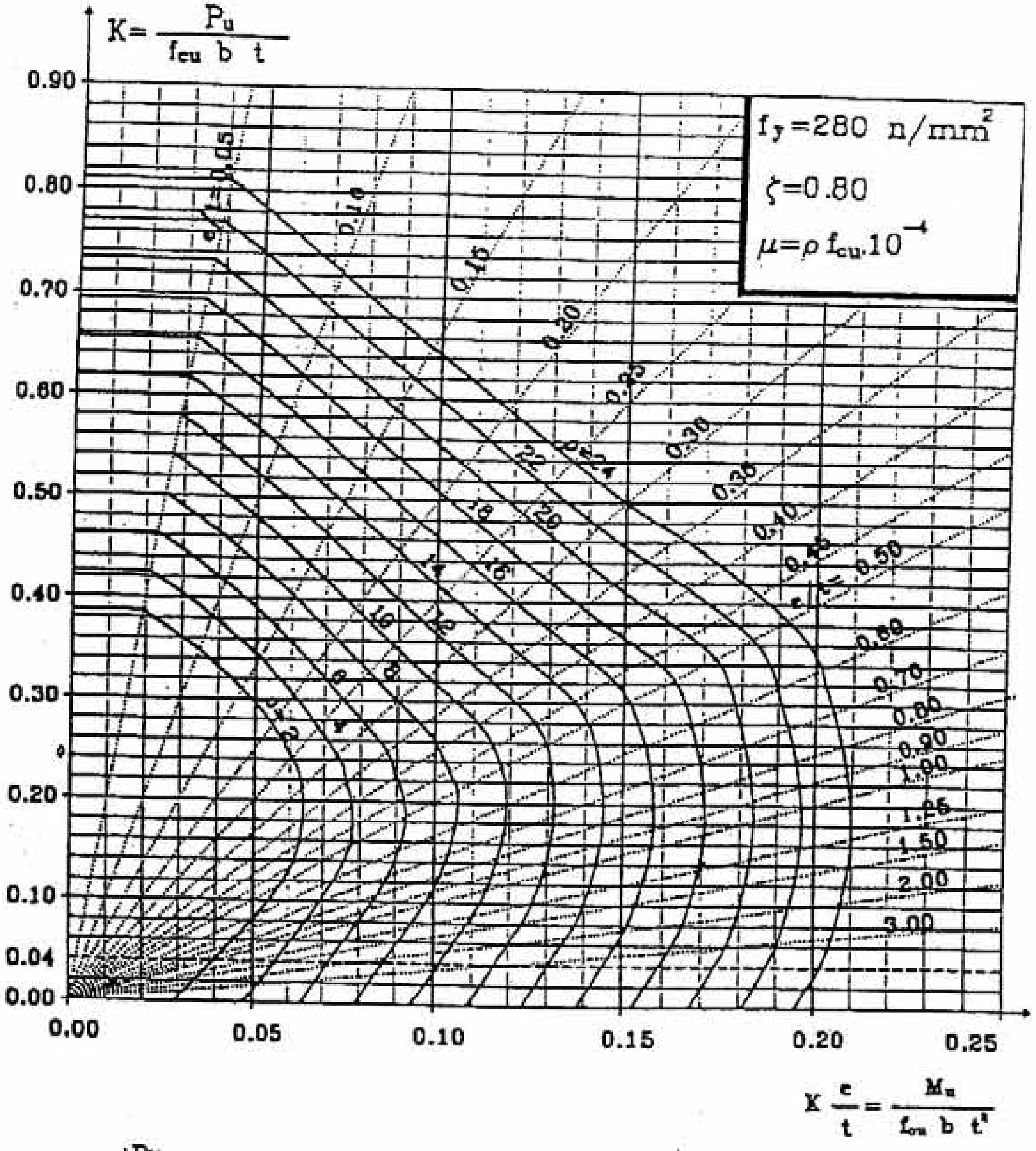
R-Sec.

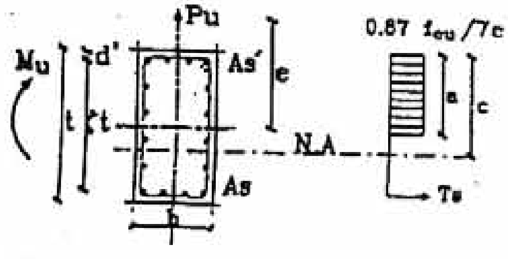
5/8

Chart (4-30): INTERACTION DIAGRAMS

FOR DESIGN OF SECTIONS SUBJECTED TO ECCENTRIC COMP. FORCES

(Uniformly Distributed Steel)

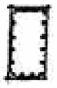




$\mu = \rho\ f_{cu}.10^{-4}$

$As_{(total)} = \mu\ b\ t$

$\zeta = \frac{d - d'}{t}$



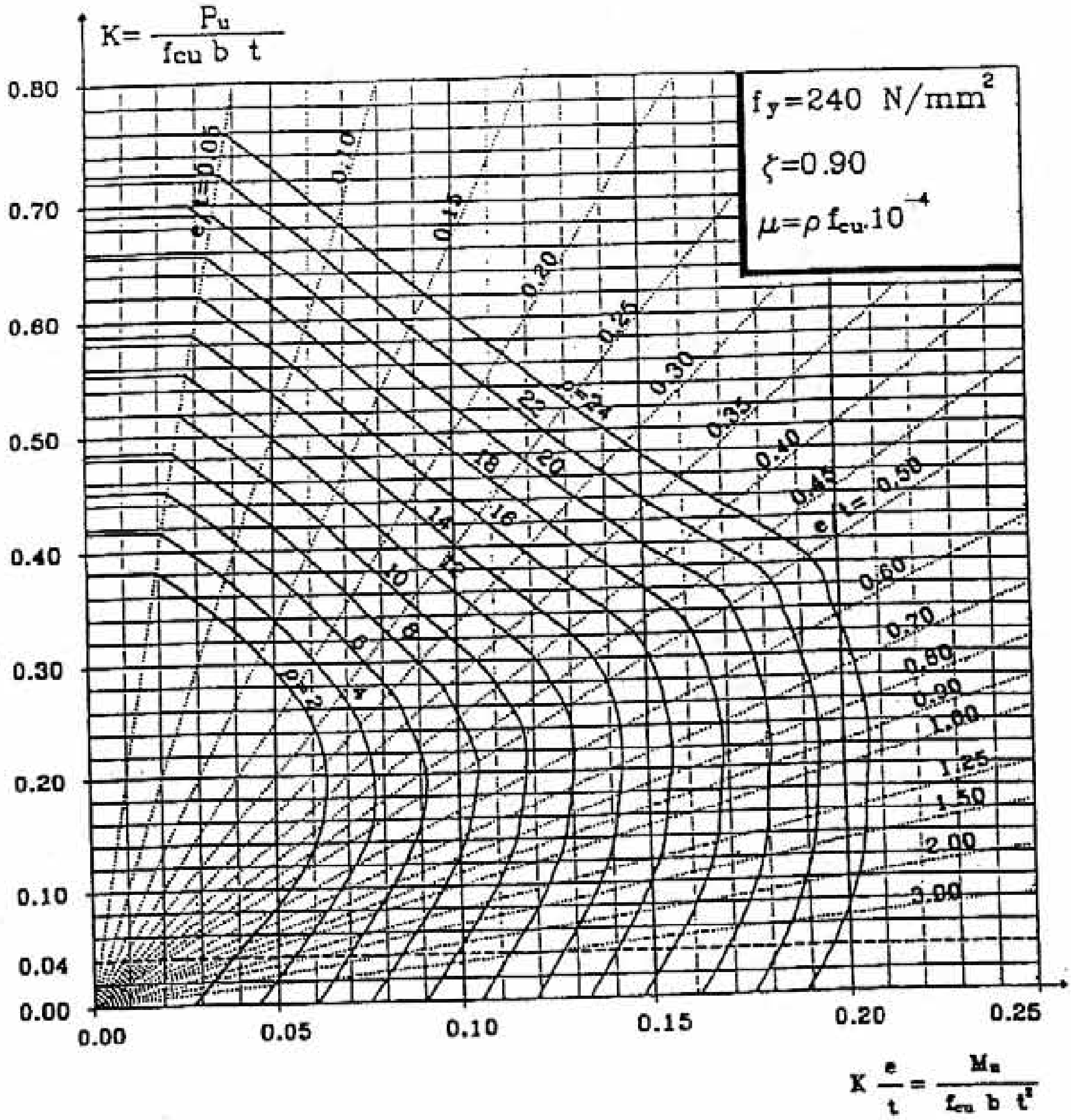
R-Sec.

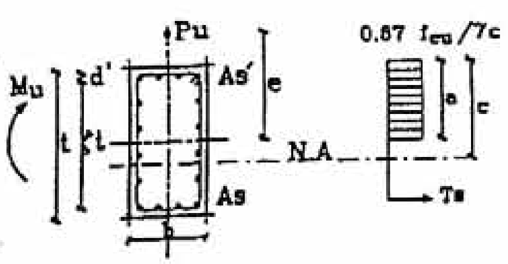
6/8

Chart (4-31): INTERACTION DIAGRAMS

FOR DESIGN OF SECTIONS SUBJECTED TO ECCENTRIC COMP. FORCES

(Uniformly Distributed Steel)

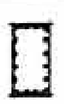




$\mu = \rho\, f_{cu} . 10^{-4}$

$As_{(total)} = \mu\, b\; t$

$\zeta = \dfrac{d - d'}{t}$



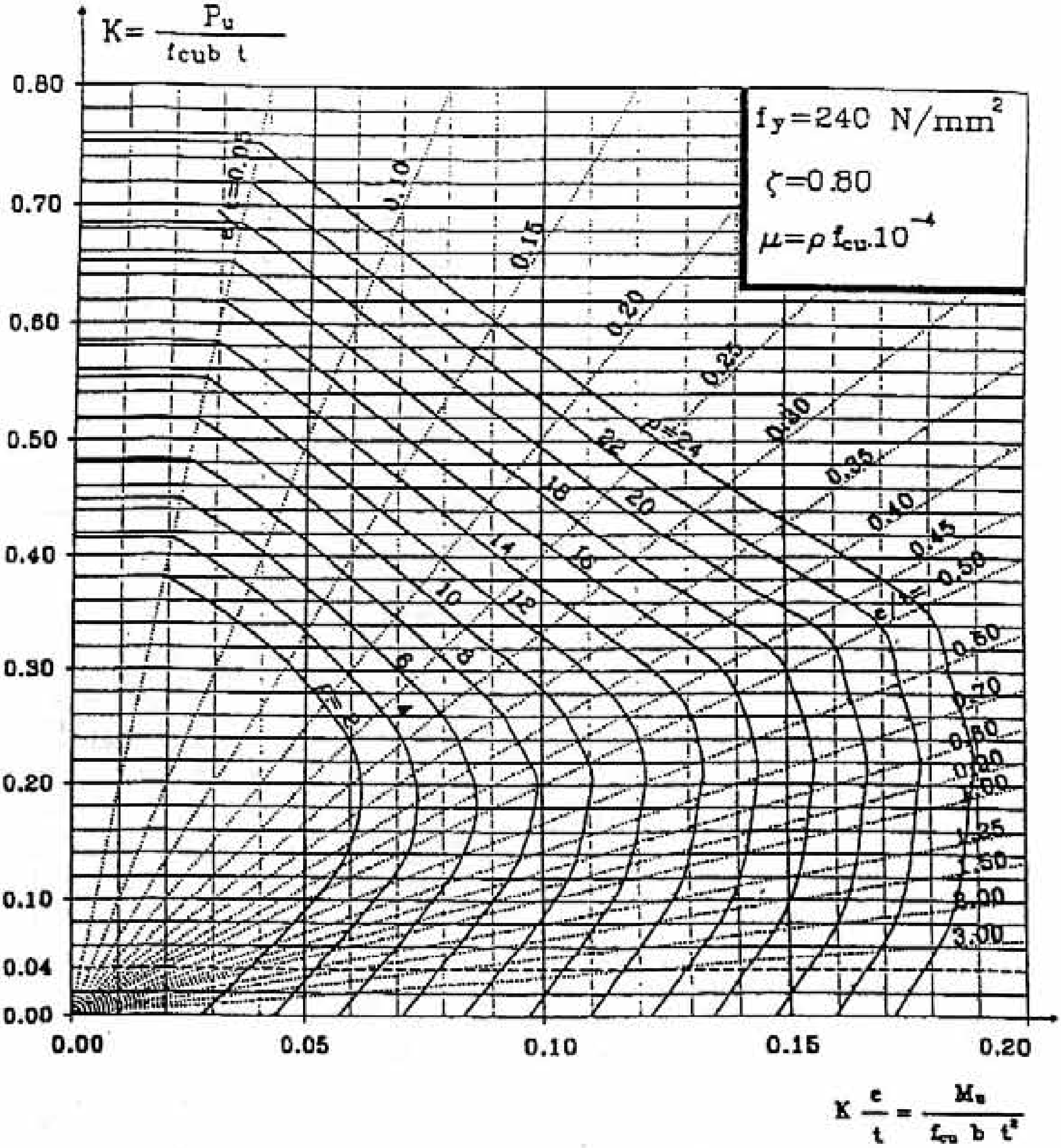
R–Sec.

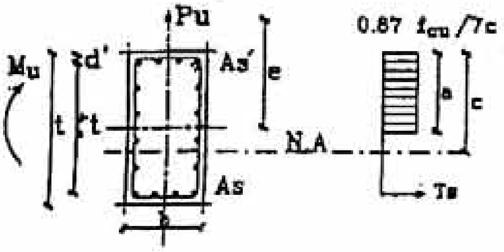
7/8

Chart (4-32): INTERACTION DIAGRAMS

FOR DESIGN OF SECTIONS SUBJECTED TO ECCENTRIC COMP. FORCES

(Uniformly Distributed Steel)





$$\mu = \rho\, f_{cu} . 10^{-4}$$

$$As = \mu\, b\, t$$

(total)

$$\zeta = \frac{d - d'}{t}$$

R–Sec.

8 /8

Chart (4-33): INTERACTION DIAGRAMS

FOR DESIGN OF SECTIONS SUBJECTED TO ECCENTRIC COMP. FORCES

(Circular Sections)

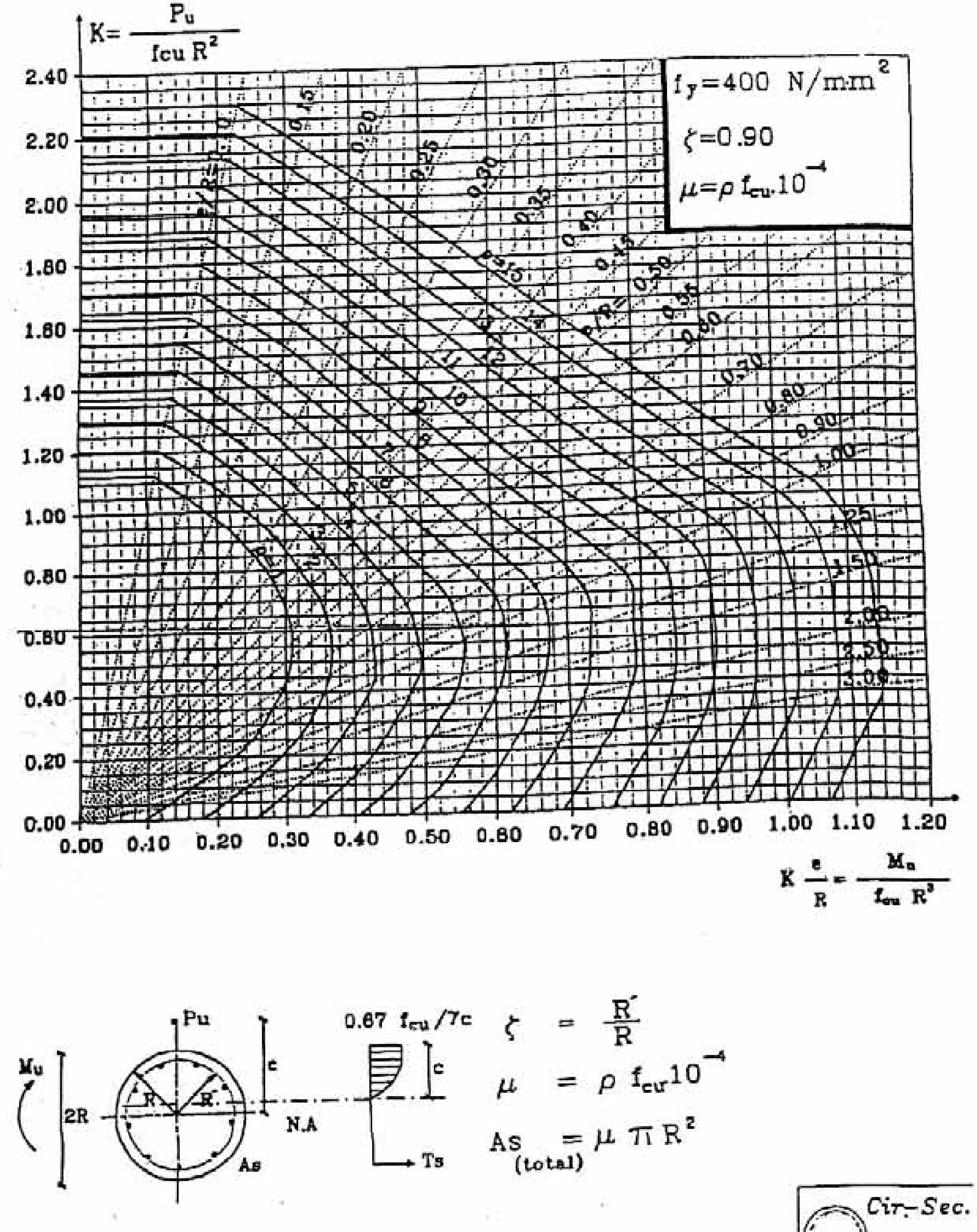
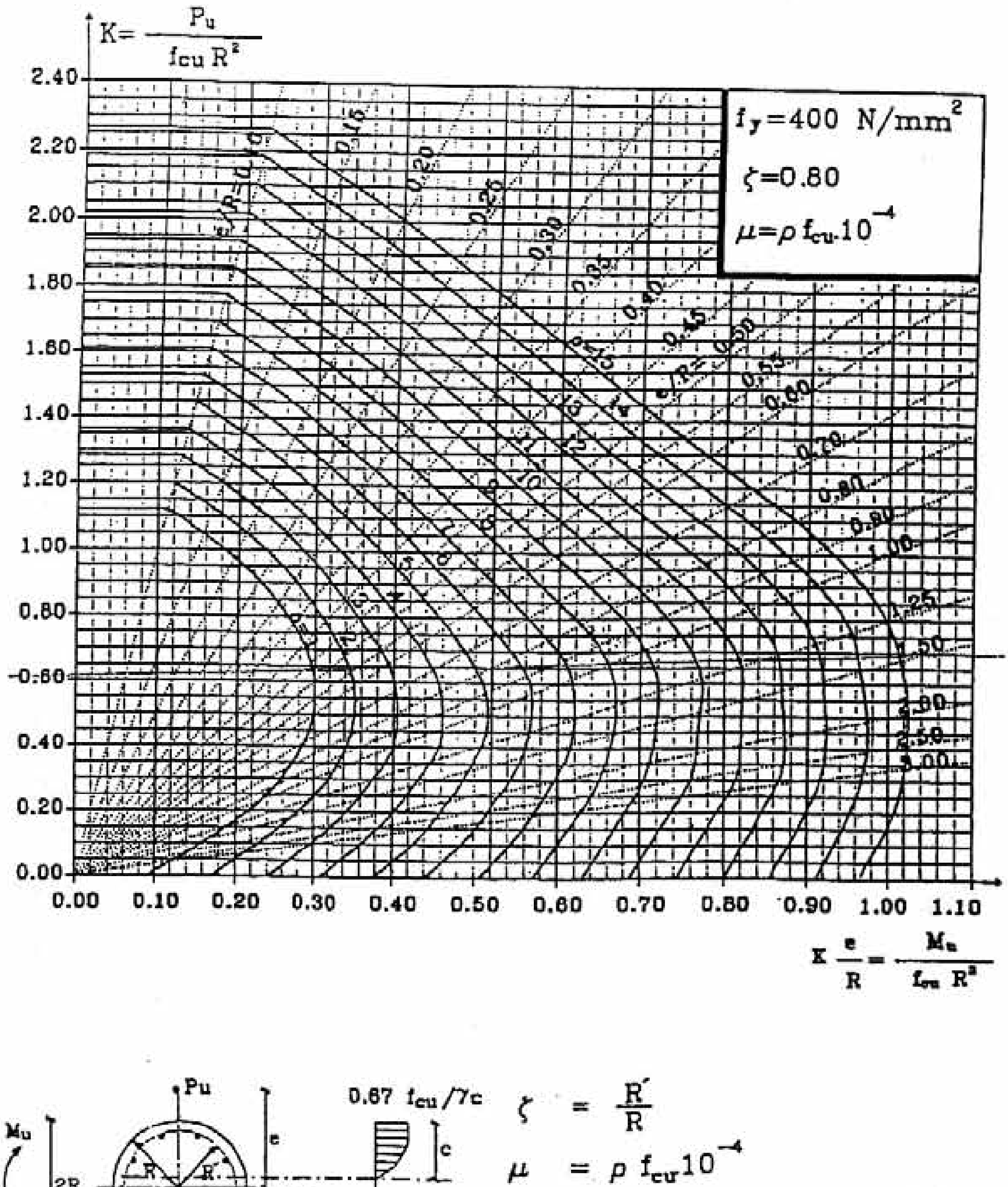
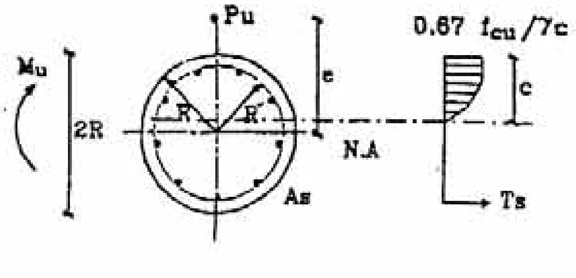


Chart (4-34): INTERACTION DIAGRAMS

FOR DESIGN OF SECTIONS SUBJECTED TO ECCENTRIC COMP. FORCES

(Circular Sections)





$\zeta = \frac{R'}{R}$

$\mu = \rho f_{cu} 10^{-4}$

$As_{(total)} = \mu \pi R^2$

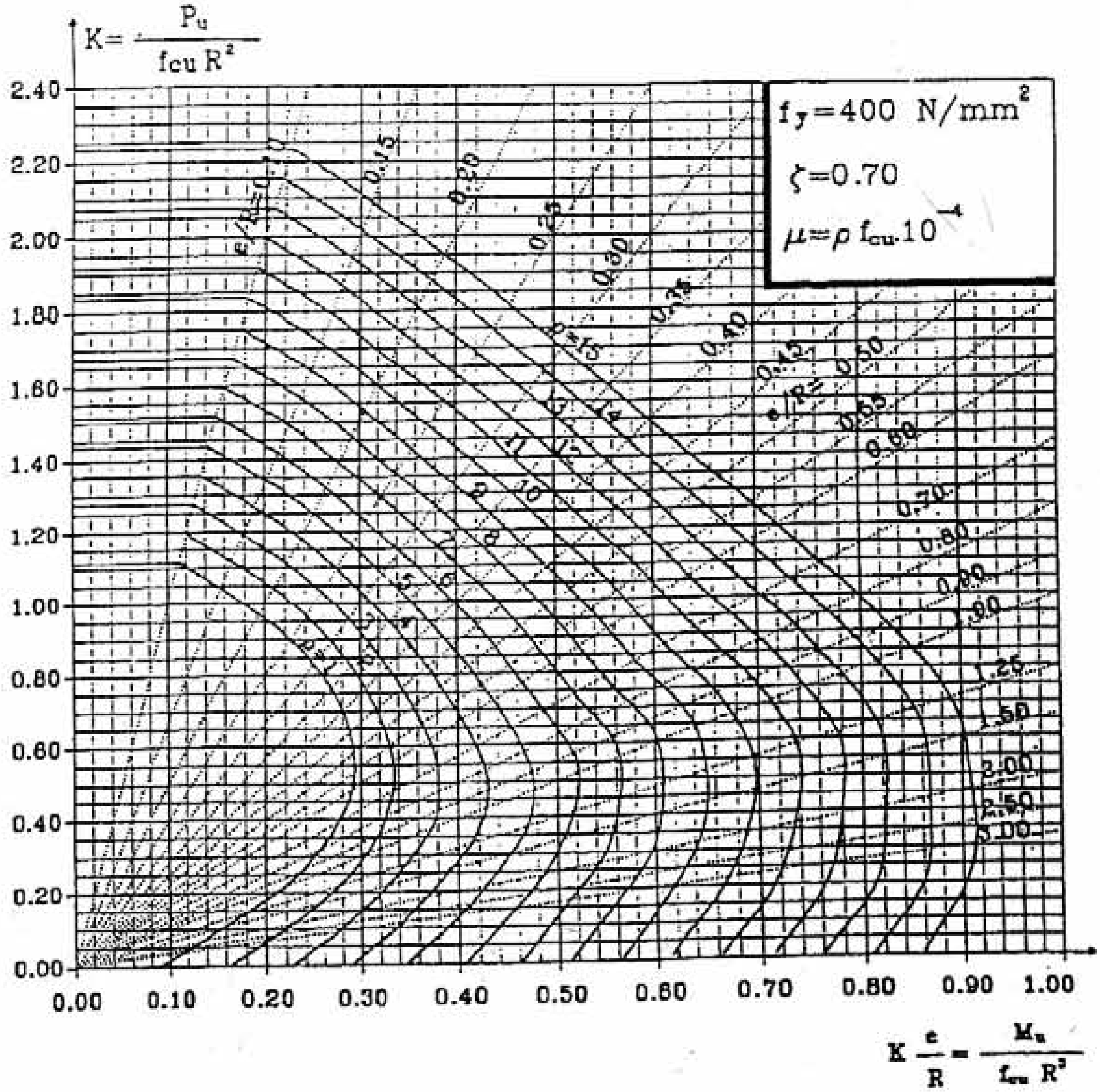
Cir–Sec.

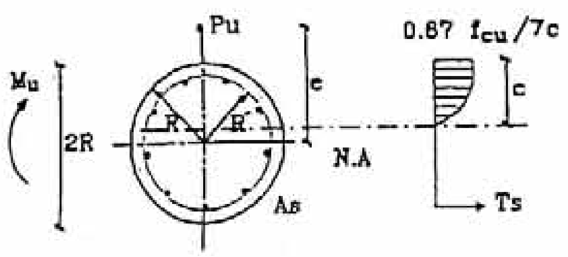
2/12

Chart (4-35): INTERACTION DIAGRAMS

FOR DESIGN OF SECTIONS SUBJECTED TO ECCENTRIC COMP. FORCES

(Circular Sections)





$\zeta = \frac{R'}{R}$

$\mu = \rho \, f_{cu} 10^{-4}$

$As_{(total)} = \mu \, \pi \, R^2$

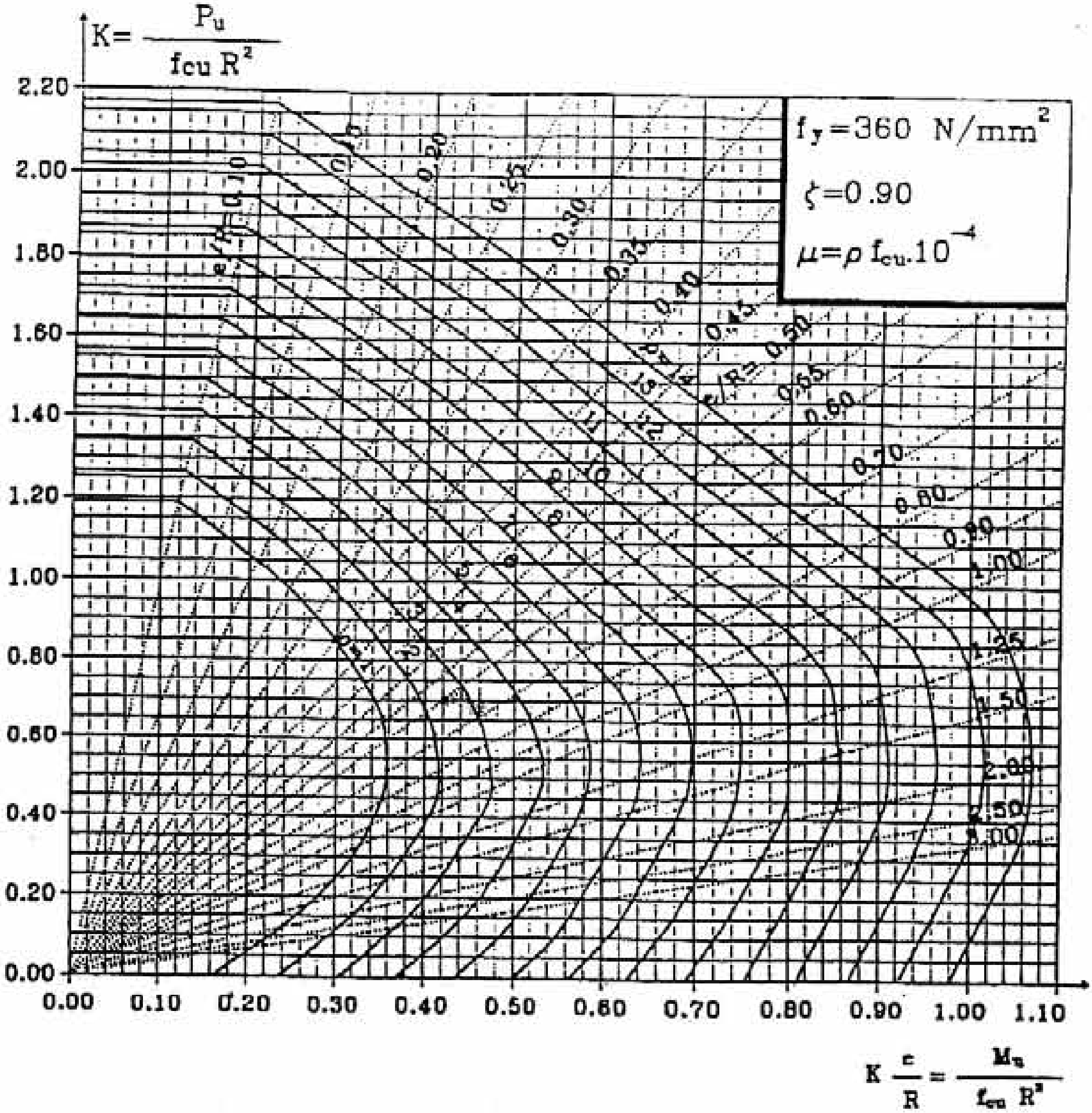
Cir–Sec.

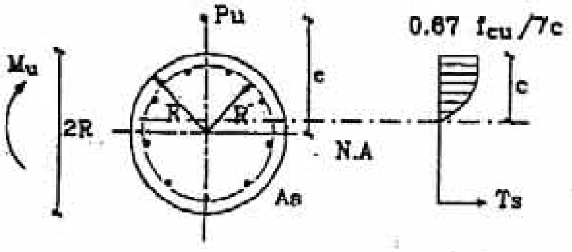
3/12

Chart (4-36): INTERACTION DIAGRAMS

FOR DESIGN OF SECTIONS SUBJECTED TO ECCENTRIC COMP. FORCES

(Circular Sections)





$\zeta = \frac{R'}{R}$

$\mu = \rho\ f_{cu} 10^{-4}$

$As_{(total)} = \mu\ \pi R^2$

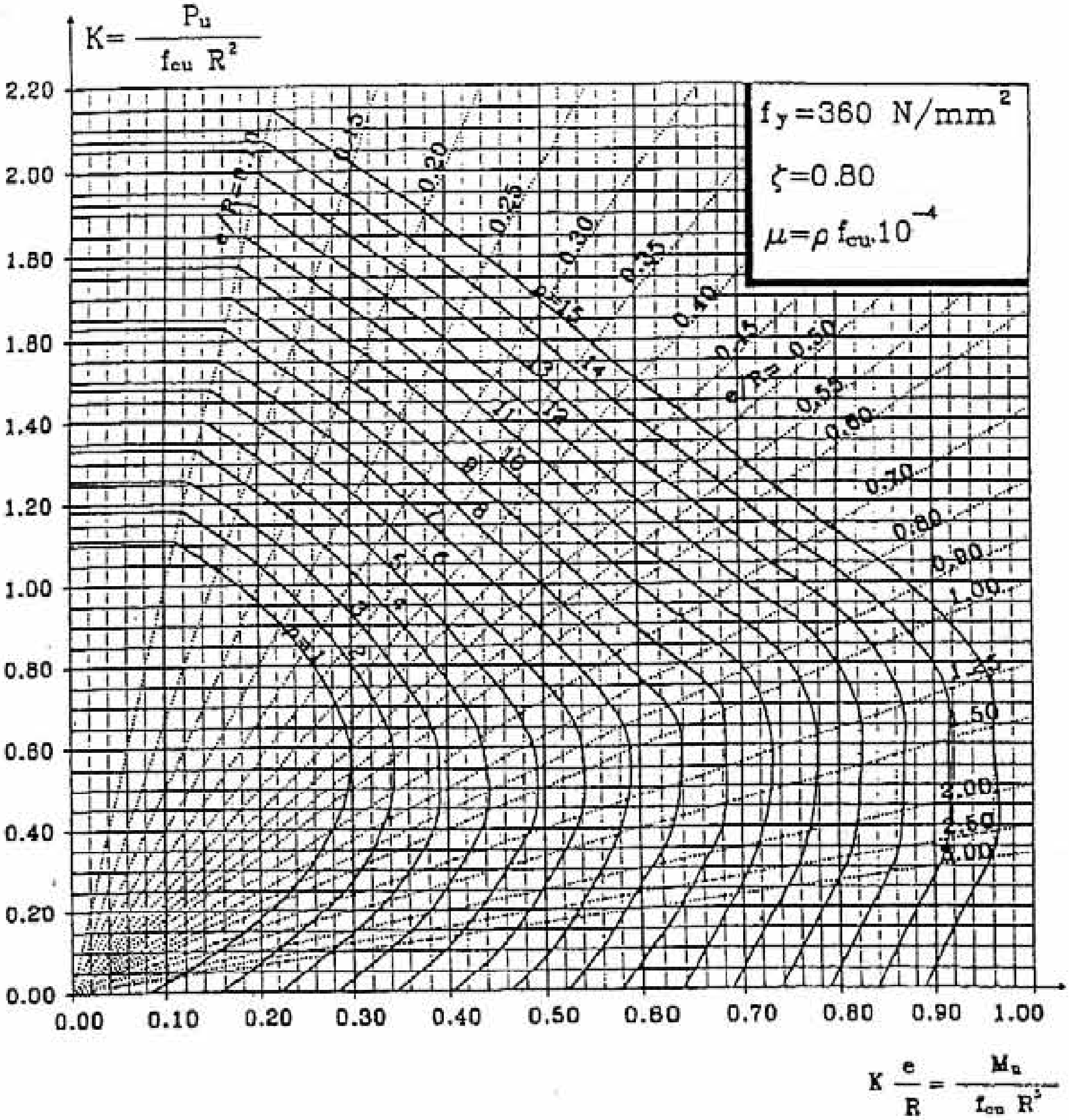
Cir–Sec.

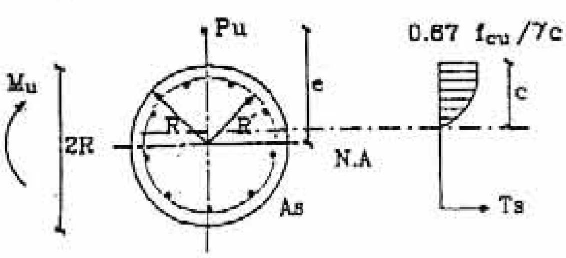
4/12

Chart (4-37): INTERACTION DIAGRAMS

FOR DESIGN OF SECTIONS SUBJECTED TO ECCENTRIC COMP. FORCES

(Circular Sections)





$\zeta = \frac{R'}{R}$

$\mu = \rho \, f_{cu} 10^{-4}$

$A_{s\,(total)} = \mu \, \pi R^2$

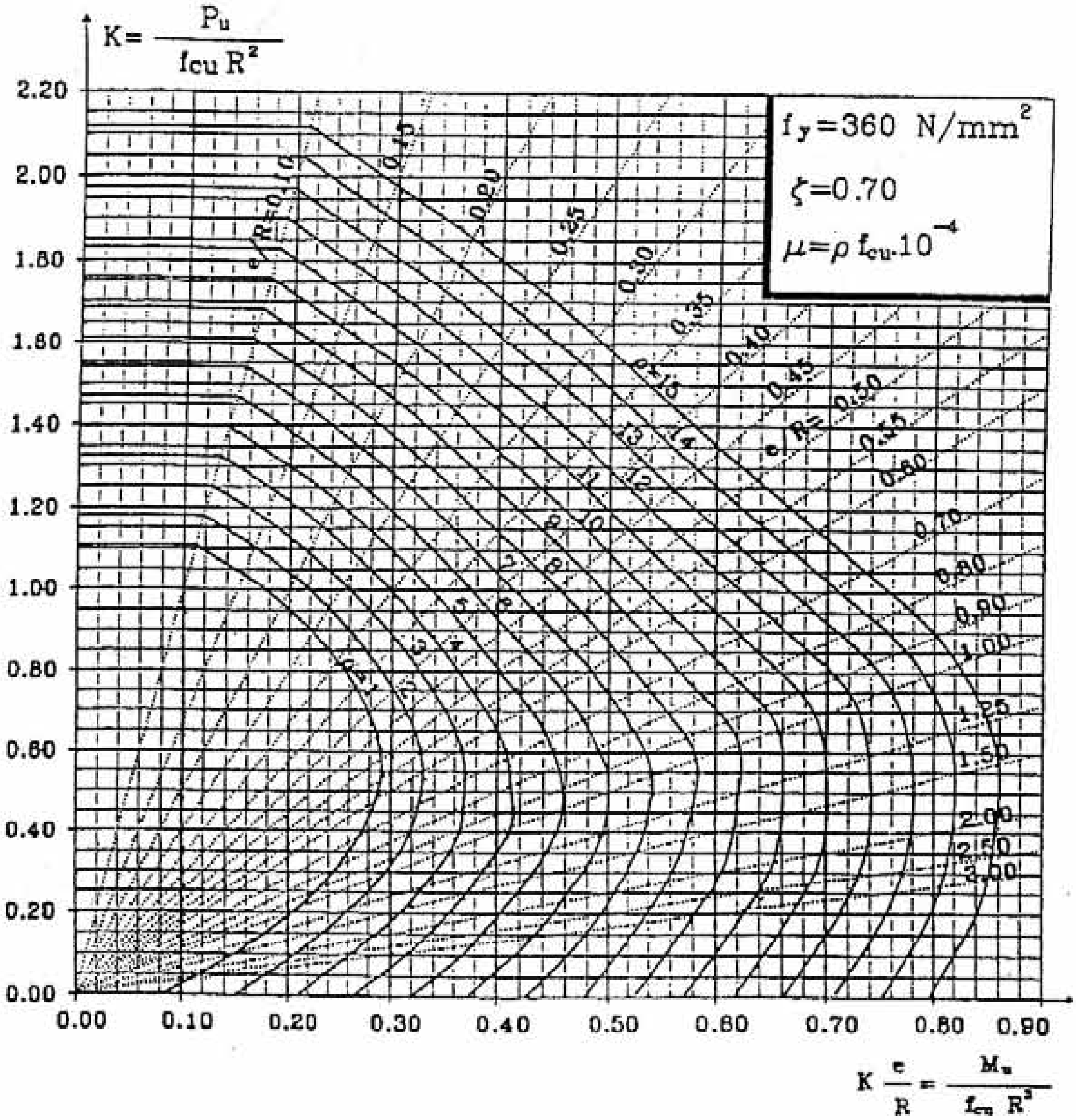
Cir–Sec.

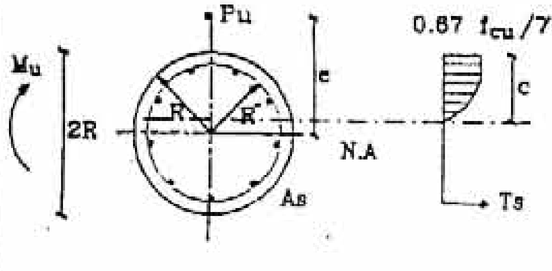
5/12

Chart (4-38): INTERACTION DIAGRAMS

FOR DESIGN OF SECTIONS SUBJECTED TO ECCENTRIC COMP. FORCES

(Circular Sections)





$\zeta = \frac{R'}{R}$

$\mu = \rho\ f_{cu} 10^{-4}$

$As_{(total)} = \mu\ \pi\ R^2$

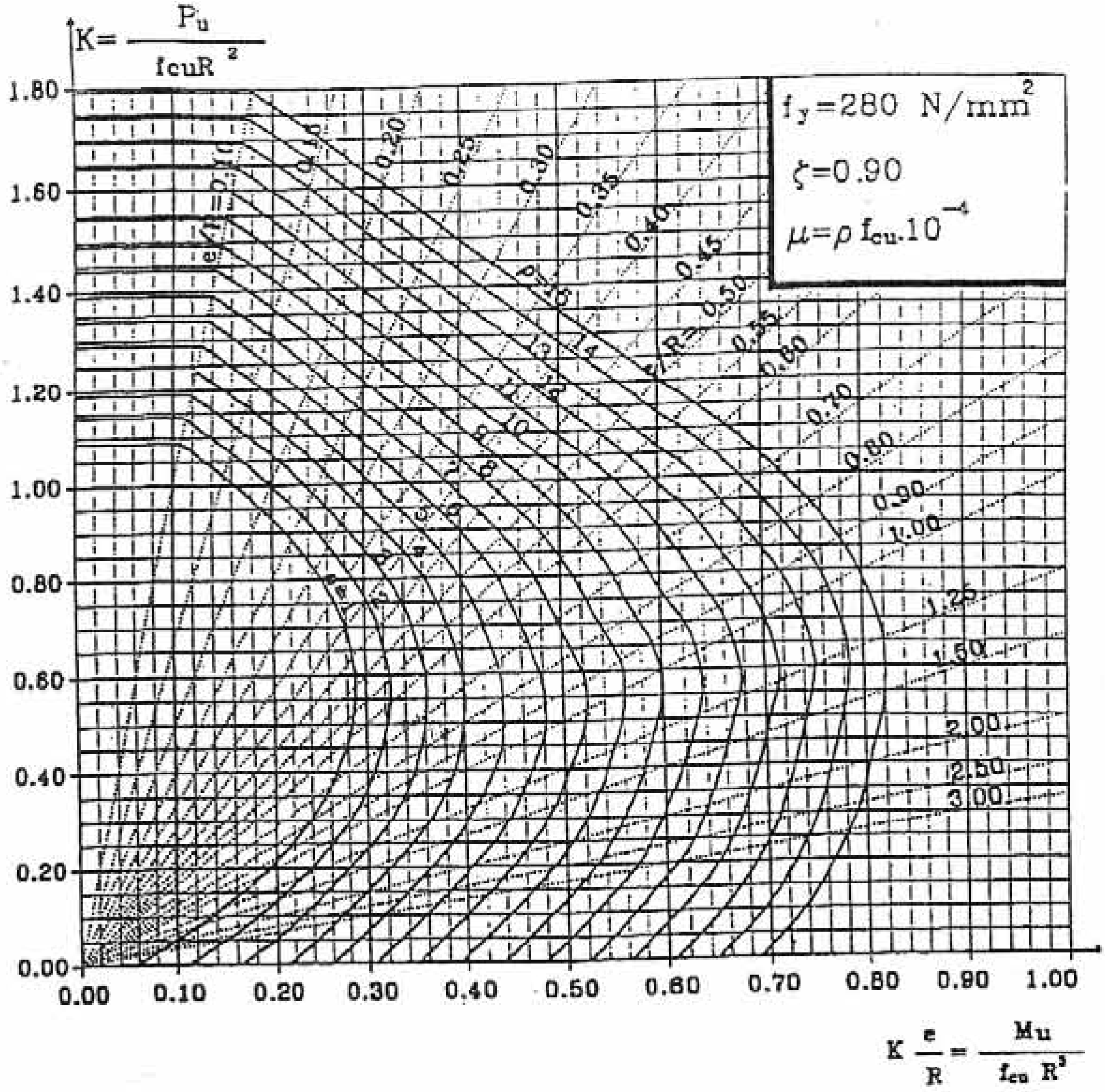
Cir–Sec.

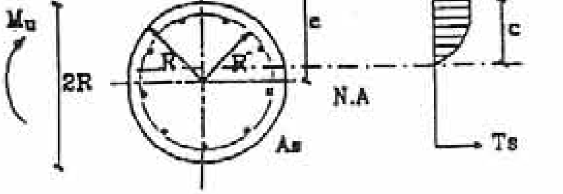
6 / 12

Chart (4-39): INTERACTION DIAGRAMS

FOR DESIGN OF SECTIONS SUBJECTED TO ECCENTRIC COMP. FORCES

(Circular Sections)





$$\zeta = \frac{R'}{R}$$

$$\mu = \rho\ f_{cu} 10^{-4}$$

$$As_{(total)} = \mu\, \pi\, R^2$$

Cir–Sec.

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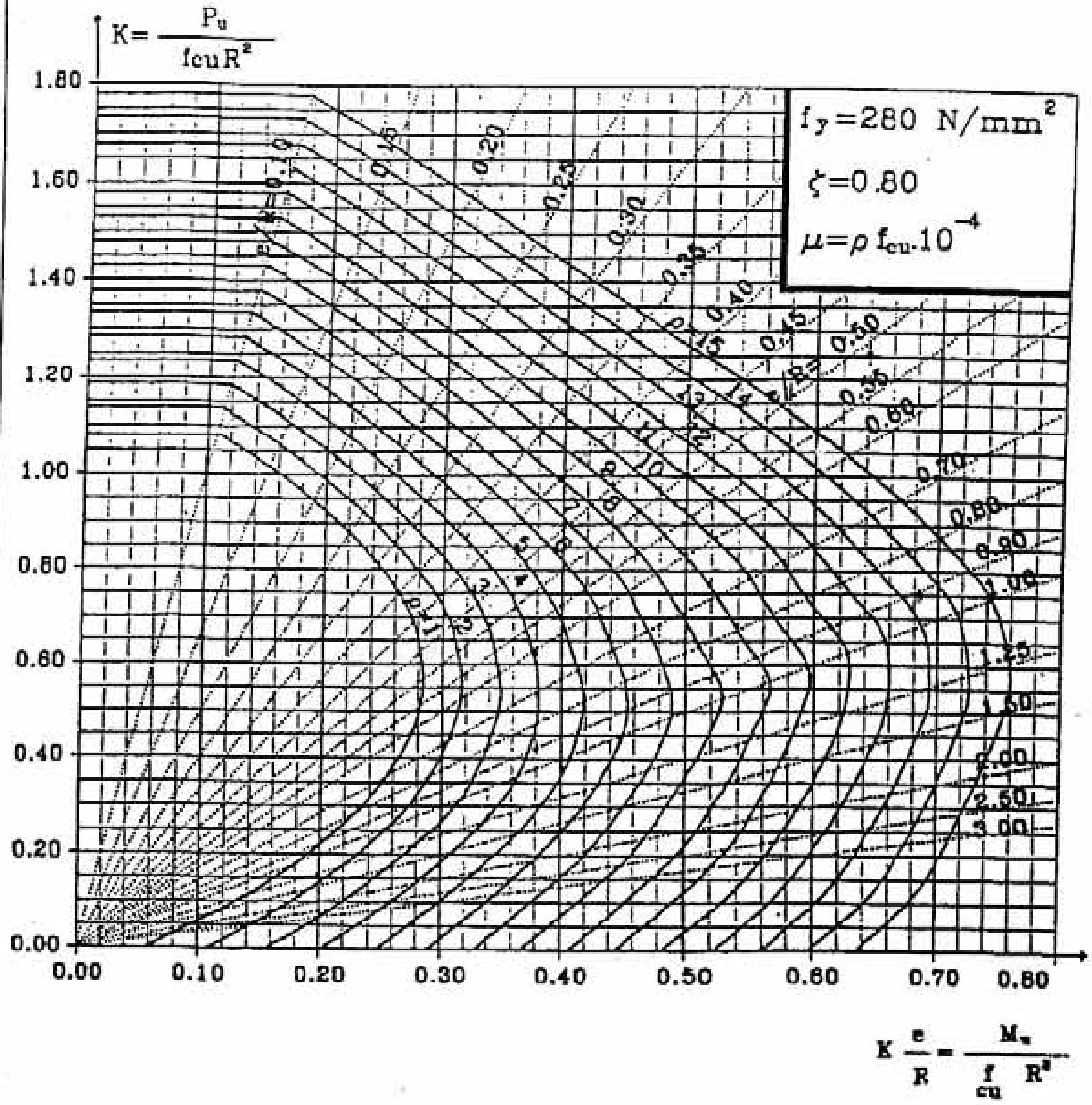
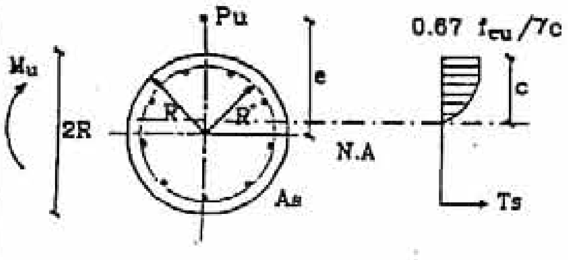
Chart (4-40): INTERACTION DIAGRAMS

FOR DESIGN OF SECTIONS SUBJECTED TO ECCENTRIC COMP. FORCES

(Circular Sections)





$\zeta = \frac{R'}{R}$

$\mu = \rho\ f_{cu} 10^{-4}$

$As_{(total)} = \mu\ \pi\ R^2$

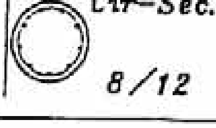
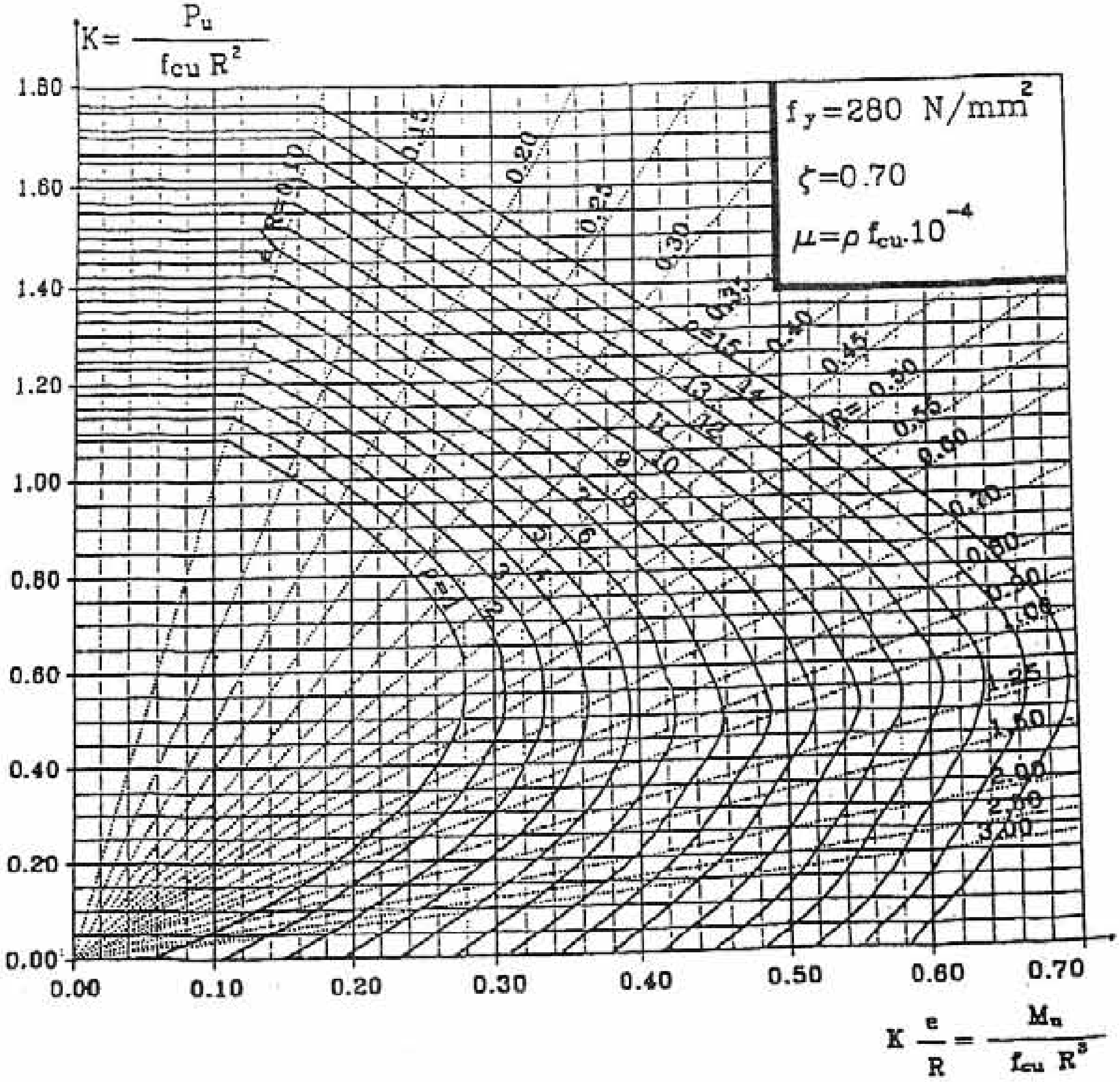
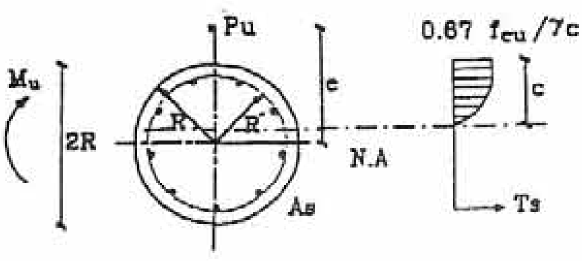


Chart (4-41): INTERACTION DIAGRAMS

FOR DESIGN OF SECTIONS SUBJECTED TO ECCENTRIC COMP. FORCES

(Circular Sections)





$\zeta = \frac{R'}{R}$

$\mu = \rho\ f_{cu} 10^{-4}$

$As_{(total)} = \mu\ \pi\ R^2$

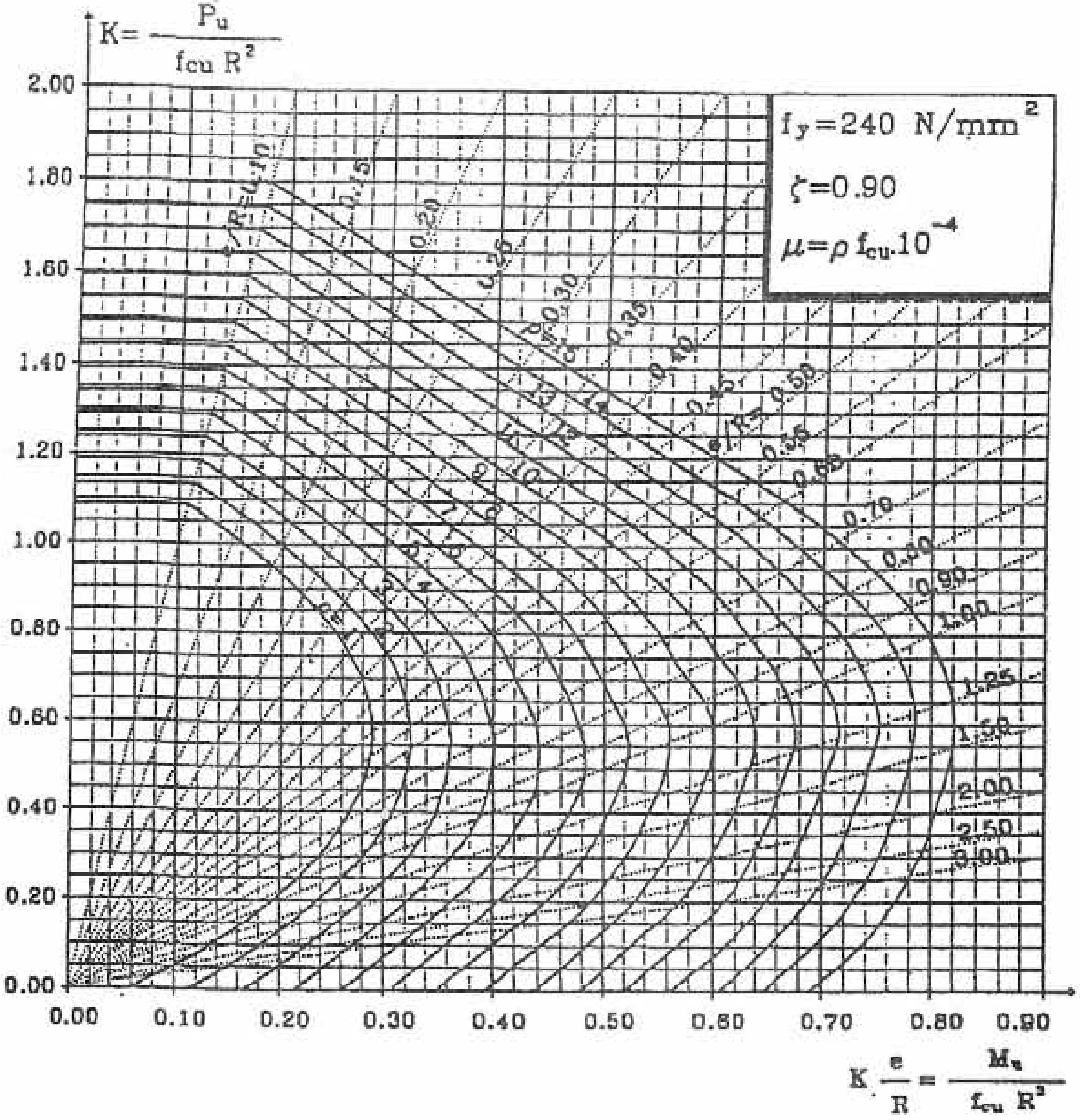
Cir-Sec.

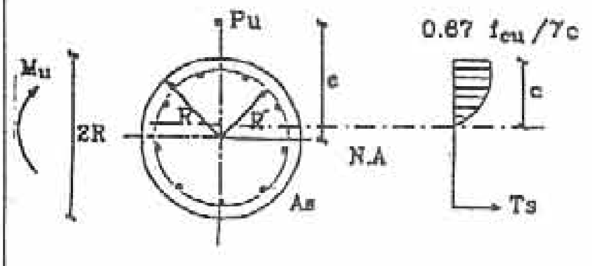
9 / 12

Chart (4-42): INTERACTION DIAGRAMS

FOR DESIGN OF SECTIONS SUBJECTED TO ECCENTRIC COMP. FORCES

(Circular Sections)





$\zeta = \frac{R'}{R}$

$\mu = \rho\ f_{cu} 10^{-4}$

$As_{(total)} = \mu\ \pi\ R^2$

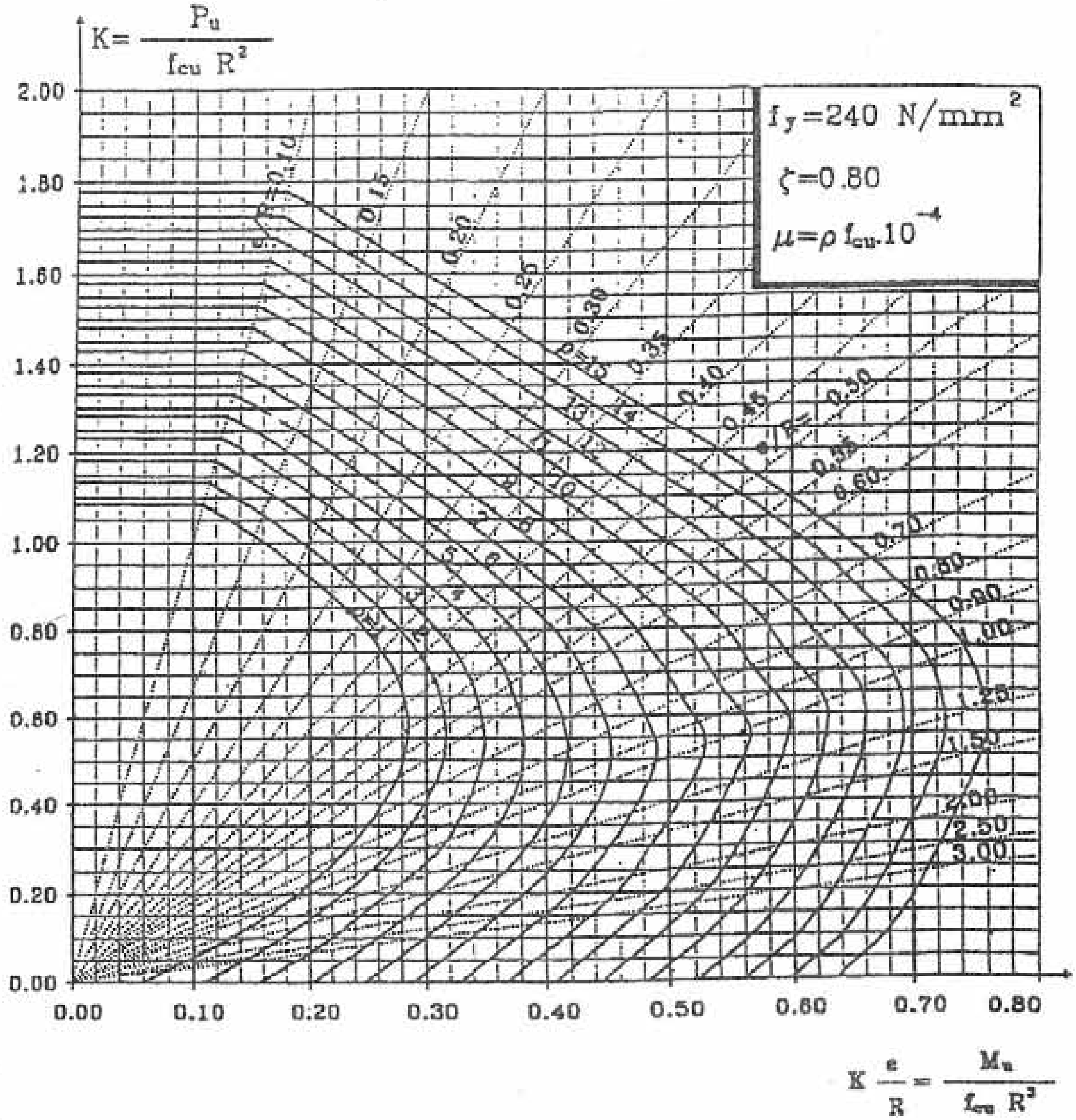
Cir–Sec.

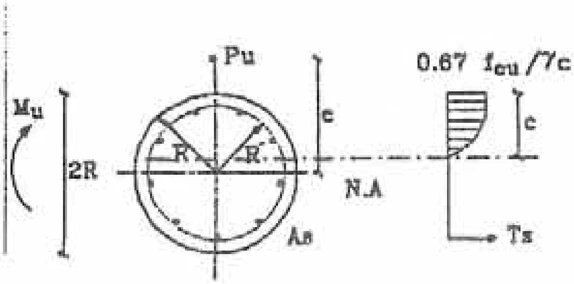
10/12

Chart (4-43): INTERACTION DIAGRAMS

FOR DESIGN OF SECTIONS SUBJECTED TO ECCENTRIC COMP. FORCES

(Circular Sections)





$\zeta = \frac{R'}{R}$

$\mu = \rho\ f_{cu} 10^{-4}$

$As_{(total)} = \mu\ \pi\ R^2$

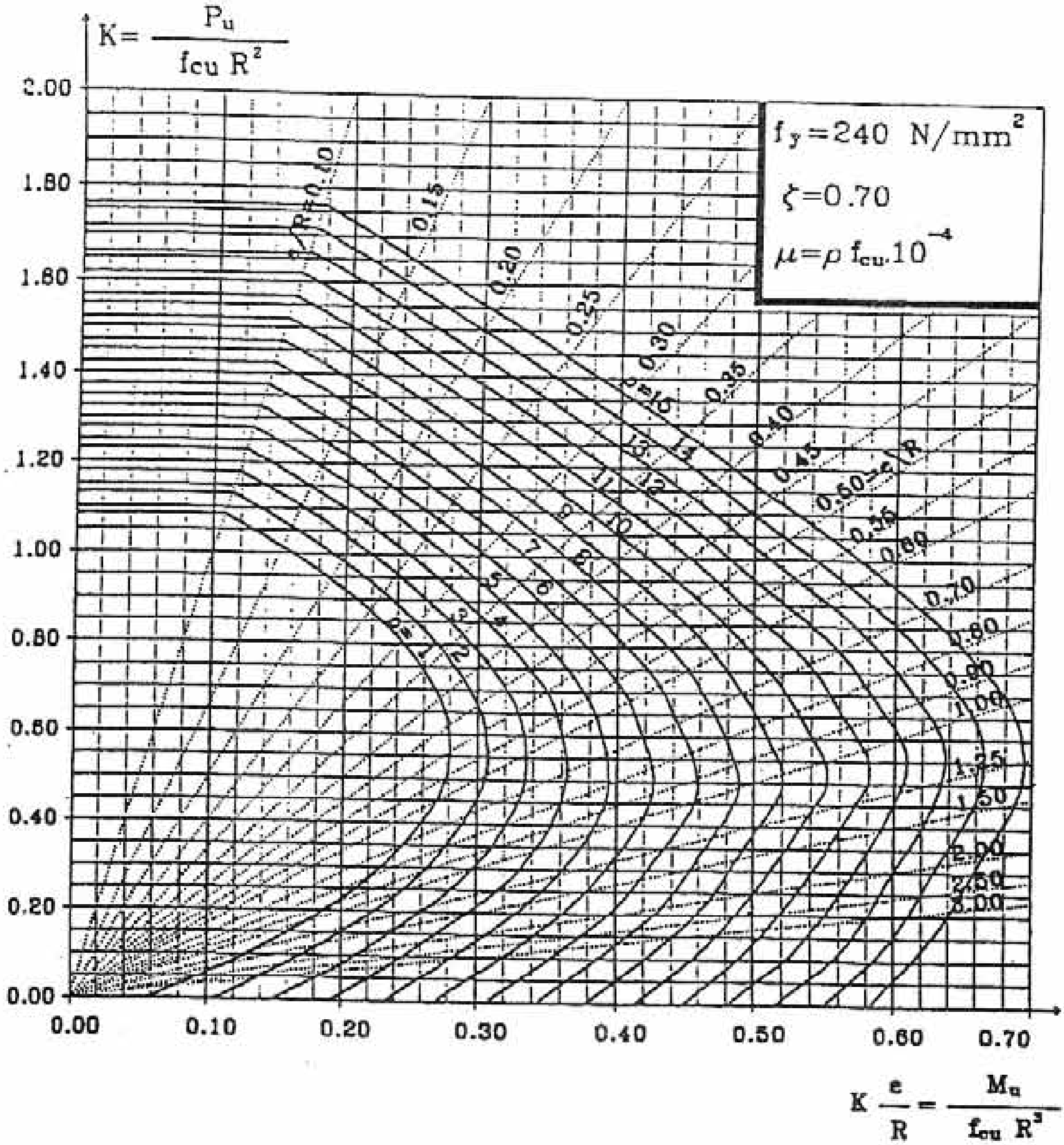
Cir–Sec.

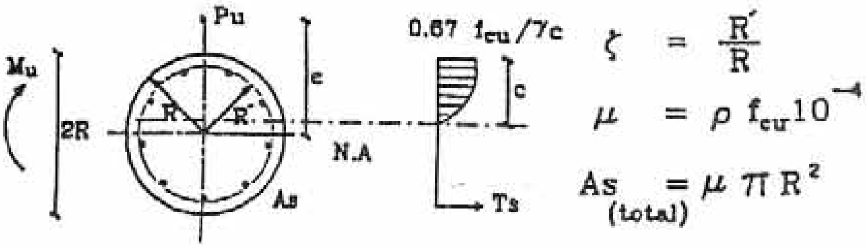
11/12

Chart (4-44): INTERACTION DIAGRAMS

FOR DESIGN OF SECTIONS SUBJECTED TO ECCENTRIC COMP. FORCES

(Circular Sections)





Cir–Sec.

12/12

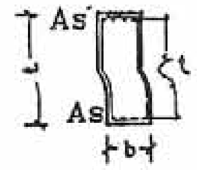
D E S I G N   A I D S

INTERACTION DIAGRAMS FOR ECCENTRIC FORCES

RECTANGULAR SECTION

Top & Bott. R.F.T Only

$\alpha = \frac{As'}{As}$



$\alpha = 1.00$

$f_y = 240$ — f_{cu} < 20, 25 > ζ < 0.9, 0.8

$f_y = 360$ — f_{cu} < 25, 30, 35 > ζ < 0.9, 0.8

$f_y = 400$ — f_{cu} < 25, 30, 35 > ζ < 0.9, 0.8

How To Use Interaction Diagrams

$\frac{P_u}{b.t}$ (N/mm^2)

$\frac{P_u.e}{b.t^2}$ (N/mm^2)

$\mu = \frac{As}{b.t}$

$As = \mu.b.t.$

$As' = As$

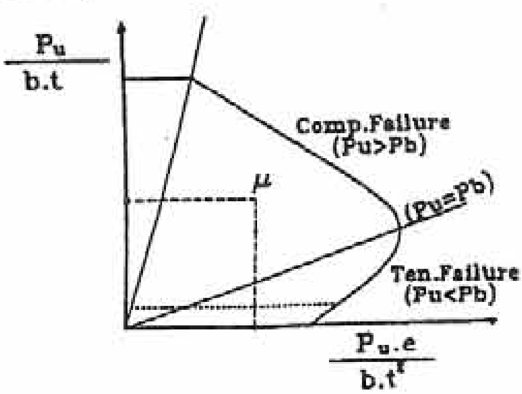


Chart (4-45) Interaction Diagrams for Design of Sections Subjected to Eccentric Forces

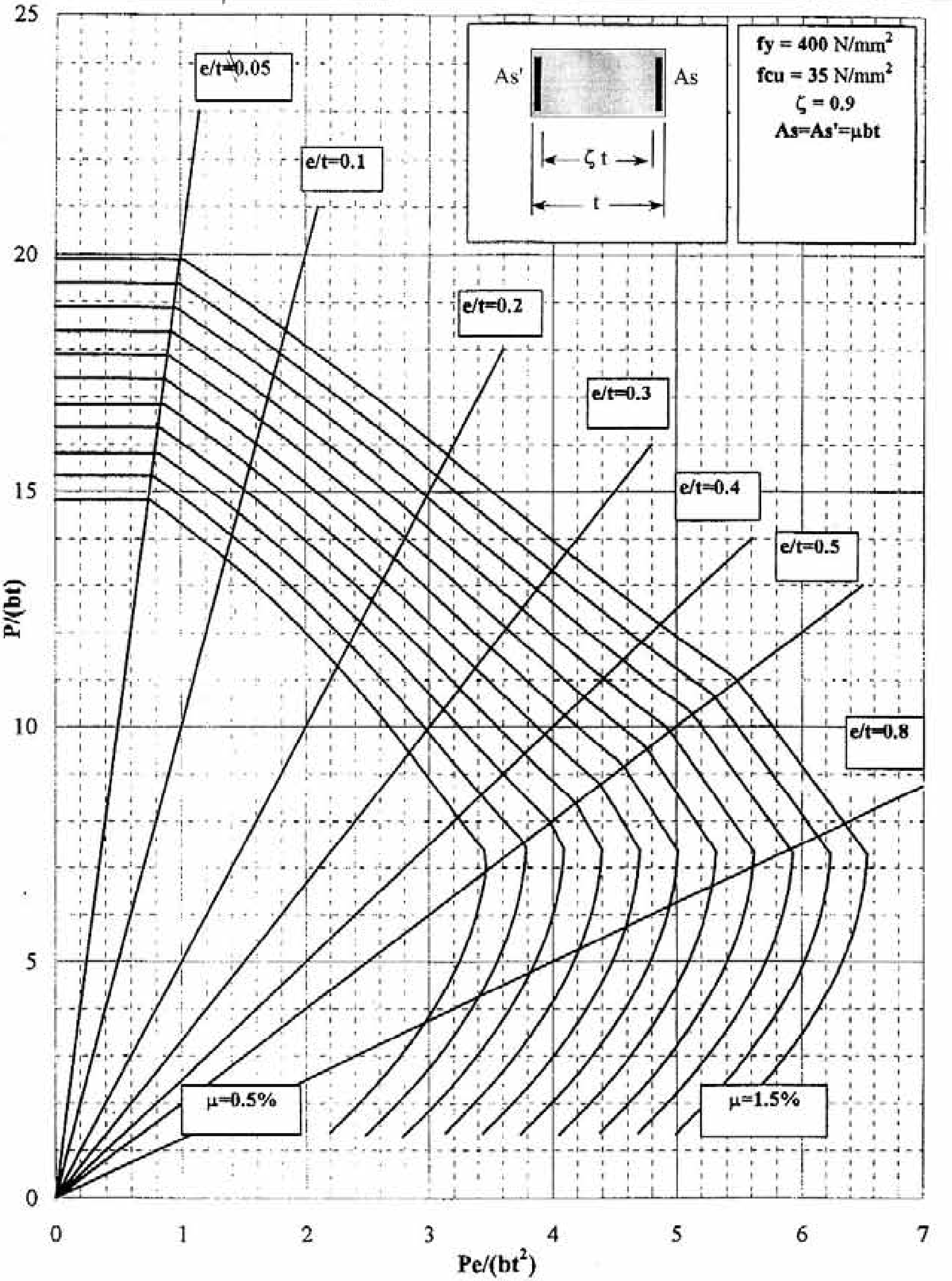


Chart (4-46) Interaction Diagrams for Design of Sections Subjected to Eccentric Forces

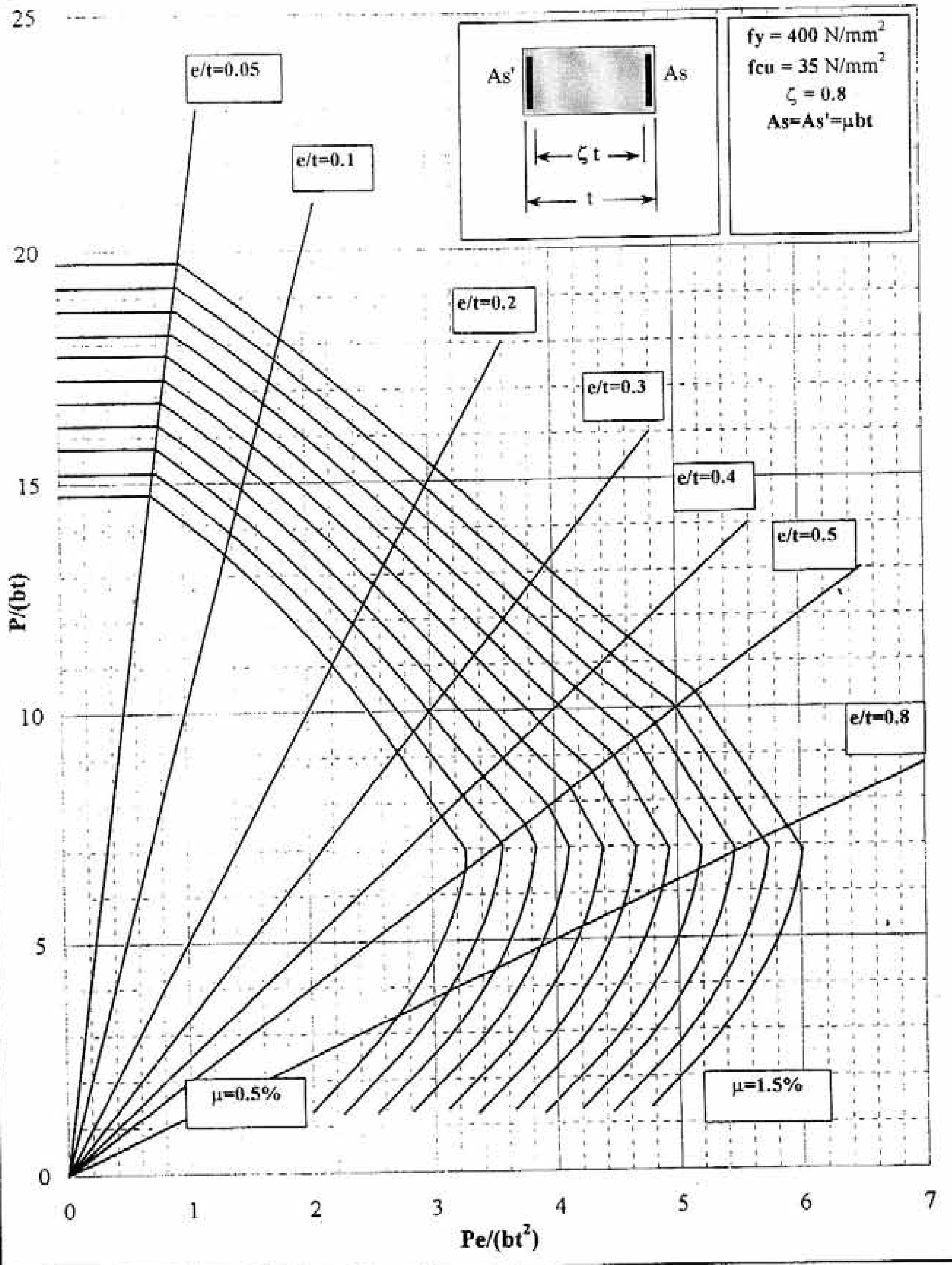


Chart (4-47) Interaction Diagrams for Design of Sections Subjected to Eccentric Forces

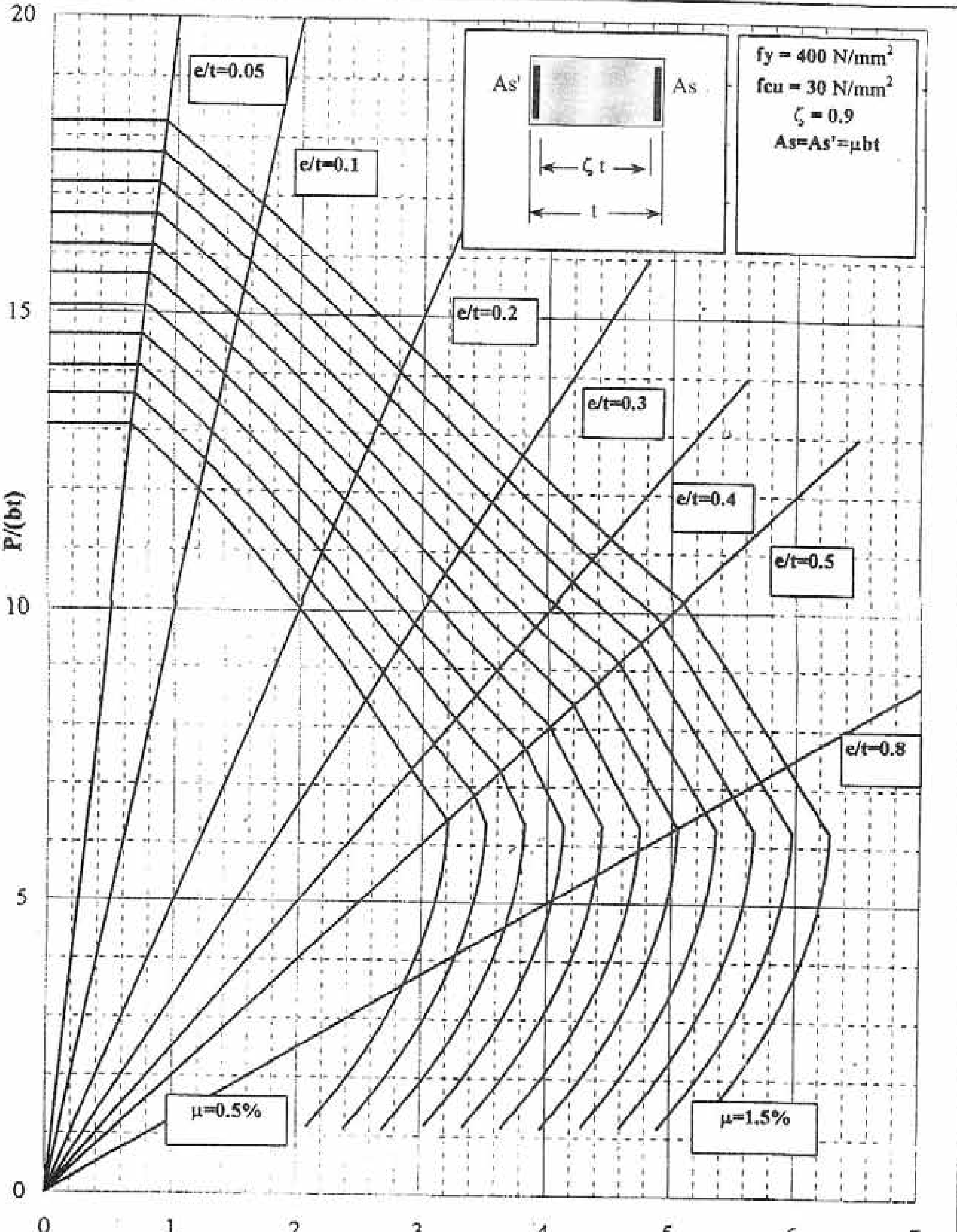


Chart (4-48) Interaction Diagrams for Design of Sections Subjected to Eccentric Forces

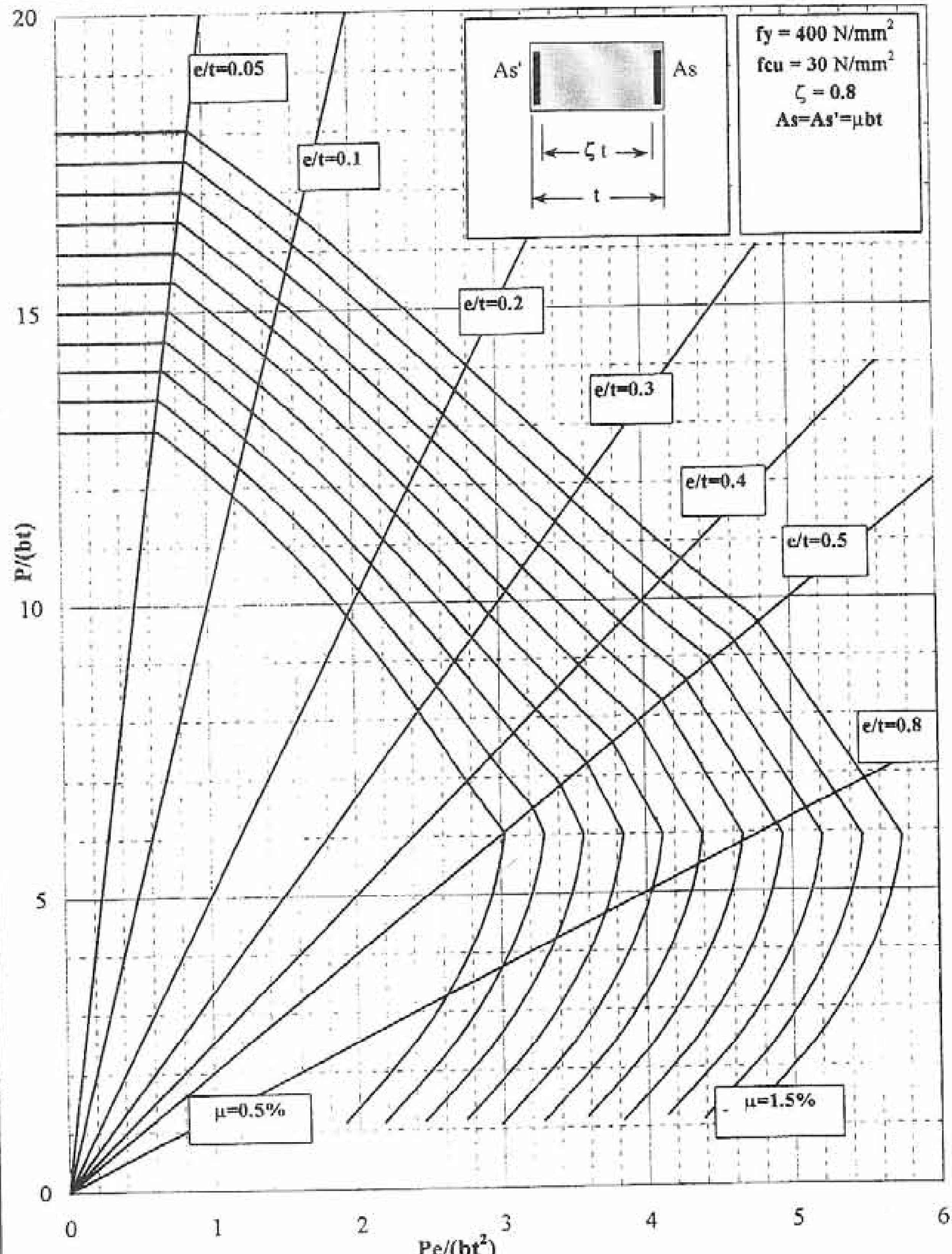


Chart (4-49) Interaction Diagrams for Design of Sections Subjected to Eccentric Forces

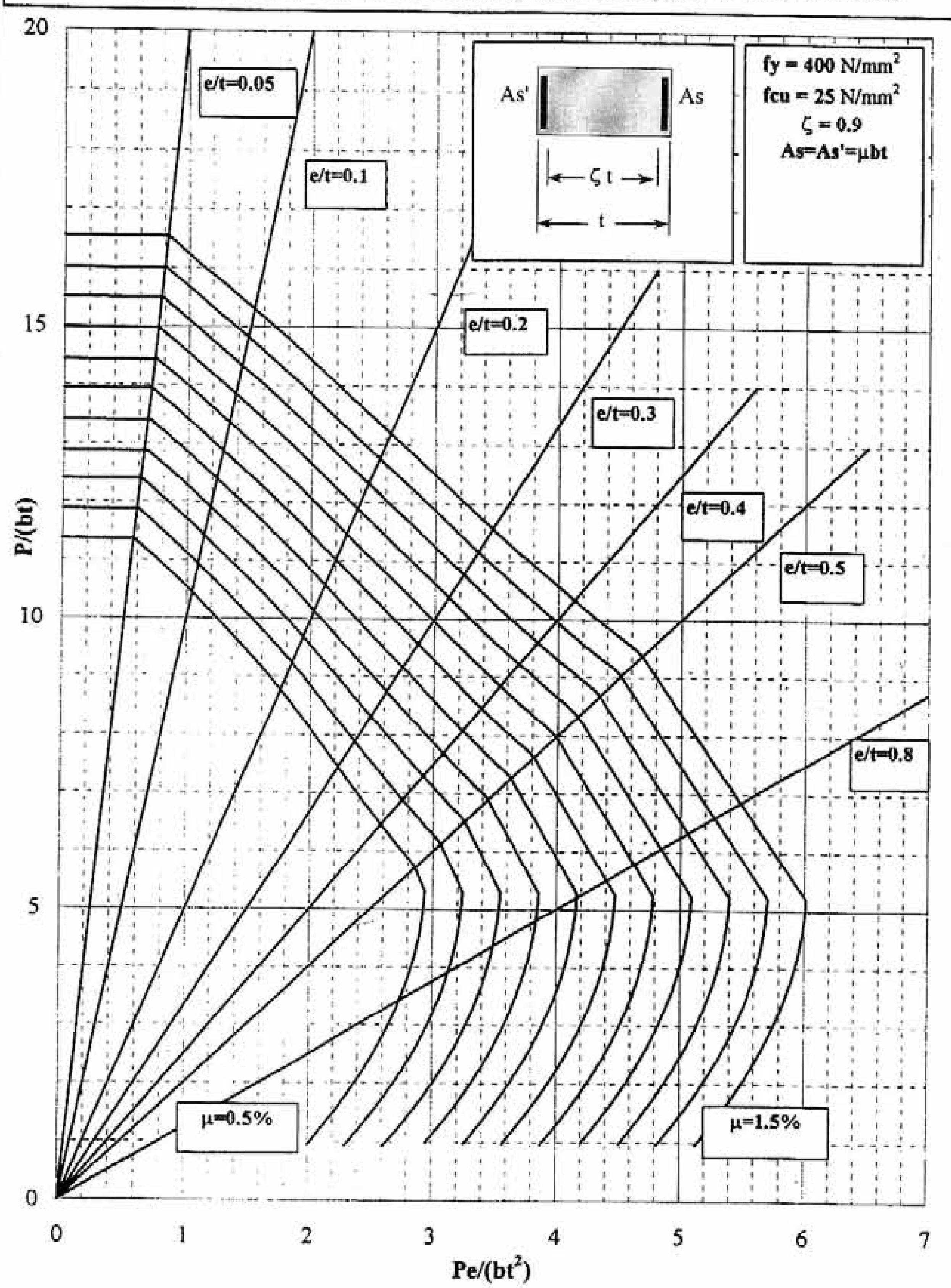


Chart (4-50) Interaction Diagrams for Design of Sections Subjected to Eccentric Forces

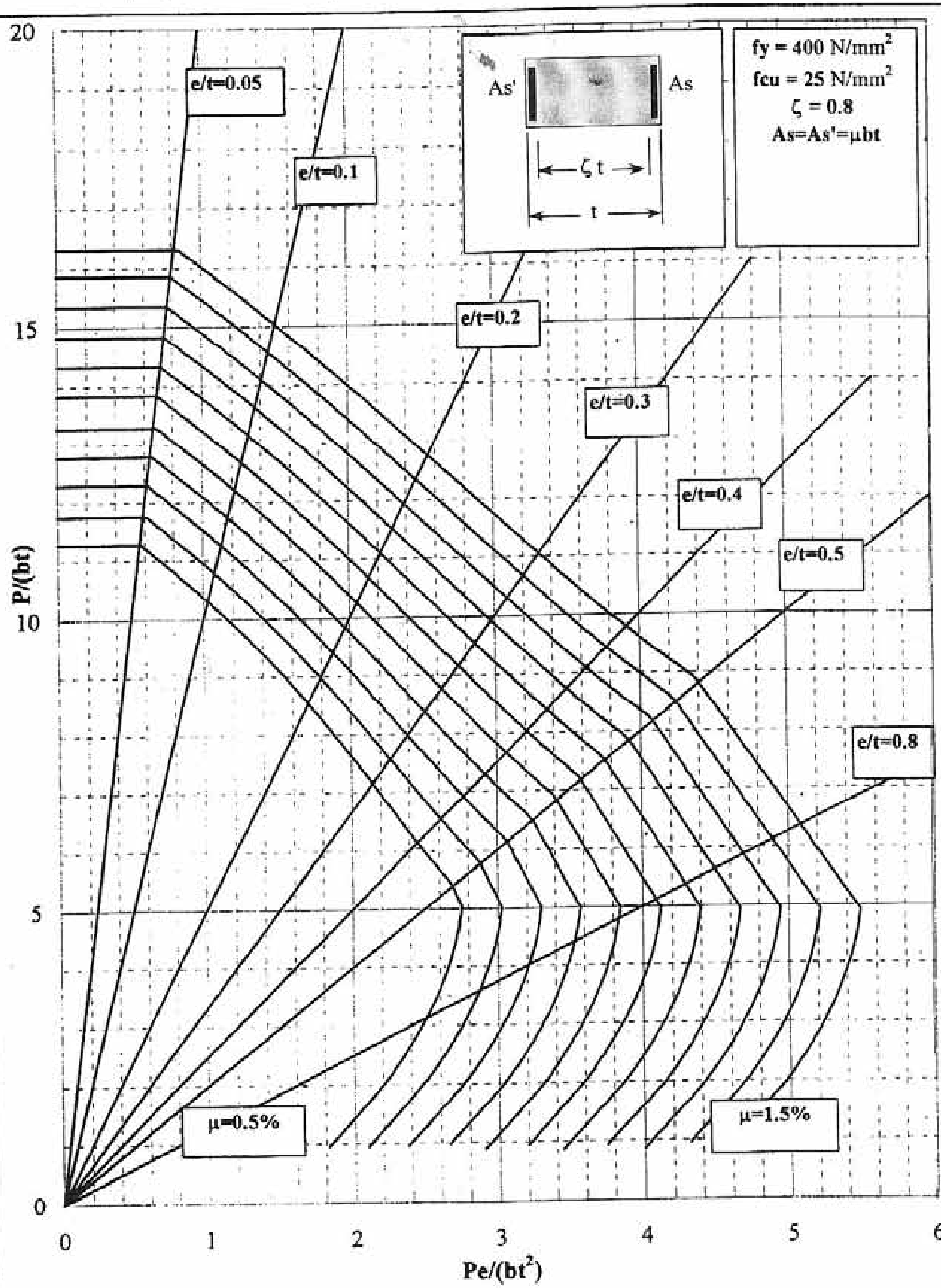


Chart (4-51) Interaction Diagrams for Design of Sections Subjected to Eccentric Forces

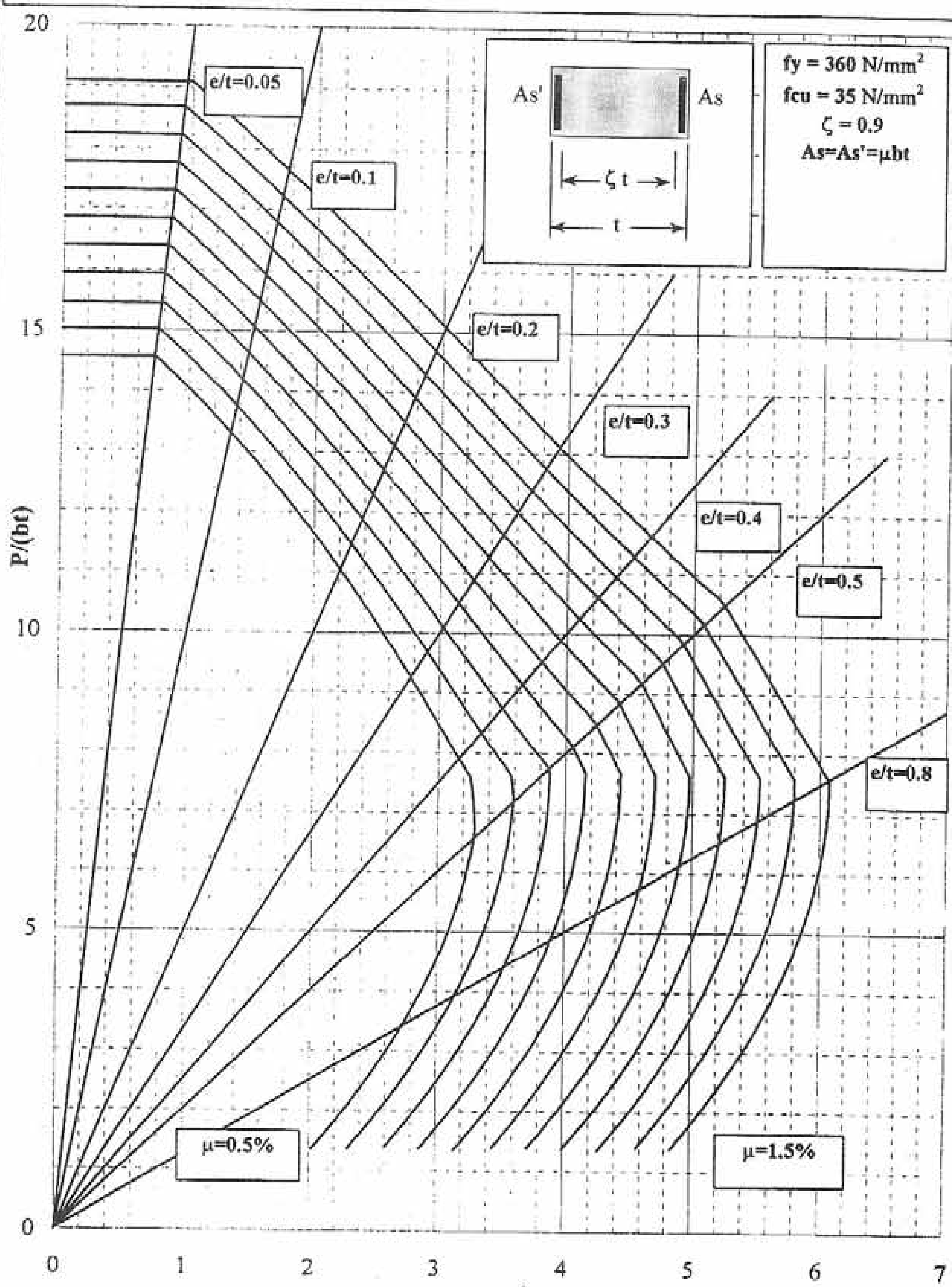


Chart (4-52) Interaction Diagrams for Design of Sections Subjected to Eccentric Forces

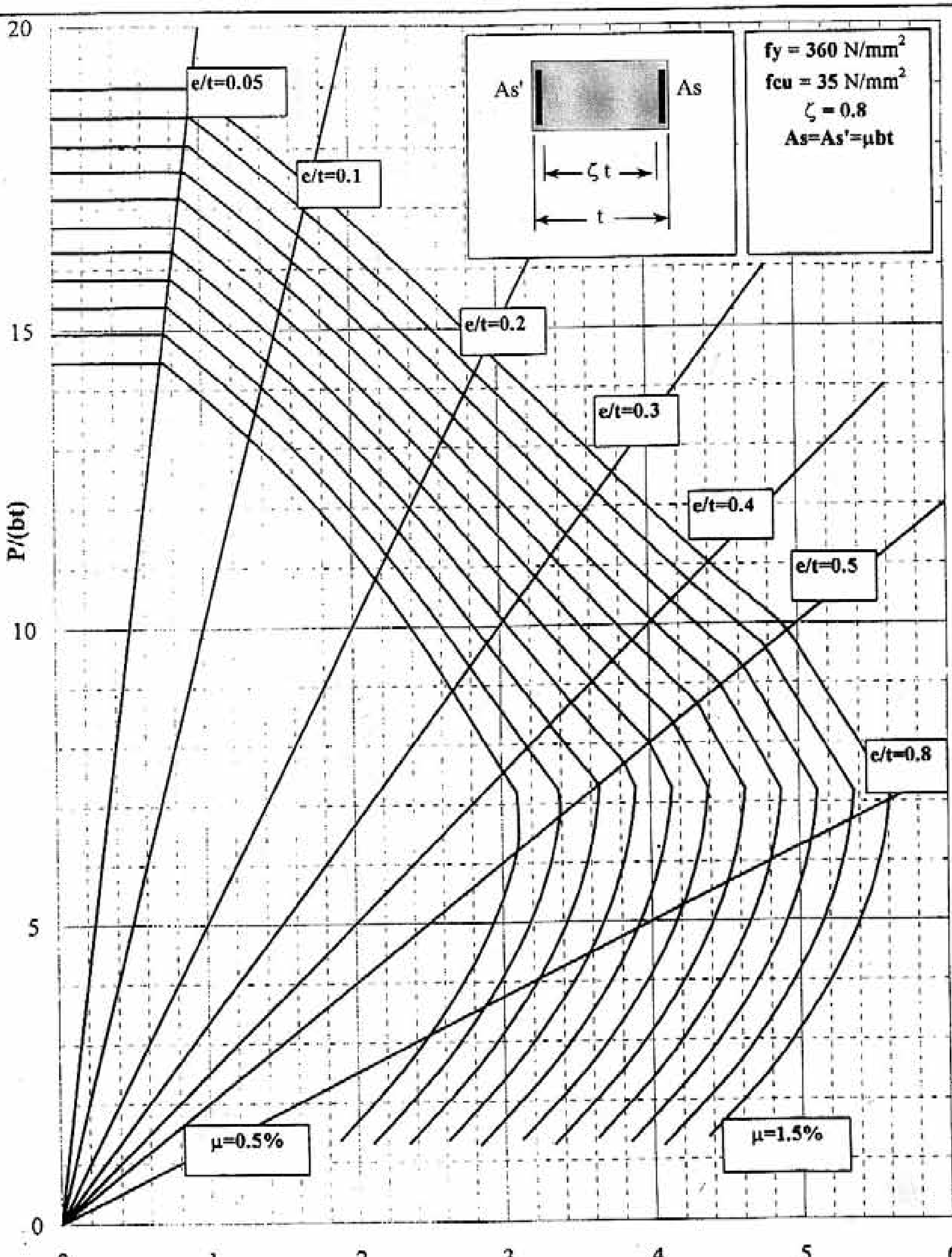


Chart (4-53) Interaction Diagrams for Design of Sections Subjected to Eccentric Forces

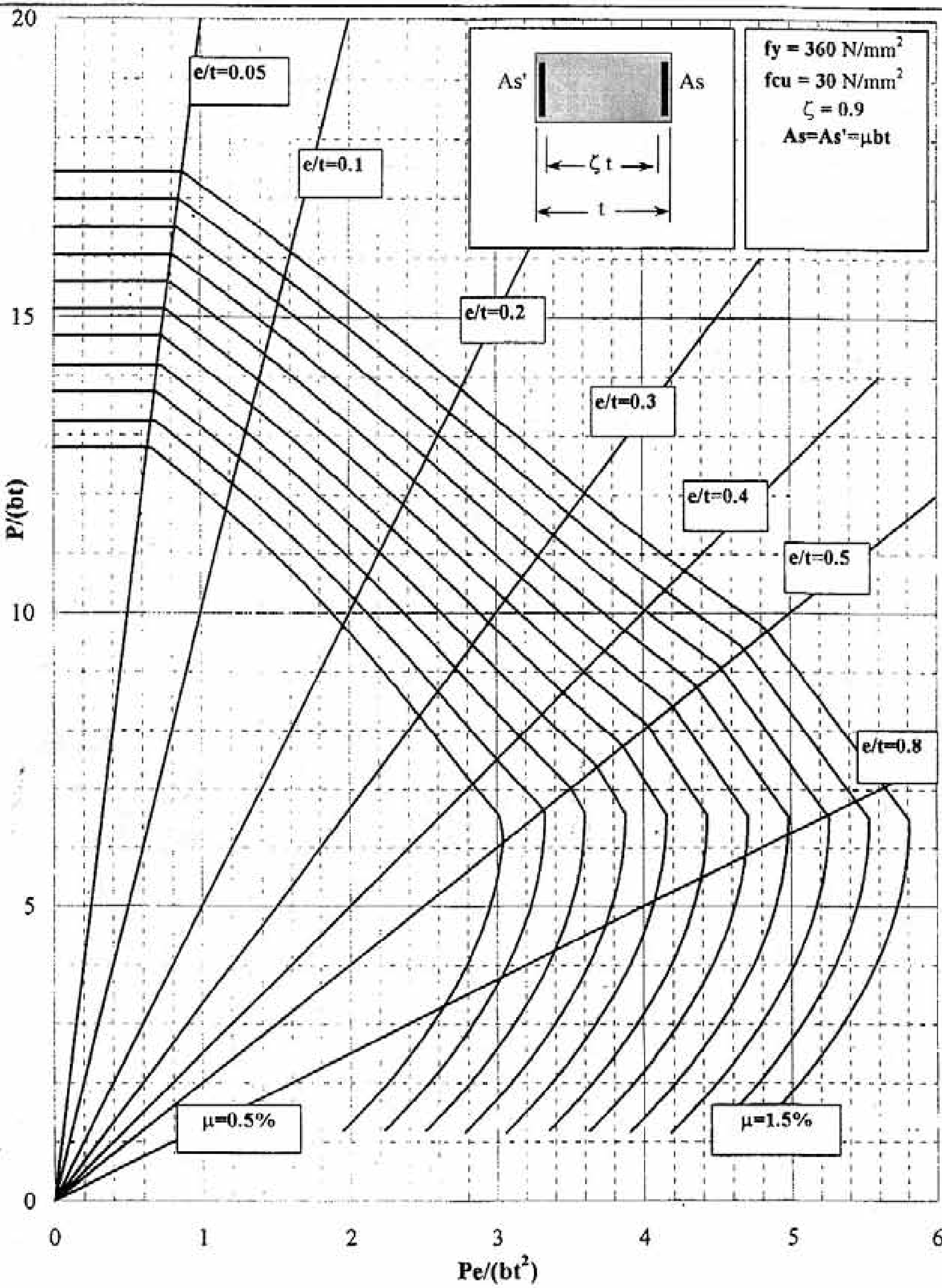


Chart (4-54) Interaction Diagrams for Design of Sections Subjected to Eccentric Forces

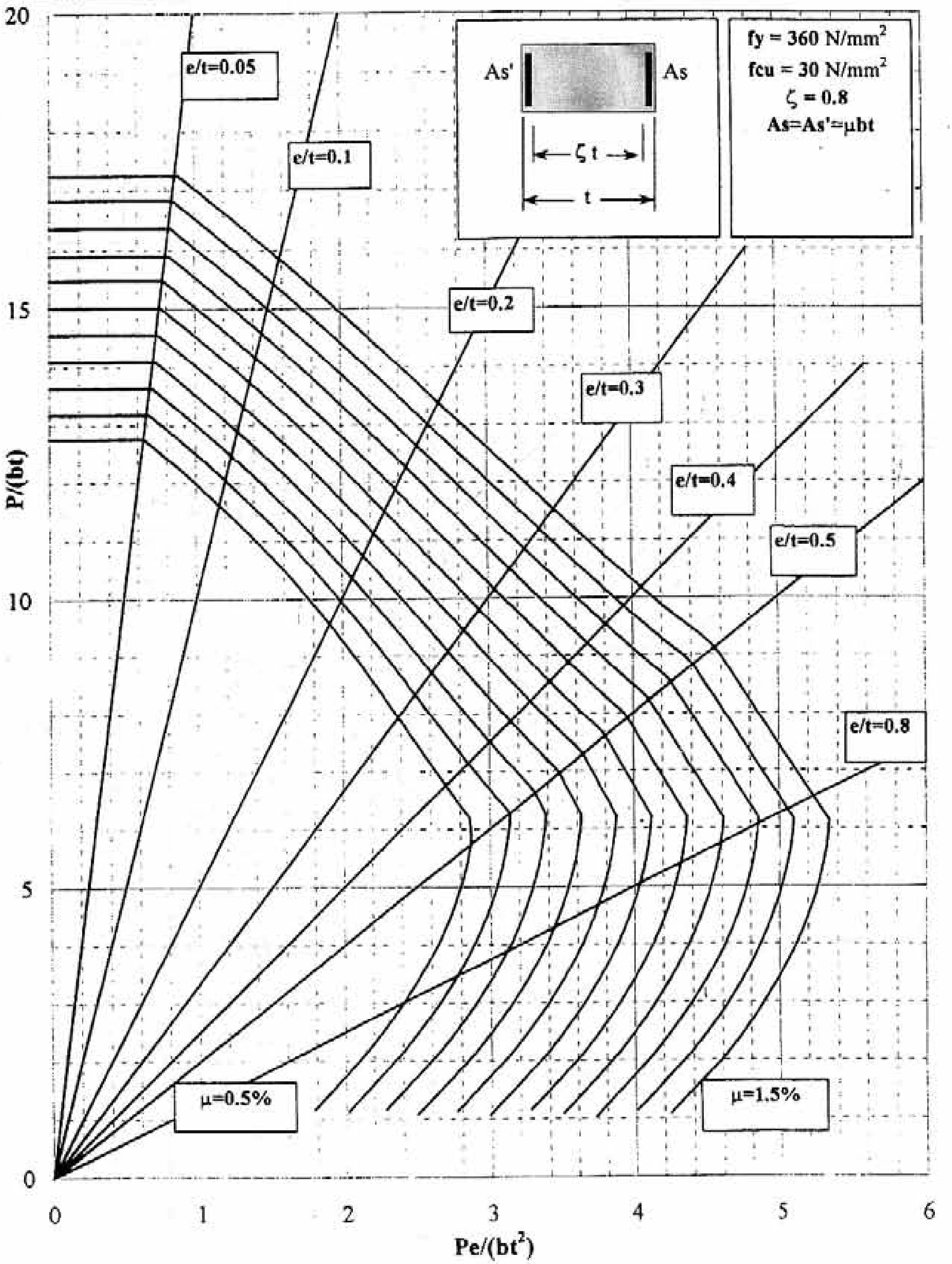


Chart (4-55) Interaction Diagrams for Design of Sections Subjected to Eccentric Forces

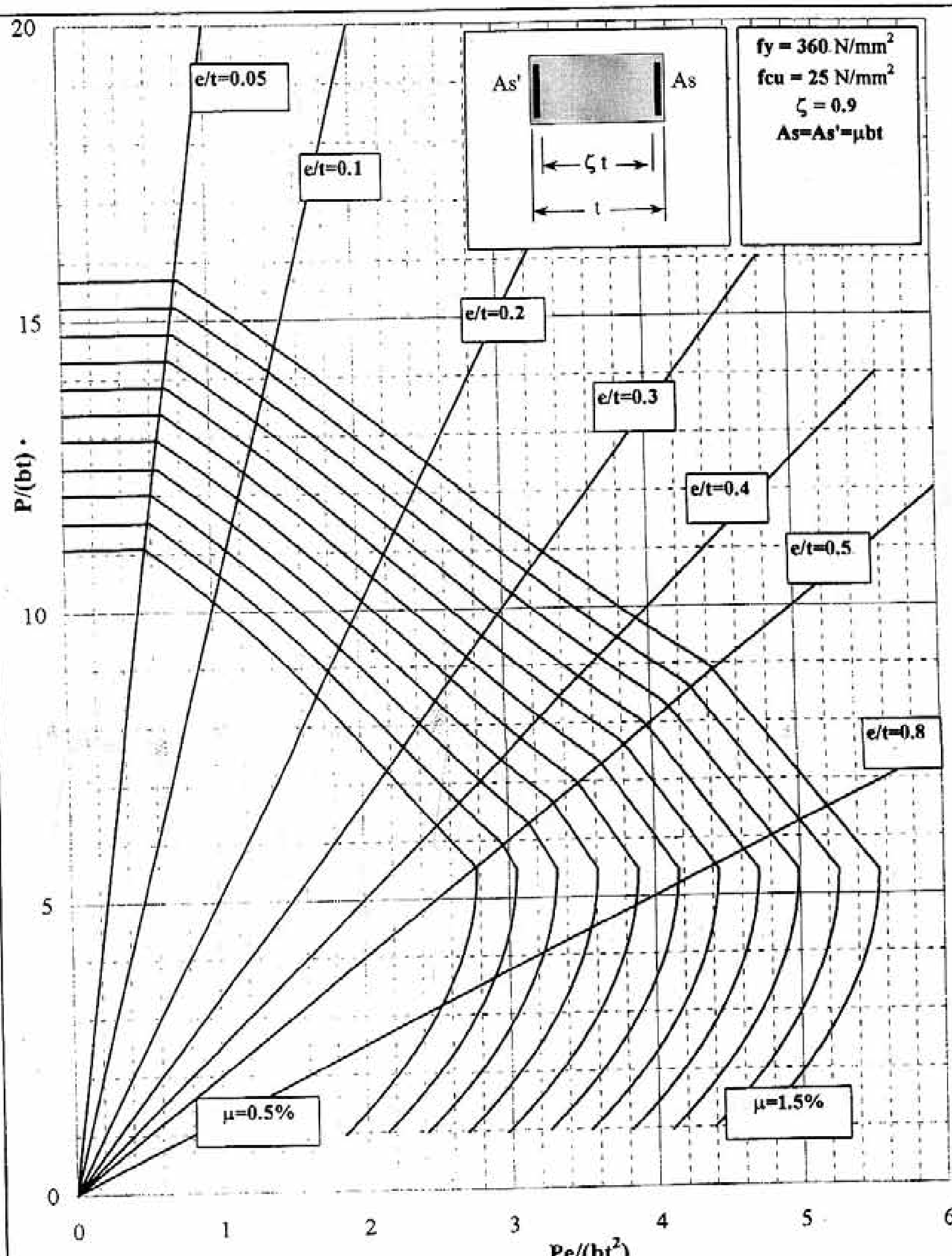


Chart (4-56) Interaction Diagrams for Design of Sections Subjected to Eccentric Forces

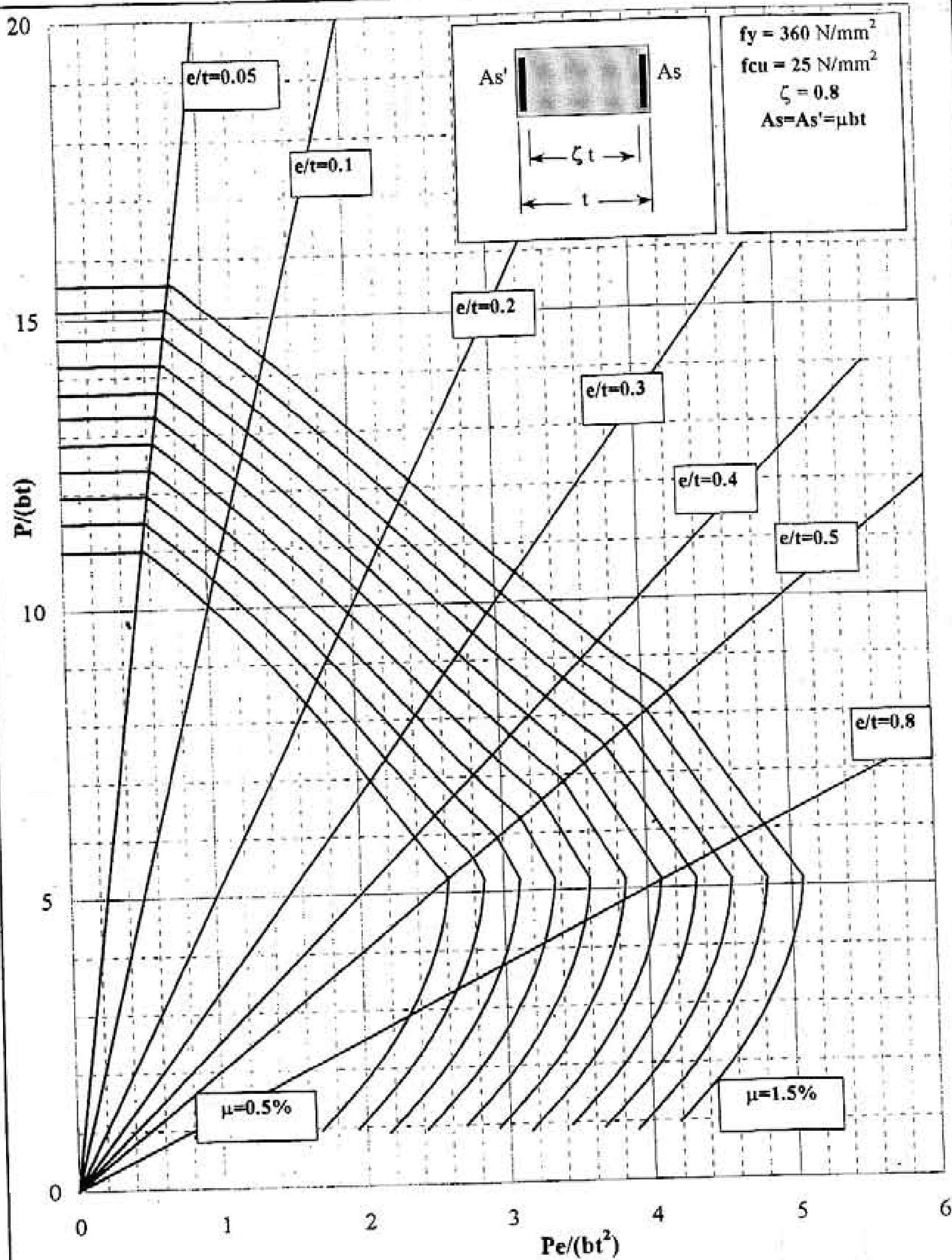


Chart (4-57) Interaction Diagrams for Design of Sections Subjected to Eccentric Forces

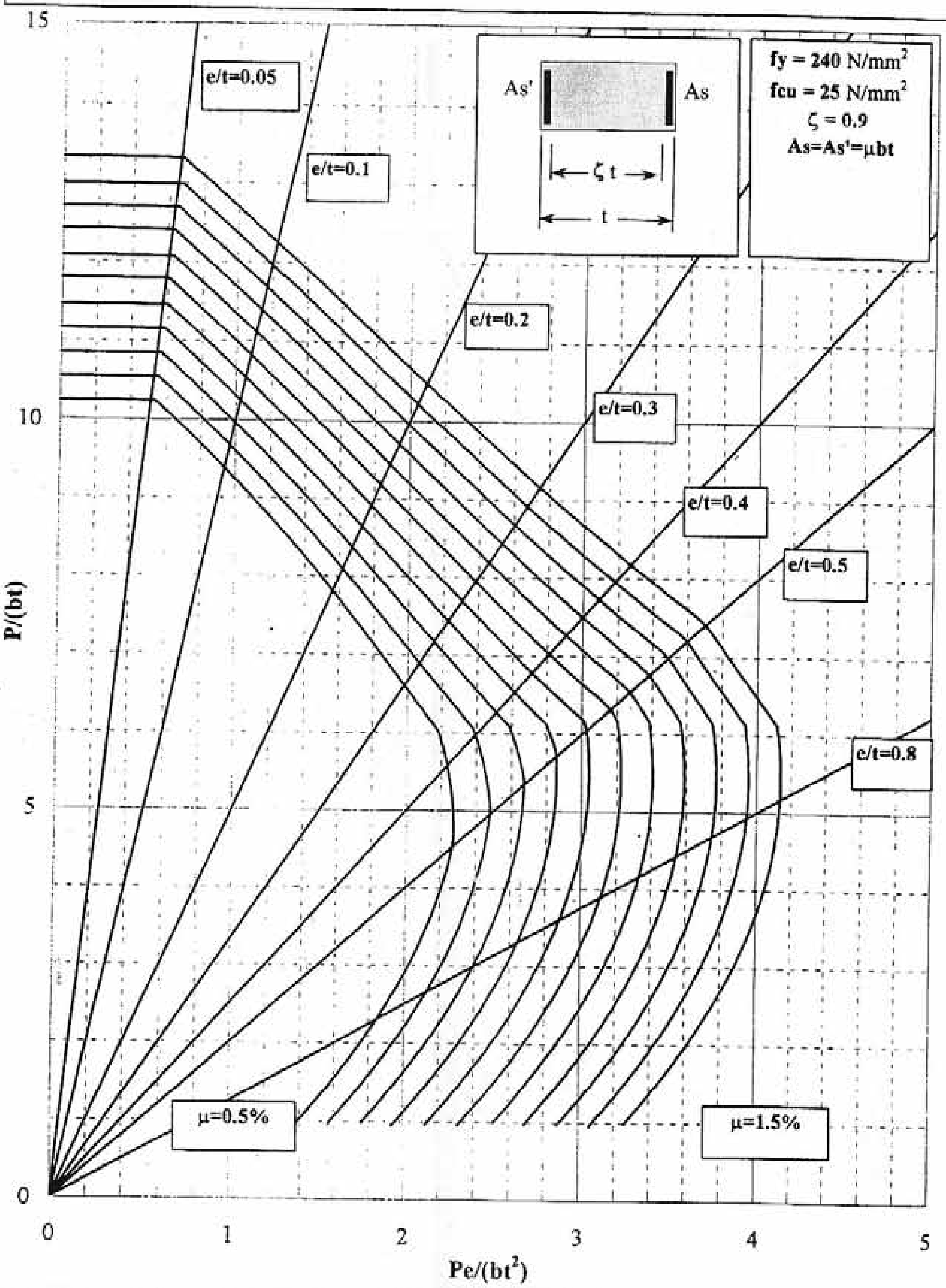


Chart (4-58) Interaction Diagrams for Design of Sections Subjected to Eccentric Forces

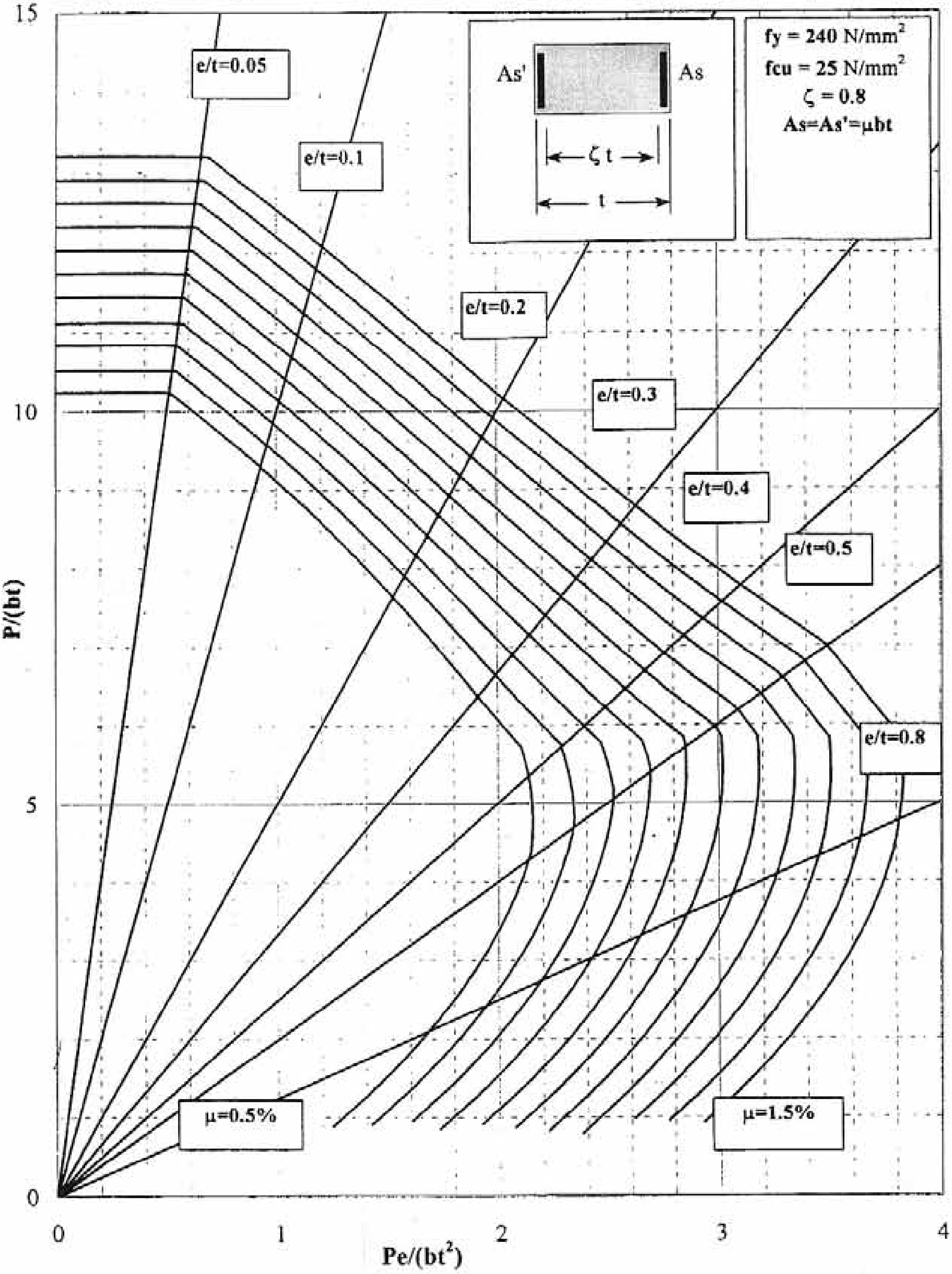


Chart (4-59) Interaction Diagrams for Design of Sections Subjected to Eccentric Forces

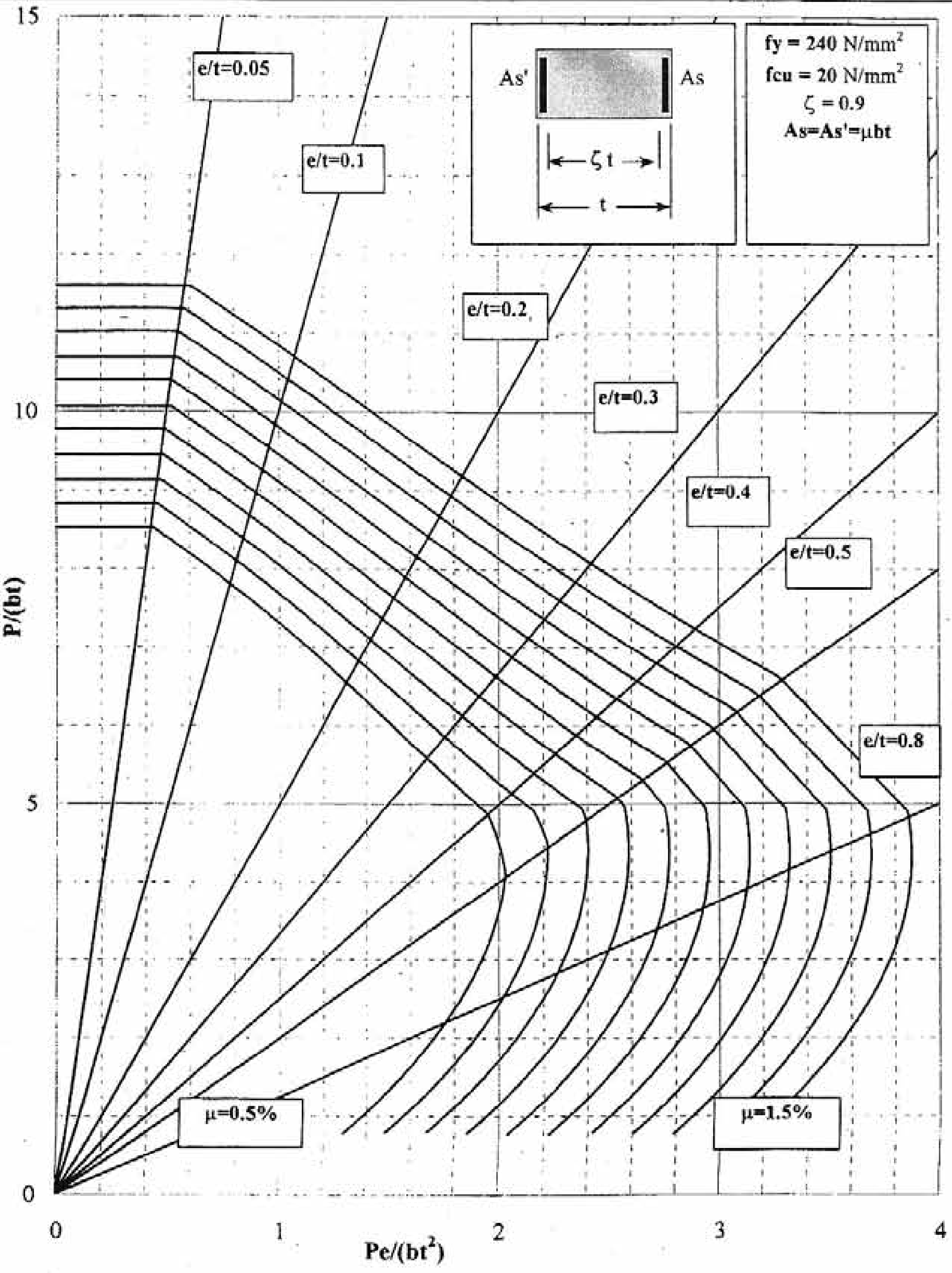
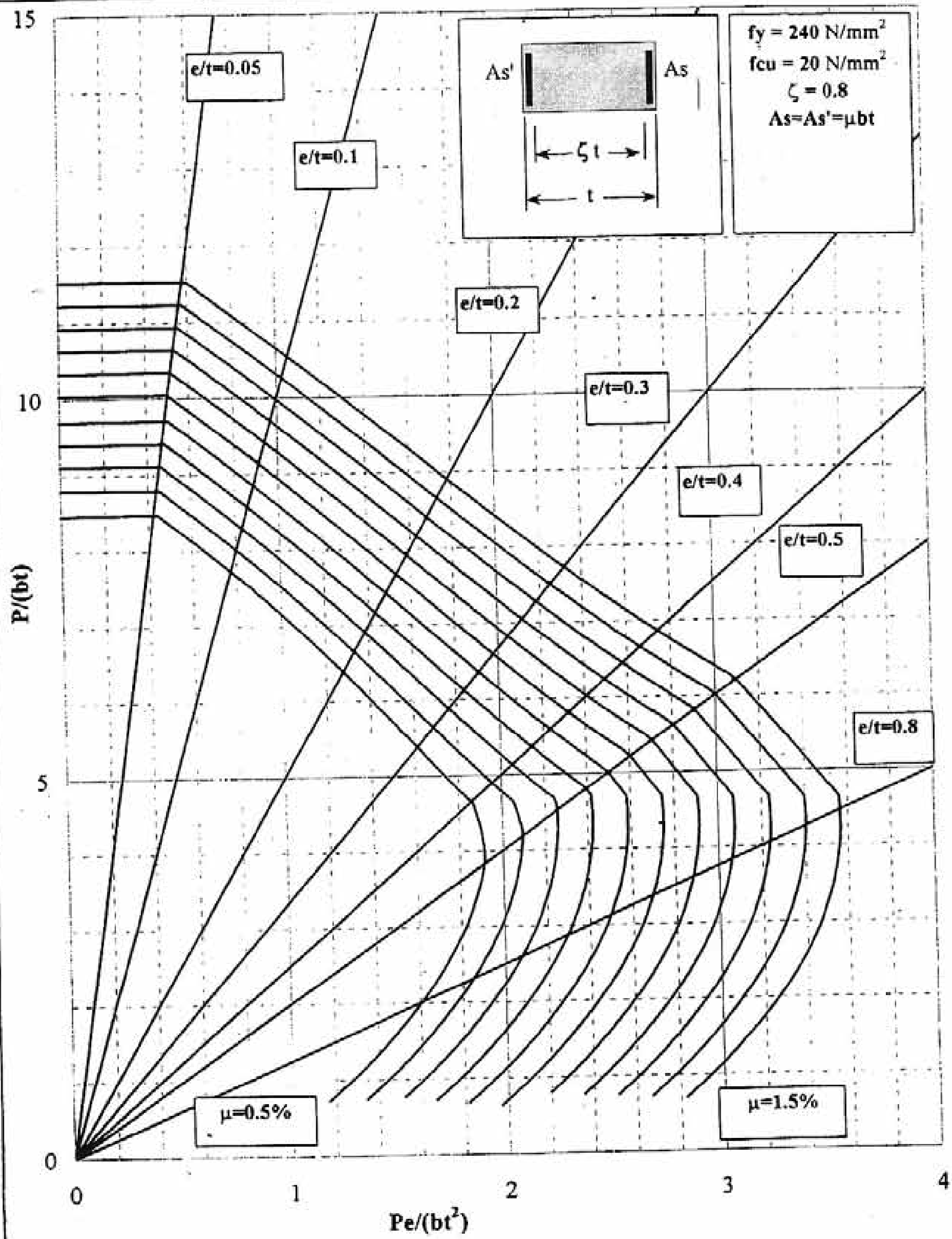


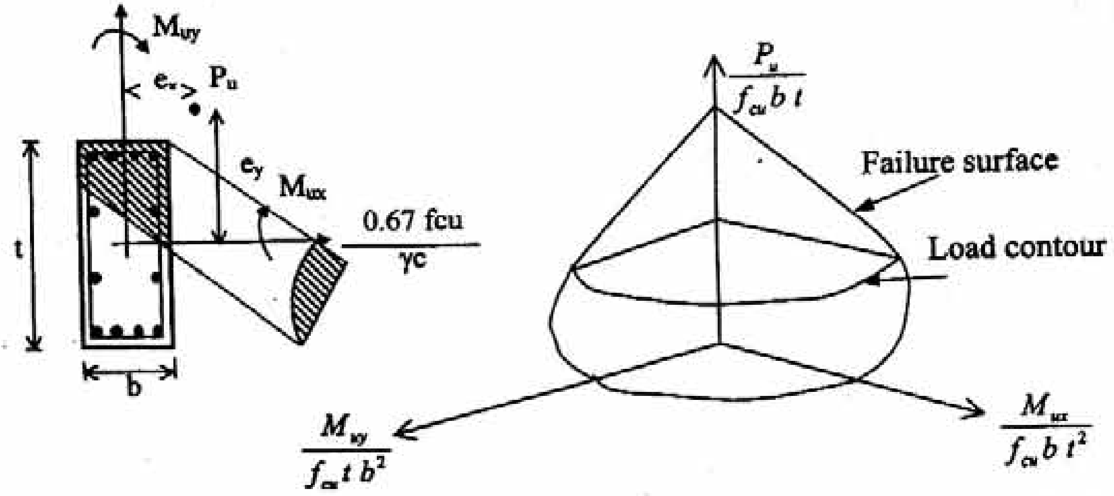
Chart (4-60) Interaction Diagrams for Design of Sections Subjected to Eccentric Forces



5. COMPRESSION MEMBERS SUBJECTED TO BIAXIAL BENDING

5.1 Introduction

Designing a rectangular column subjected to biaxial bending and axial loads is a complicated process because the position and direction of the neutral axis are difficult to establish. Furthermore, since the strain on the cross section varies linearly in both directions, considerable computation time is required to calculate the strain for every reinforcement bar in the cross section. According to the Egyptian code the strain compatibility method must be used to calculate the forces and moments for members subjected to biaxial bending. A computer program was used to generate the interaction diagrams for biaxially loaded members shown in the following pages. Since the use of the rectangular stress block is not permitted by the code for sections subjected to biaxial bending, the force and the moment in the concrete zone were determined by integrating the stress-strain curve of the concrete over the compressed area. These interaction curves are produced by cutting a horizontal plan R_b (load contour) through the failure surface as shown in the figure below where $R_b = \frac{P_u}{f_{cu}\, b\, t}$



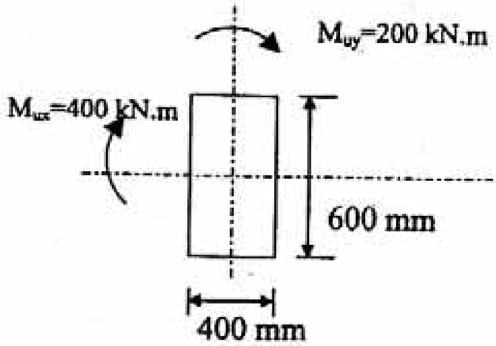
It should be noted that the reinforcement determined from these interaction curves should be distributed equally on the four sides. The use of these interaction curves is explained by the following examples

5.2 Solved Examples

Example 1: Rectangular section with Biaxial Bending

Determine the reinforcement to be provided in a short column subjected to biaxial bending with the following data:

Column size	400 x 600 mm
f_{cu}	25 N/mm^2
f_y	360 N/mm^2
P_u	1800 KN
M_{ux}	400 kN.m
M_{uy}	200 kN.m

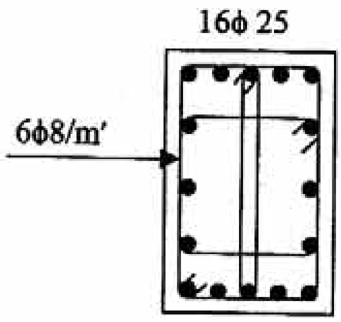


Solution: calculate the following terms

$$R_b = \frac{P_u}{f_{cu}\, b\, t} = \frac{1800 \times 1000}{25 \times 400 \times 600} = 0.30$$

$$\frac{M_{ux}}{f_{cu}\, b\, t^2} = \frac{400 \times 1000 \times 1000}{25 \times 400 \times 600^2} = 0.11$$

$$\frac{M_{uy}}{f_{cu}\, t\, b^2} = \frac{200 \times 1000 \times 1000}{25 \times 600 \times 400^2} = 0.083$$



Assume ζ=0.9

Referring to chart with R_b=0.3 Chart (5-5), the reinforcement index ρ can be obtained

$\rho = 11.8$

$\mu = \rho\ f_{cu}\, 10^{-4} = 11.8 \times 25 \times 10^{-4} = 0.0295$

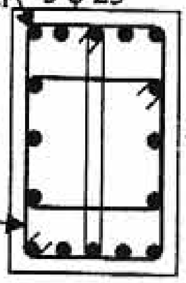
$A_{s,total} = \mu\ b\ t = 0.0295 \times 400 \times 600 = 7080$ mm^2 =70.8 cm^2

$A_{s,chosen}$= 16 ϕ 25. This reinforcement will be distributed equally on the four sides.

Example 2: Rectangular section with Biaxial Bending

Determine the reinforcement in the previous example if the normal force has been increased to 2200 kN

Solution: calculate the following terms



$$R_b = \frac{P_u}{f_{cu}\, b\, t} = \frac{2200 \times 1000}{25 \times 400 \times 600} = 0.366$$

From the previous example

$$\frac{M_{ux}}{f_{cu}\, b\, t^2} = \frac{400 \times 1000 \times 1000}{25 \times 400 \times 600^2} = 0.11$$

$$\frac{M_{uy}}{f_{cu}\, t\, b^2} = \frac{200 \times 1000 \times 1000}{25 \times 600 \times 400^2} = 0.083$$

Since the biaxial interaction diagrams do not have a value of R_b=0.366 interpolation will be performed between R_b=0.30 and R_b=0.40

Referring to chart with R_b=0.30 Chart (5-5), the reinforcement index ρ can be obtained

$\rho = 11.8$

Referring to chart with R_b=0.40 Chart (5-6), the reinforcement index ρ can be obtained

$\rho = 15$

Therefore for R_b=0.366, $\rho = 13.94$

$\mu = \rho\ f_{cu}\, 10^{-4} = 13.94 \times 25 \times 10^{-4} = 0.03486$

$A_{s,total} = \mu\ b\ t = 0.03486 \times 400 \times 600 = 8366\ mm^2 = 83.66\ cm^2$

$A_{s,chosen}$= 4 ϕ 28 +12 ϕ 25.

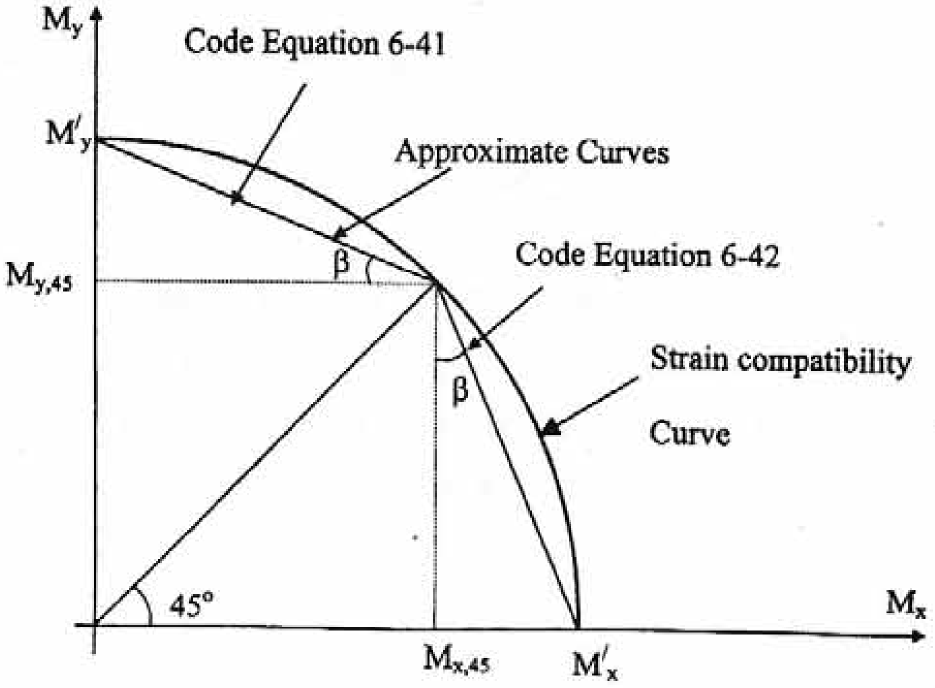
The 4 ϕ 28 bars will be placed in the corners according to article 6-4-6-3-c and the 12 ϕ 25 bars will be distributed equally on the four sides.

5.3 Design of Biaxially Loaded Members Using Code Provision 6-4-6

5.3.1 Case of Symmetrical Arrangement of Reinforcement (clause 6-4-6-3)

The code offers a simplified method for the case of a rectangular section with symmetrical reinforcement by approximating the curved shape by two straight lines as shown in the figure below. The biaxial state will be transformed to a case of uniaxial eccentric compression. The section will be designed as if it is subjected to an increased moment about one axis only. The magnification factor β depends on the value of the applied normal force. It should be noted that the values of β has been changed in the current edition than 1995 edition to account for load contour changes at high normal loads.

Note: Interaction diagrams with uniformly distributed steel reinforcement must be used in conjunction with this simplified method.



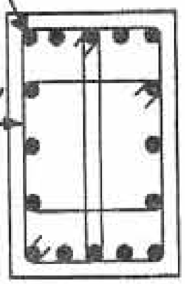
Example 3: Rectangular Section Subjected to Biaxial Bending with symmetrical Reinforcement

Design the column in example 1 using the simplified method in the code

Solution

Determine the load level R_b using the following equation

$$R_b = \frac{P_u}{f_{cu}\, b\, t} = \frac{1800 \, x \, 1000}{25 \, x \, 400 \, x \, 600} = 0.30$$



From Table 6-12-a in the code

$\beta = 0.75$

Assume cover = 40 mm

a' =560 mm , b' =360 mm

sine M_x /a'= (400/560) > (M_y/b') =(200/360), the design moment will be taken about x.

using code Equation (6-42)

M'_x = 400+ 0.75 x (560/360) 200 = 633 kN.m

$$\frac{M'_x}{f_{cu}\, b\, t^2} = \frac{633 \, x \, 1000 \, x \, 1000}{25 \, x \, 400 \, x \, 600^2} = 0.176$$

Locate a point in the ***uniaxial*** interaction diagram Chart 4-27 (uniformly distributed steel)

ρ =13.5

μ = 13.5 x 25 x 10^{-4}= 0.033

$A_{s,total}$ = μ b t = 0.033 x 400 x 600 = 7920 mm^2 =79.2 cm^2

$A_{s,chosen}$= 4ϕ28 +12 ϕ 25. This reinforcement will be distributed equally on the four sides, thus the 4ϕ28 will be in the corners

5.3.2 Case of Unsymmetrical Arrangement of Reinforcement (clause 6-4-6-4)

The code permits the use of another approximate method for sections subjected to biaxial bending with unsymmetrical reinforcement. A magnification factor α_b was introduced to modify the design moments M_x and M_y according to the load level and moment ratio. The methodology is implemented in two stages:

a) Magnifying the bending moment about the principle axes using the magnification factor α_b obtained from table 6-12-b

b) Calculating the required reinforcement steel cross sectional area using the uniaxial interaction diagrams for the magnified moment M'_x and M'_y.

Example 4 illustrates the design procedure.

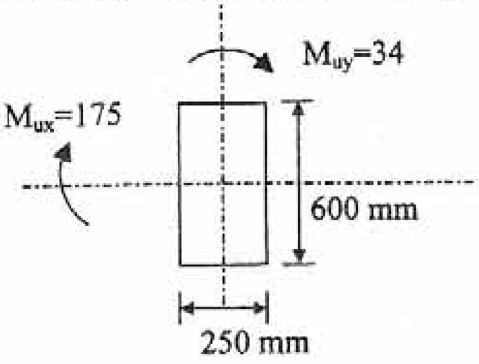
5.4. Maximum Reinforcement Ratio

The maximum vertical reinforcement ratio allowed by the ECCS 203 varies by the location of the column. The maximum ratio for interior, edge and corner columns is 4%, 5% and 6%, respectively. The designer has to check these limits when using the biaxial charts for designing reinforced concrete columns.

Example 4: Rectangular Section Subjected To Biaxial Bending With Unsymmetrical Reinforcement

Determine the reinforcement to be provided in a short column subjected to biaxial bending with the following data:

Column size	250x600 mm
f_{cu}	25 N/mm^2
f_y	360 N/mm^2
P_u	1125 KN
M_{ux}	175 kN.m
M_{uy}	34 kN.m



$$\frac{M_{ux}/a'}{M_{uy}/b'} = \frac{175/570}{34/220} = 2.0$$

From table 6-12-b in the code α_b=1.35

$M_x' = M_x \times \alpha_b = 175 \times 1.35 = 236.3$ kN.m

$M_y' = M_y \times \alpha_b = 34 \times 1.35 = 45.9$ kN.m

$$R_b = \frac{P_u}{f_{cu}\, b\, a} = \frac{1125 \times 1000}{25 \times 250 \times 600} = 0.30$$

$$\frac{M'_x}{f_{cu}\, b\, a^2} = \frac{236.3 \times 1000 \times 1000}{25 \times 250 \times 600^2} = 0.105$$

$$\frac{M'_y}{f_{cu}\, a\, b^2} = \frac{45.9 \times 1000 \times 1000}{25 \times 600 \times 250^2} = 0.049$$

Using uniaxial interaction diagrams (ζ=0.9, α=1.0) Chart 4-4, the reinforcement index (ρ) in both **X** (enter with 0.105, 0.3) and **Y** (enter with 0.049, 0.3) directions can be obtained

$\rho_x = 2.6$

$\rho_y = 1$

$\mu_x = \rho_x \ f_{cu}\ 10^{-4} = 2.6 \times 25 \times 10^{-4} = 0.0065$

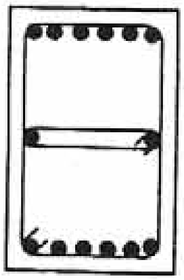
$\mu_y = \rho_y \ f_{cu}\ 10^{-4} = 1 \times 25 \times 10^{-4} = 0.0025$

$A_{sx} = \mu_x\, b\, a = 0.0065 \times 250 \times 600 = 975\ mm^2$

$A_{sy} = \mu_y\, b\, a = 0.0025 \times 250 \times 600 = 375\ mm^2$

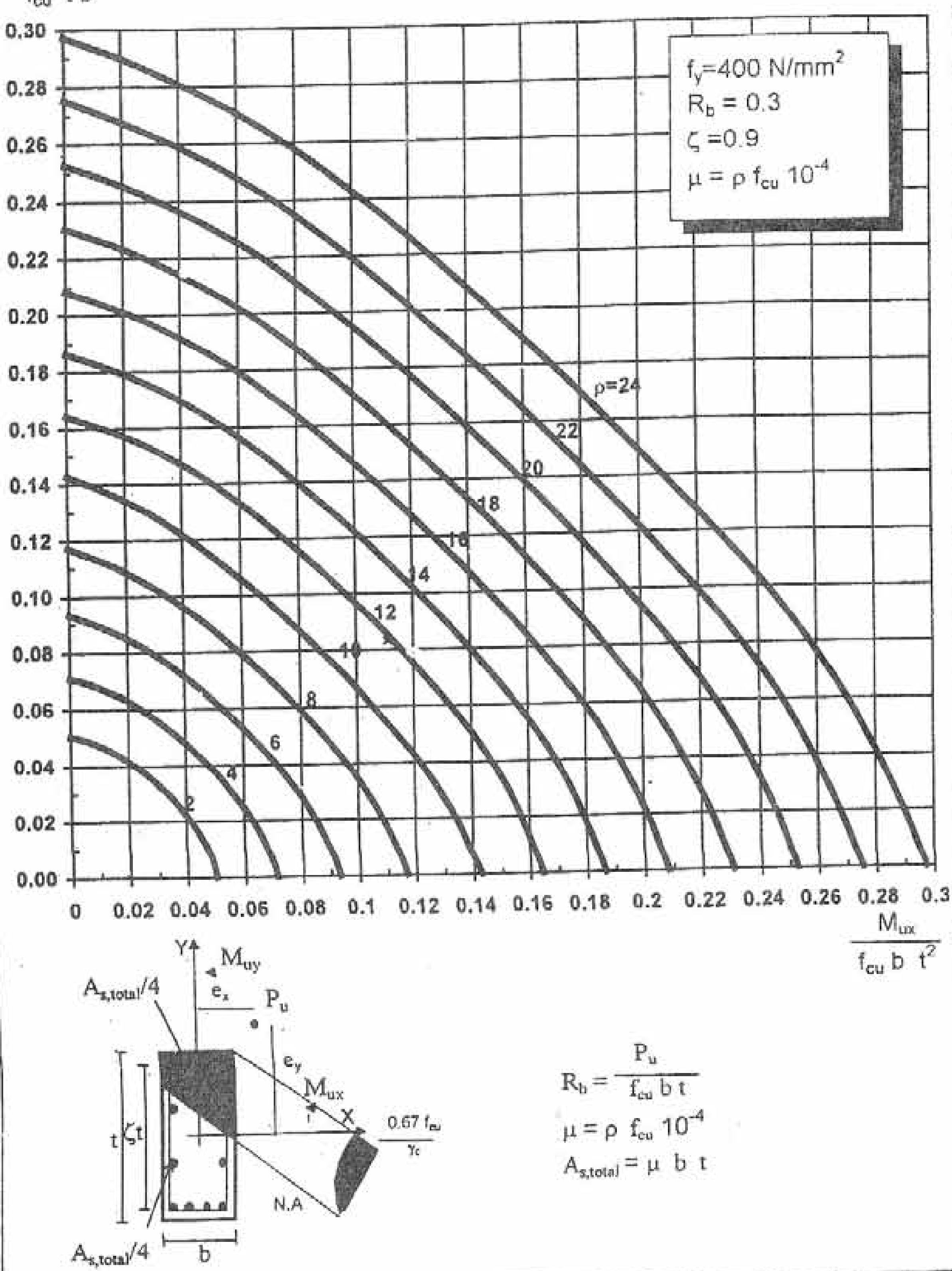
$A_{s,min} = 0.008 \times 250 \times 600 = 1200 = 12\ cm^2$

$A_{s,total} = 2\ (A_{sx} + A_{sy}) = 2700\ mm^2 = 27\ cm^2 \quad > A_{s,min}$ (choose 14 ϕ 16)



Note : The corner bar is divided between x direction and y direction [$A_{sx} = 5\ \phi\ 16$ (10 cm^2), $A_{sy} = 2\ \phi\ 16$ (4 cm^2)]

CHART (5-1): INTERACTION DIAGRAMS FOR BIAXIALLY LOADED SECTIONS

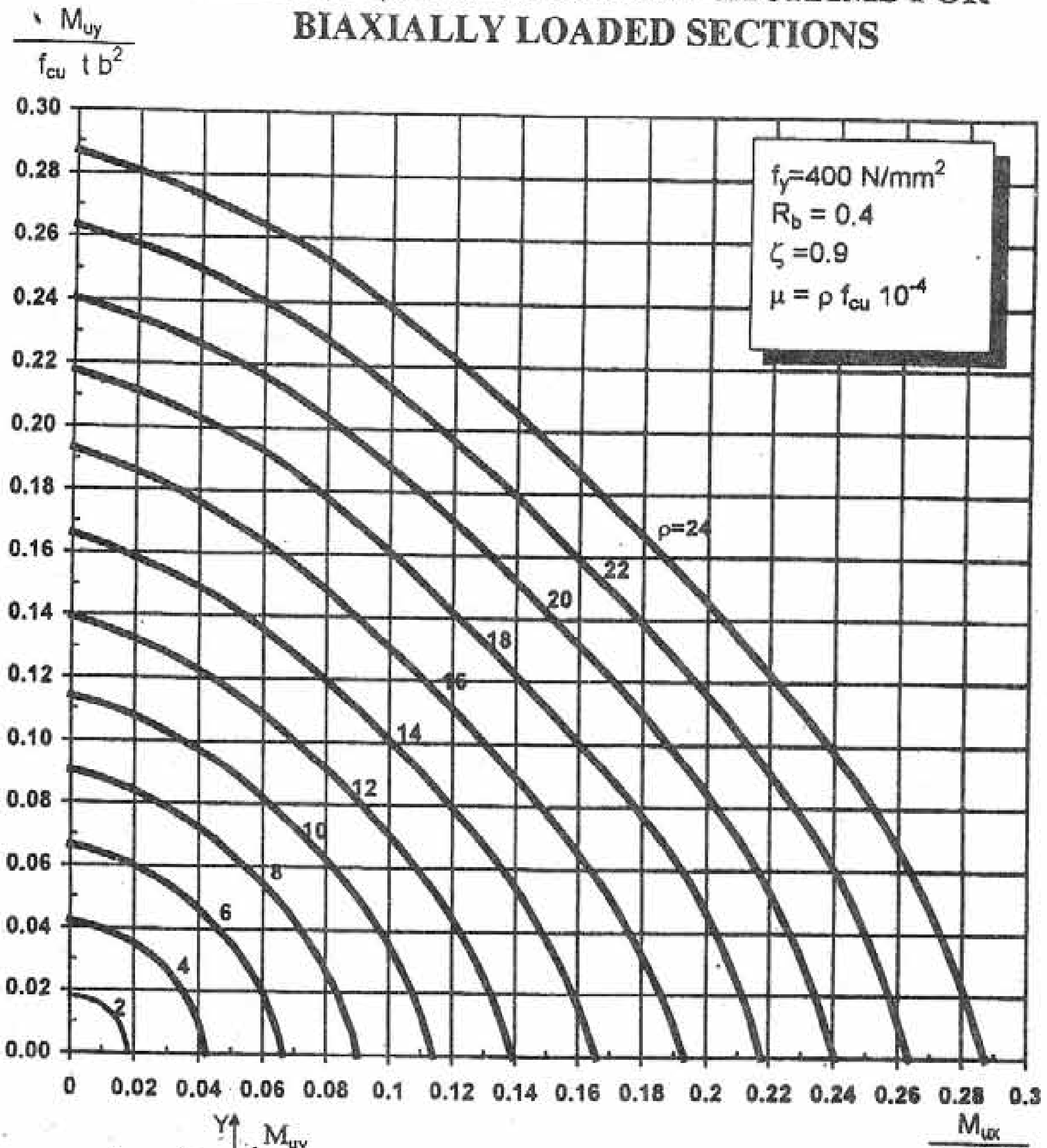


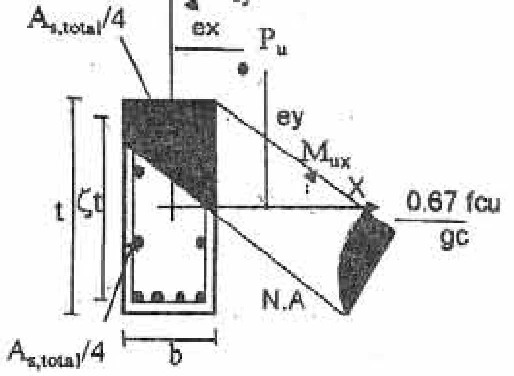
$$R_b = \frac{P_u}{f_{cu}\ b\ t}$$

$$\mu = \rho\ f_{cu}\ 10^{-4}$$

$$A_{s,total} = \mu\ b\ t$$

CHART (5-2): INTERACTION DIAGRAMS FOR BIAXIALLY LOADED SECTIONS



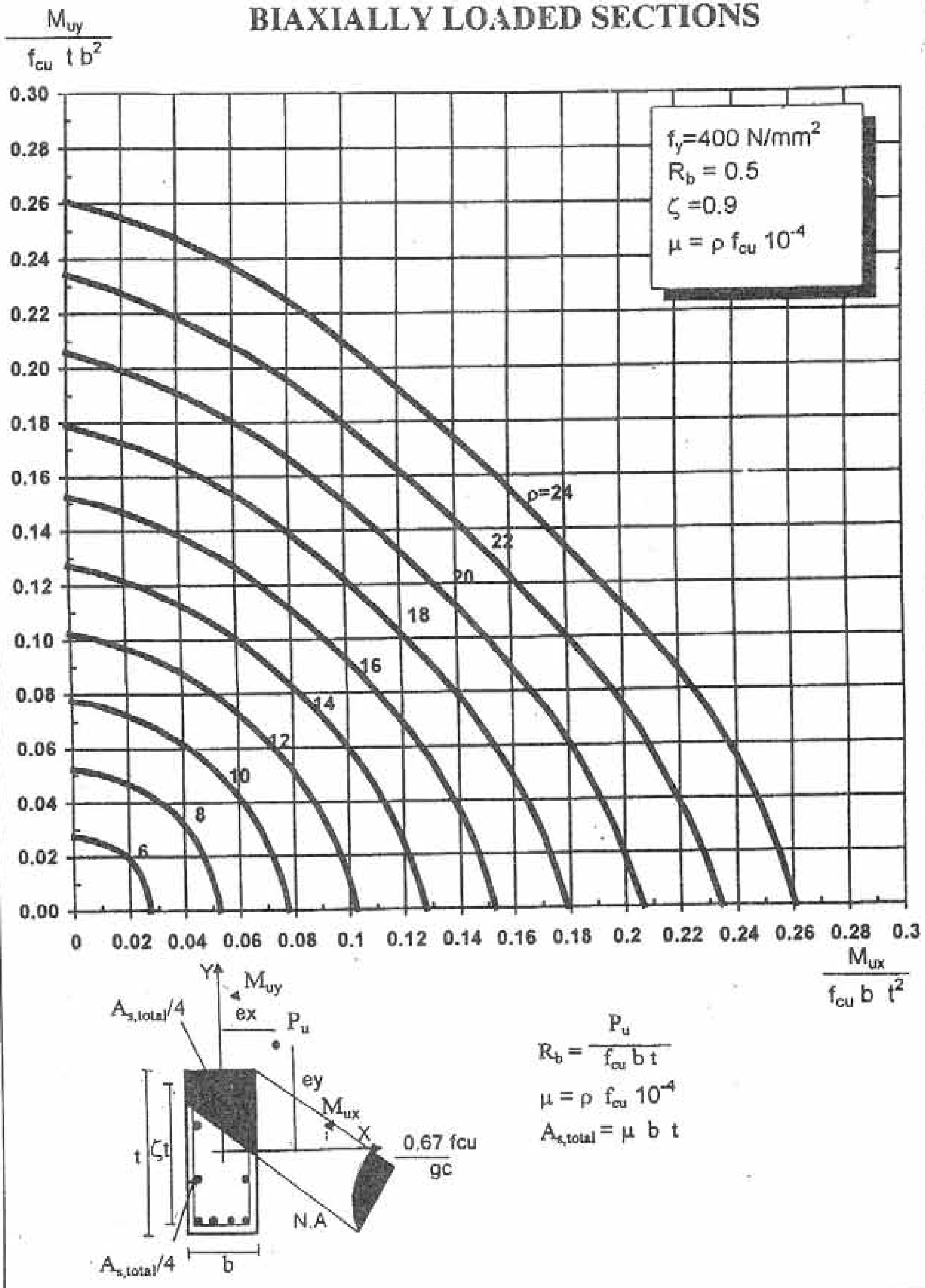


$$R_b = \frac{P_u}{f_{cu}\ b\ t}$$

$$\mu = \rho\ f_{cu}\ 10^{-4}$$

$$A_{s,total} = \mu\ b\ t$$

CHART (5-3): INTERACTION DIAGRAMS FOR BIAXIALLY LOADED SECTIONS



$$R_b = \frac{P_u}{f_{cu}\ b\ t}$$

$$\mu = \rho\ f_{cu}\ 10^{-4}$$

$$A_{s,total} = \mu\ b\ t$$

CHART (5-4): INTERACTION DIAGRAMS FOR BIAXIALLY LOADED SECTIONS

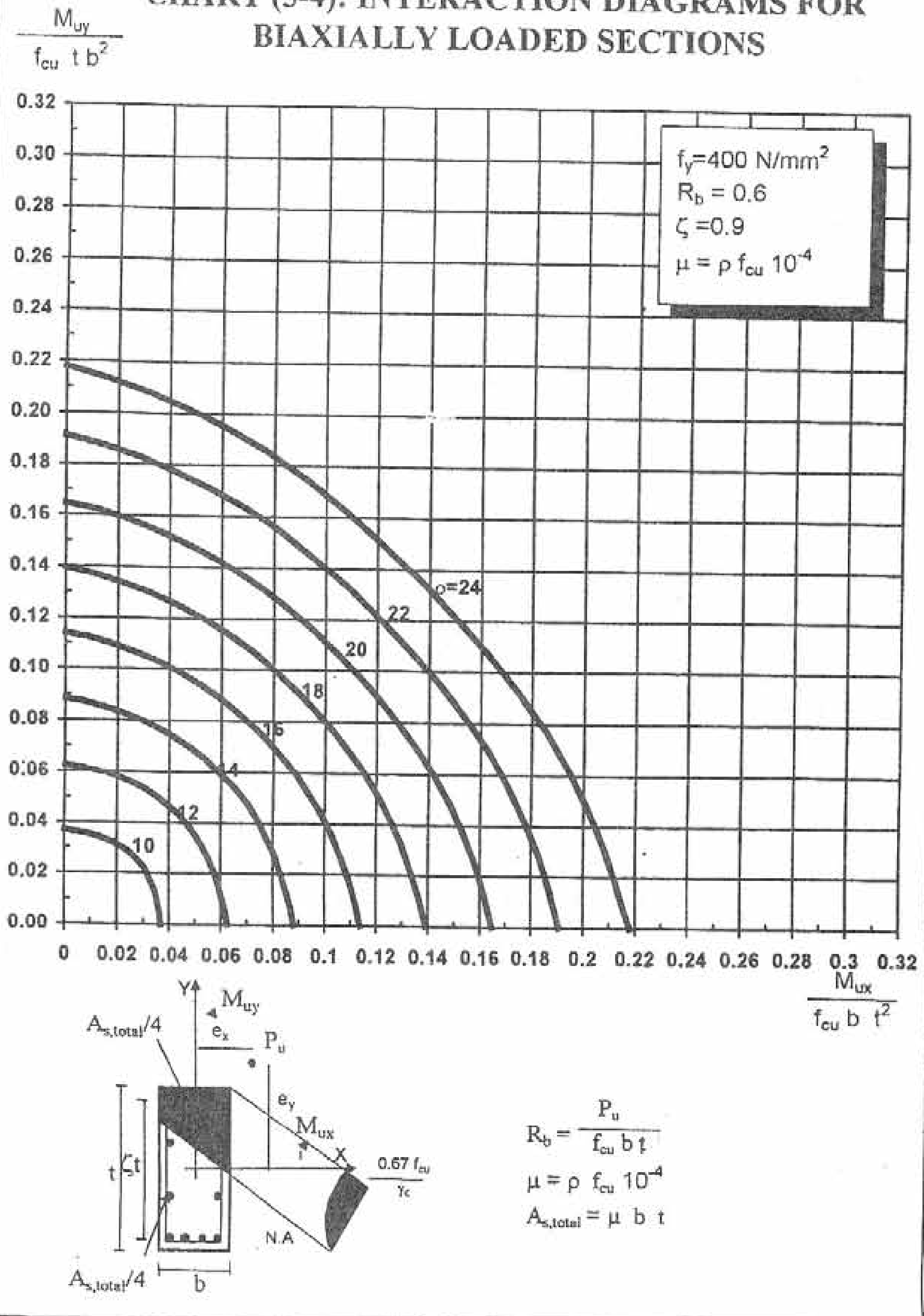
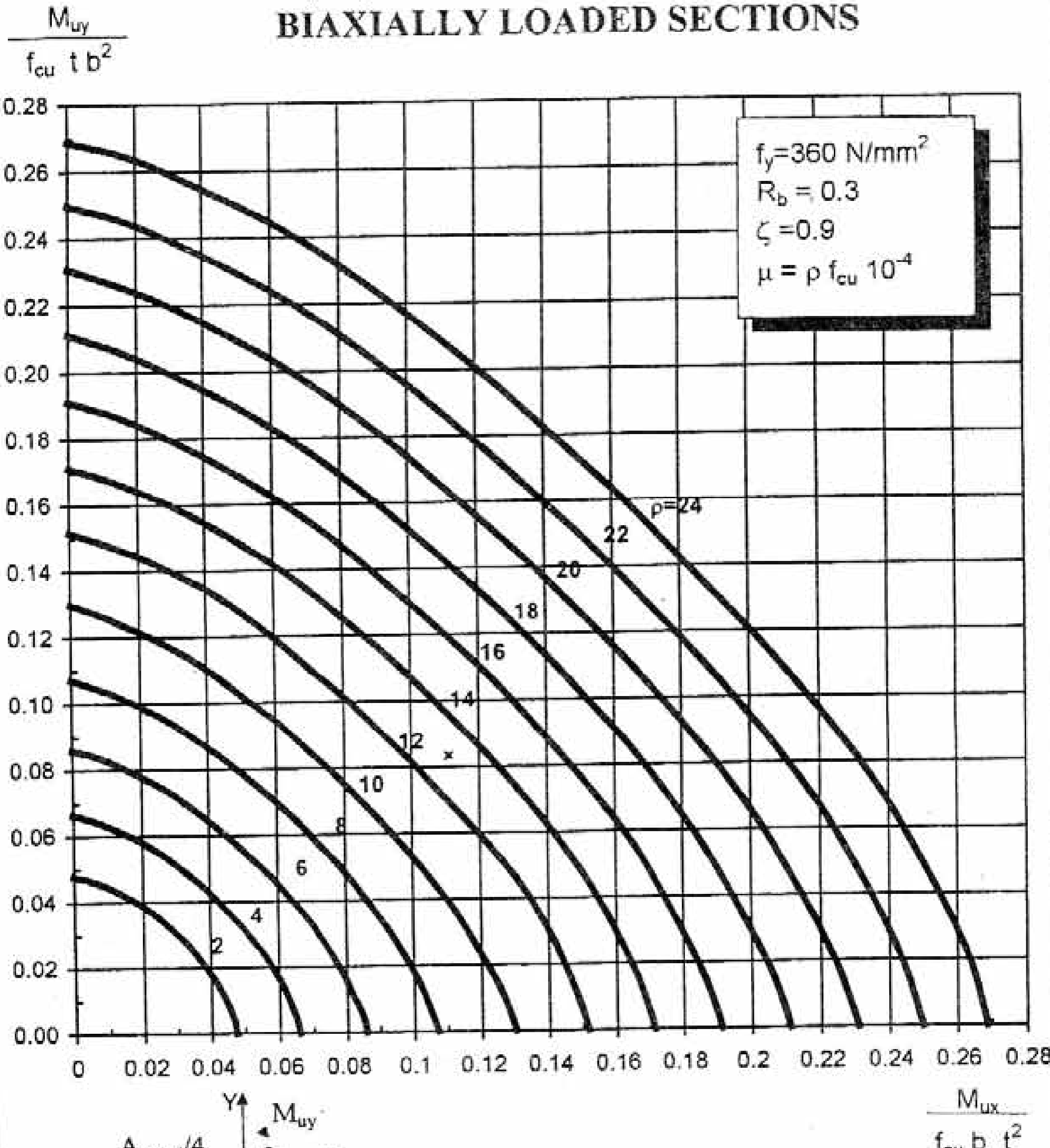
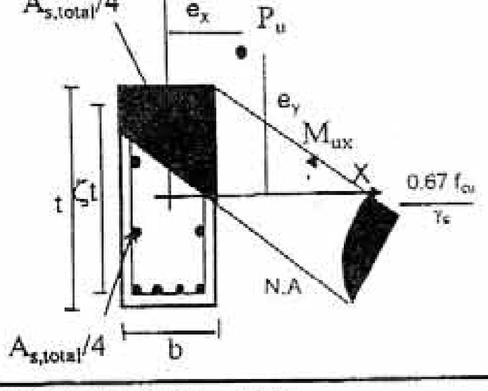


CHART (5-5): INTERACTION DIAGRAMS FOR BIAXIALLY LOADED SECTIONS



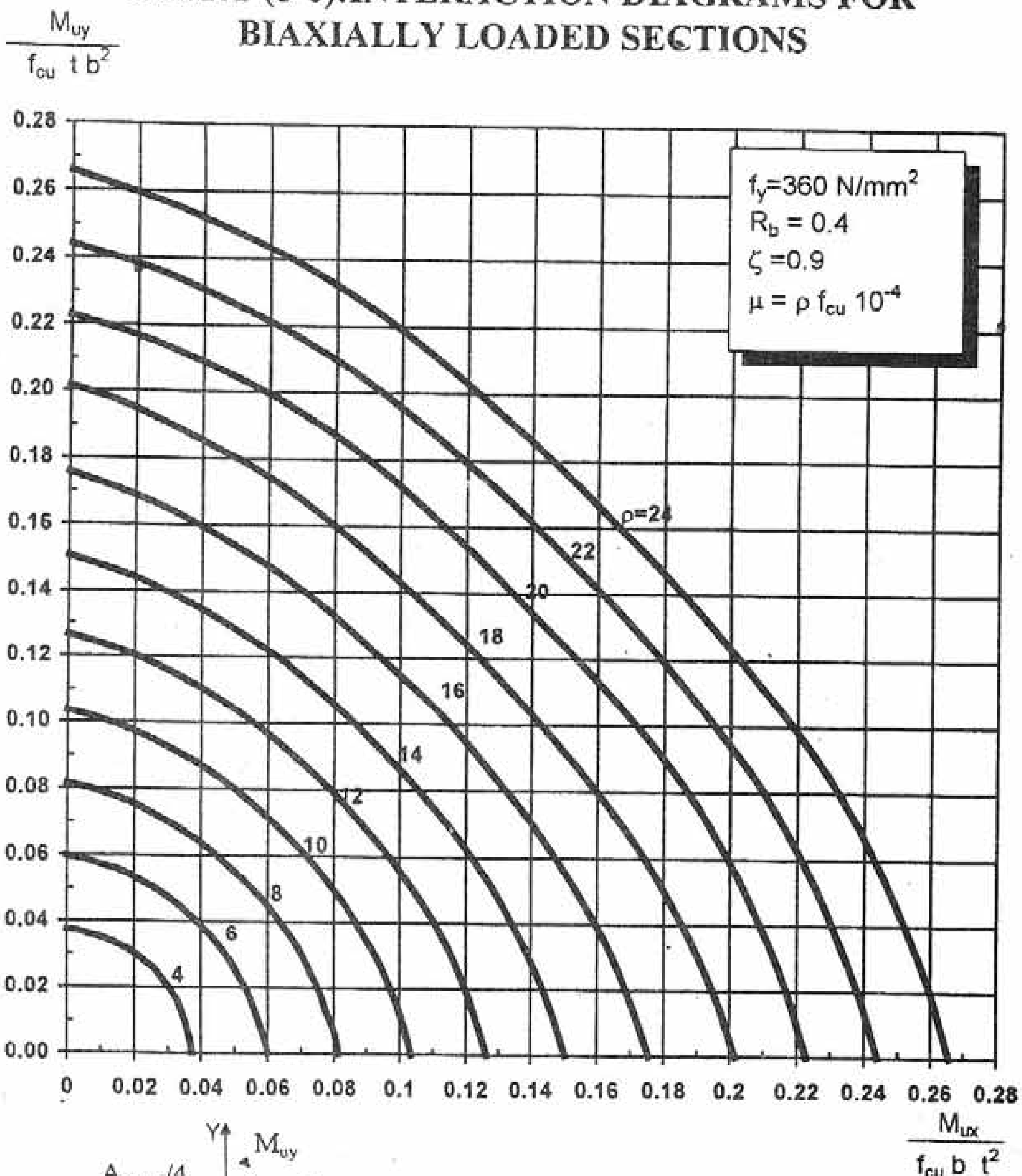


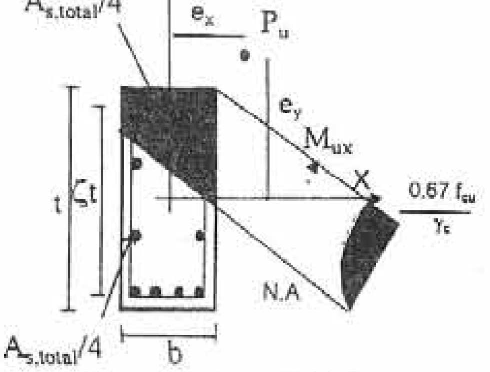
$$R_b = \frac{P_u}{f_{cu}\ b\ t}$$

$$\mu = \rho\ f_{cu}\ 10^{-4}$$

$$A_{s,total} = \mu\ b\ t$$

CHART (5-6): INTERACTION DIAGRAMS FOR BIAXIALLY LOADED SECTIONS



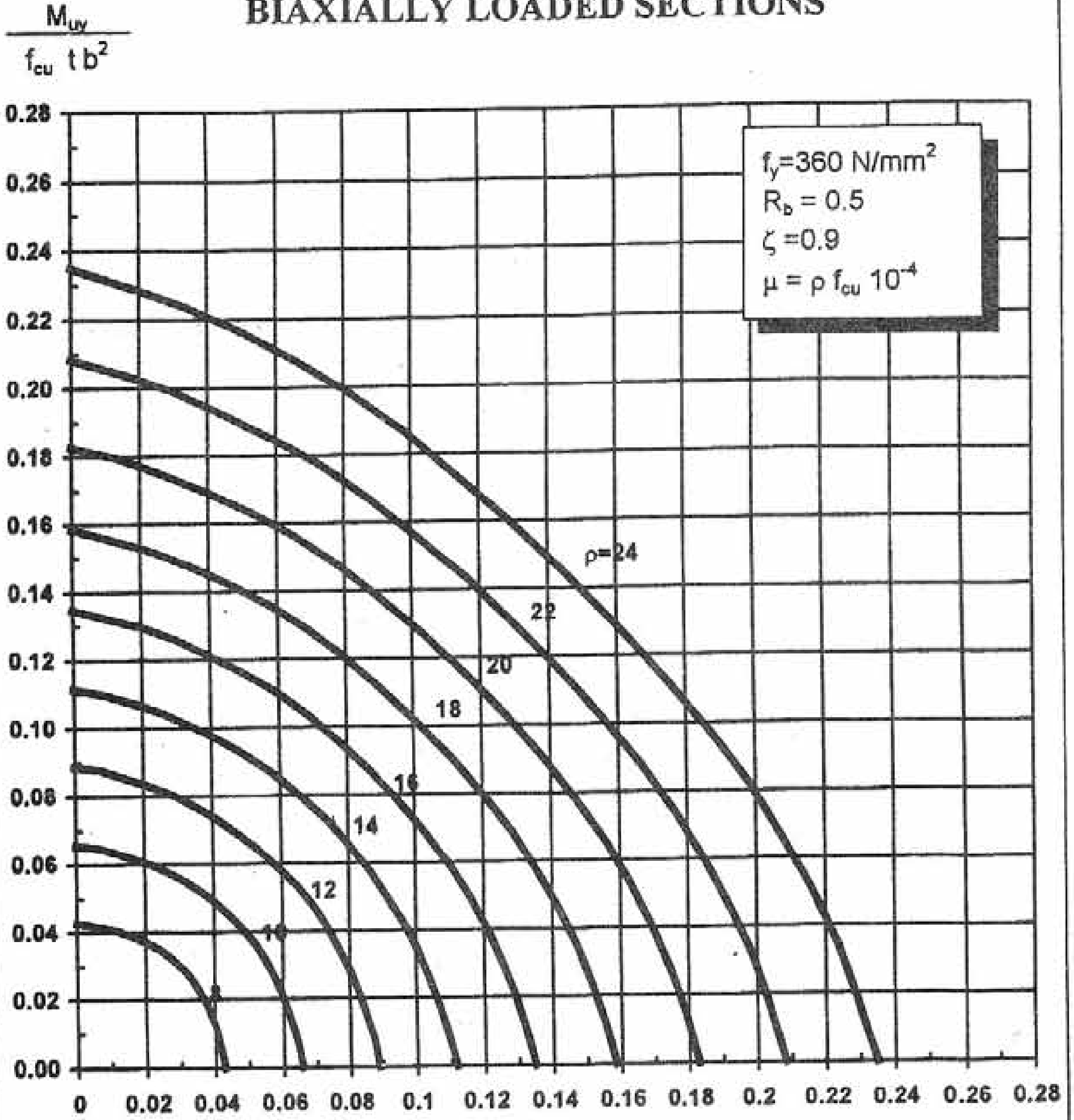


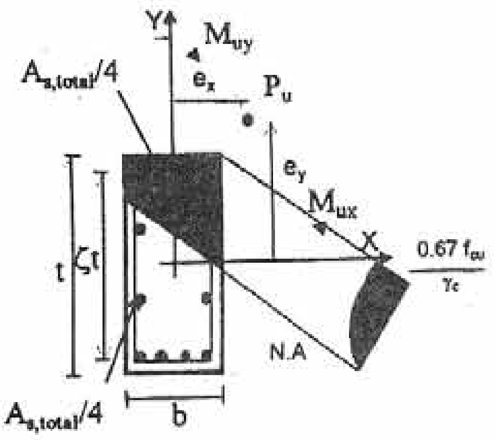
$$R_b = \frac{P_u}{f_{cu}\ b\ t}$$

$$\mu = \rho\ f_{cu}\ 10^{-4}$$

$$A_{s,total} = \mu\ b\ t$$

CHART (5-7):INTERACTION DIAGRAMS FOR BIAXIALLY LOADED SECTIONS





$$R_b = \frac{P_u}{f_{cu}\ b\ t}$$

$$\mu = \rho\ f_{cu}\ 10^{-4}$$

$$A_{s,total} = \mu\ b\ t$$

CHART (5-8): INTERACTION DIAGRAMS FOR BIAXIALLY LOADED SECTIONS

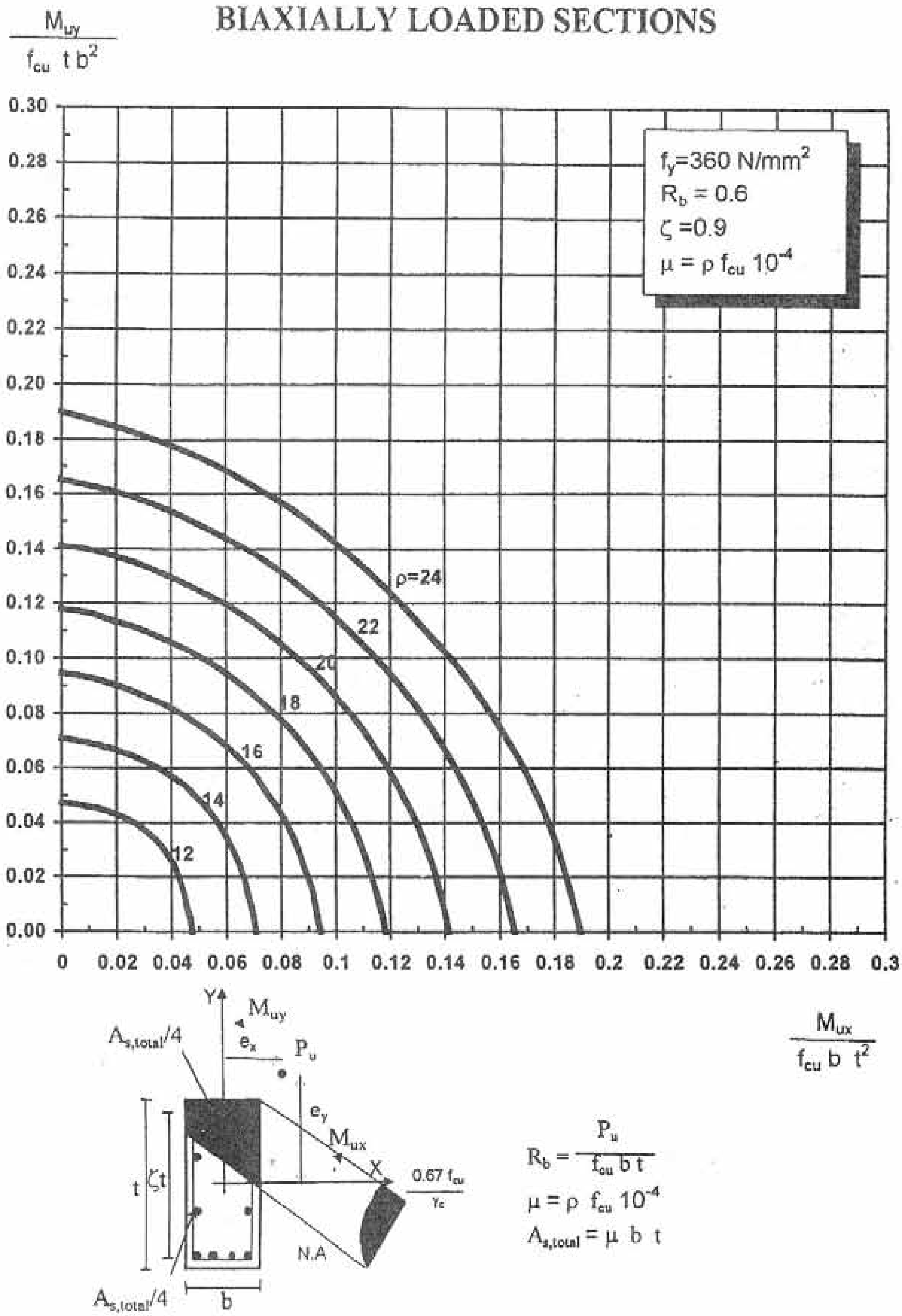
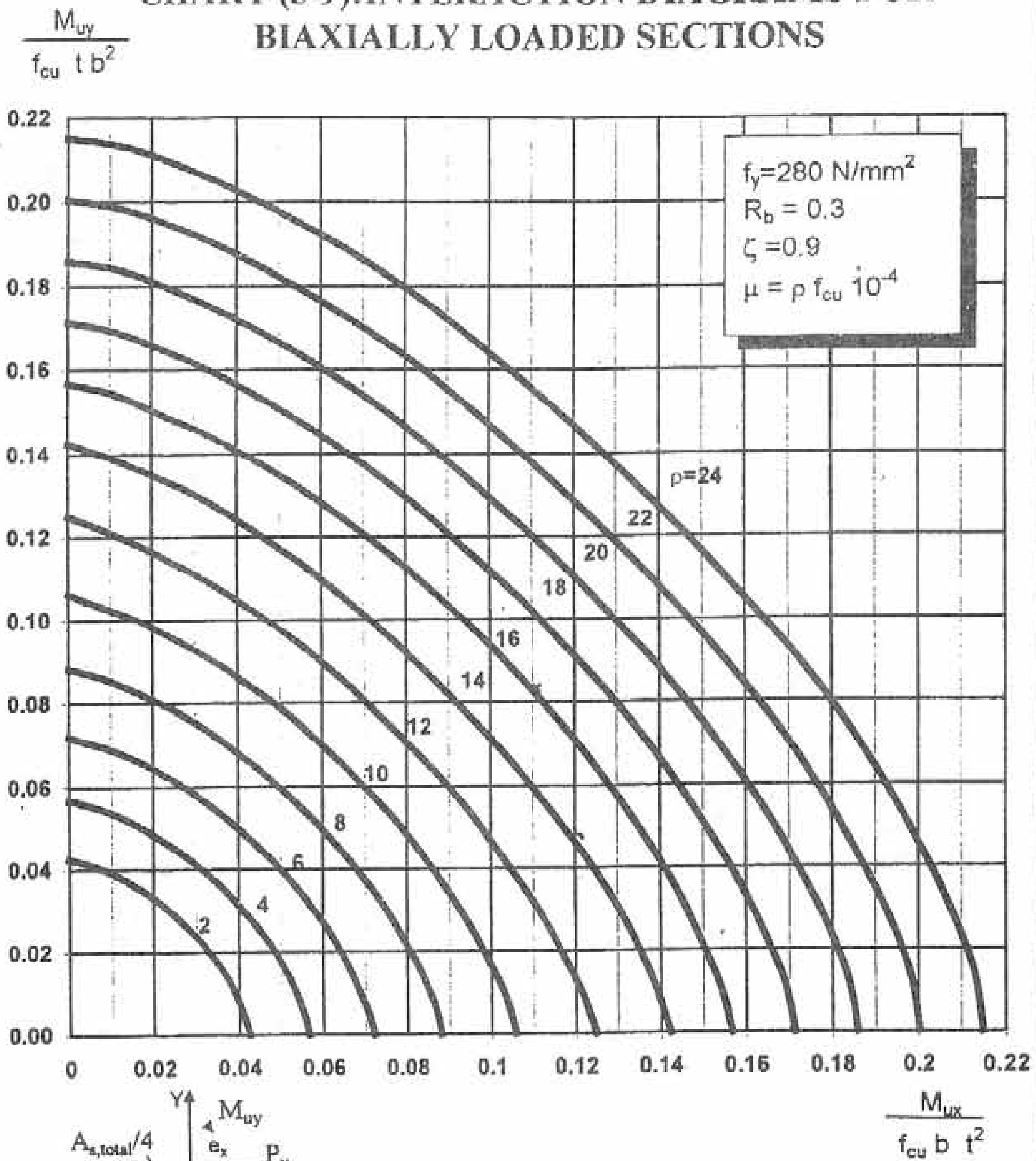
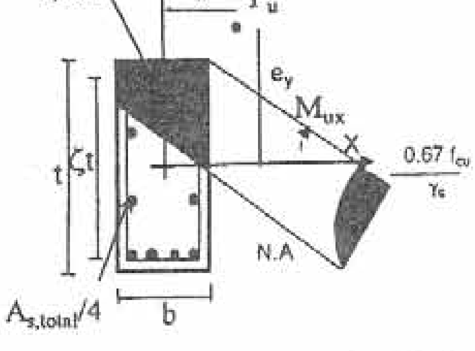


CHART (5-9):INTERACTION DIAGRAMS FOR BIAXIALLY LOADED SECTIONS



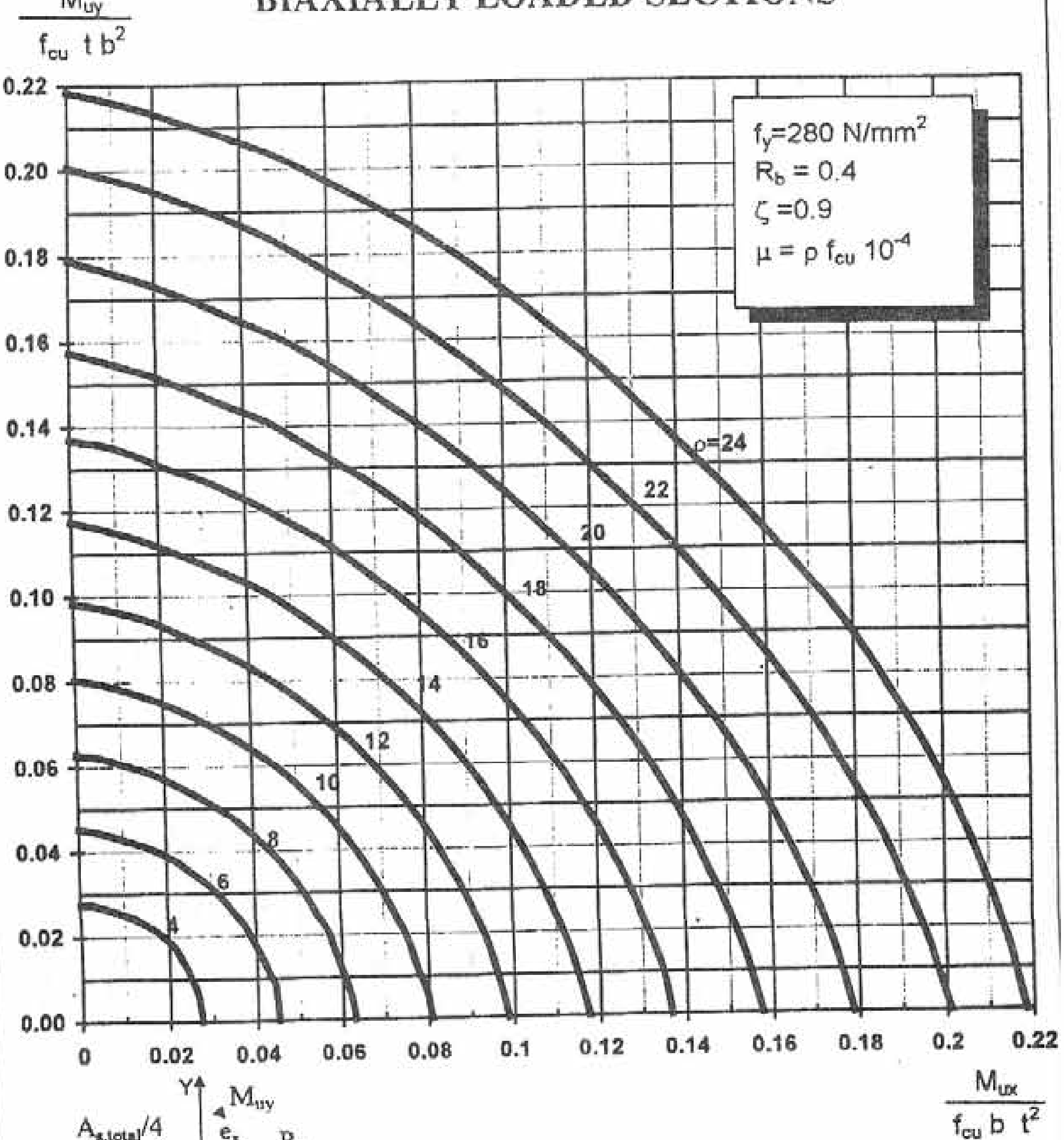


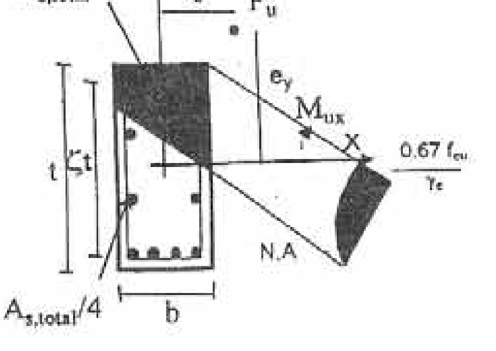
$$R_b = \frac{P_u}{f_{cu}\ b\ t}$$

$$\mu = \rho\ f_{cu}\ 10^{-4}$$

$$A_{s,total} = \mu\ b\ t$$

CHART (5-10):INTERACTION DIAGRAMS FOR BIAXIALLY LOADED SECTIONS



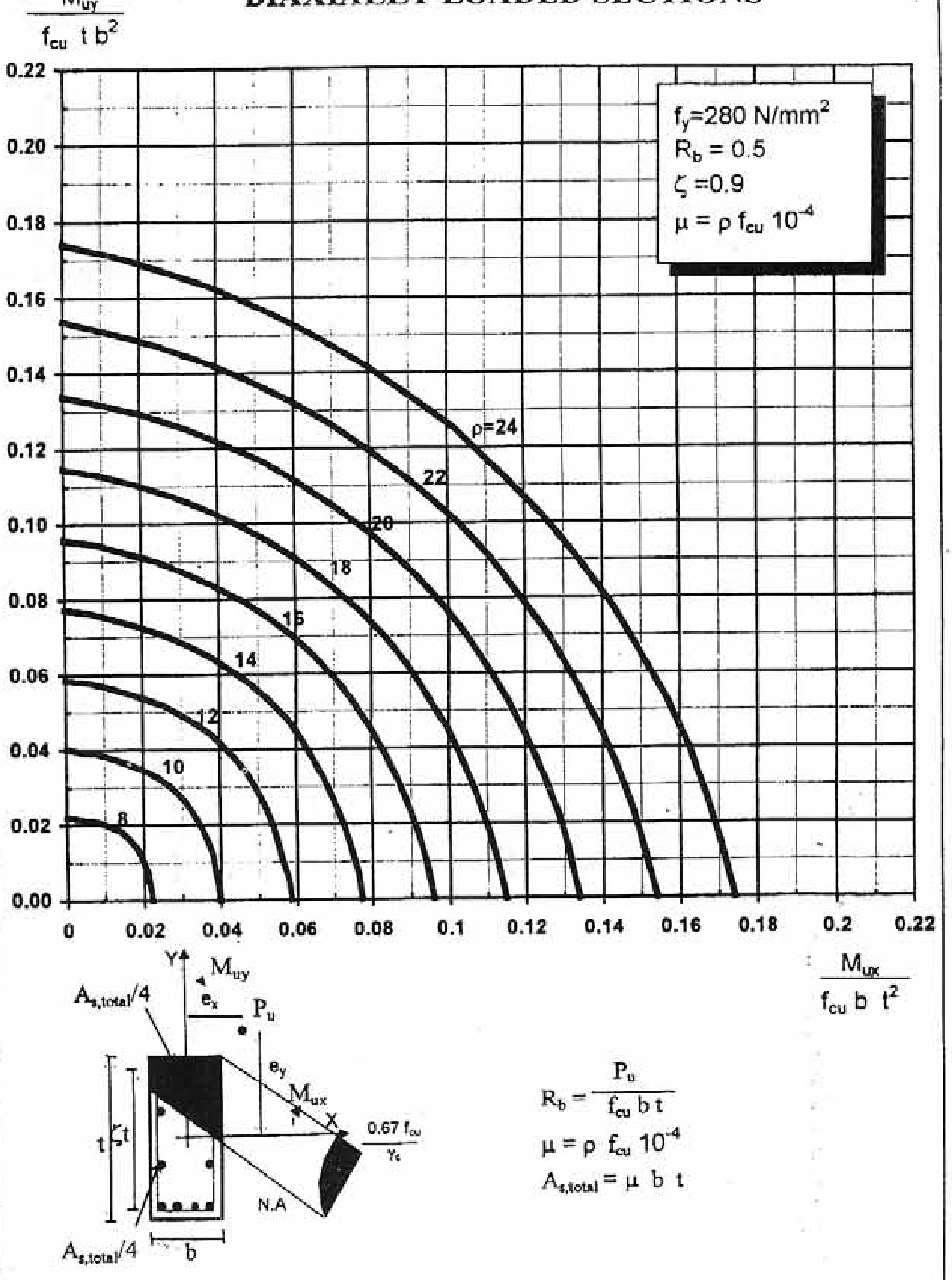


$$R_b = \frac{P_u}{f_{cu}\ b\ t}$$

$$\mu = \rho\ f_{cu}\ 10^{-4}$$

$$A_{s,total} = \mu\ b\ t$$

CHART (5-11):INTERACTION DIAGRAMS FOR BIAXIALLY LOADED SECTIONS

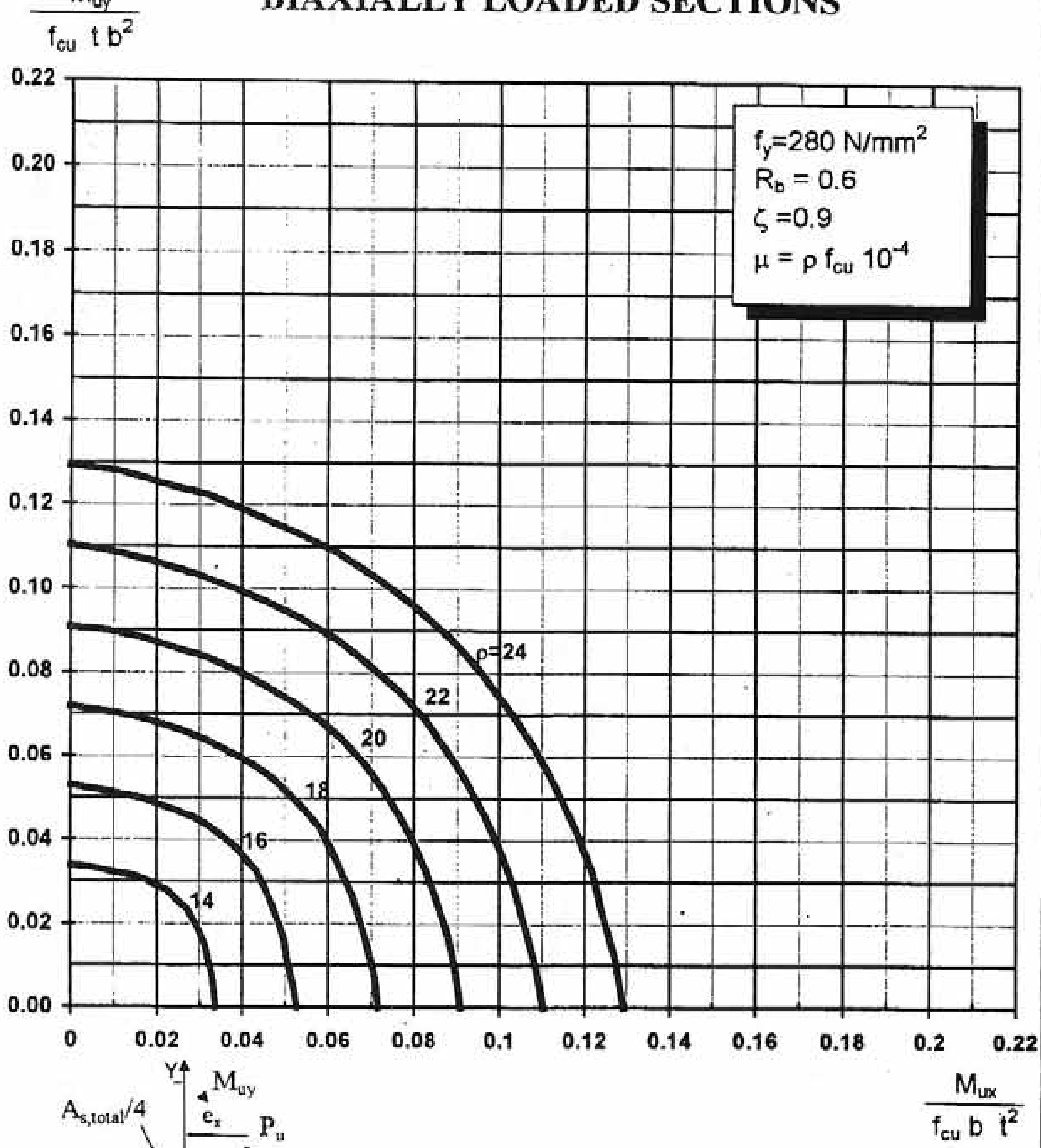


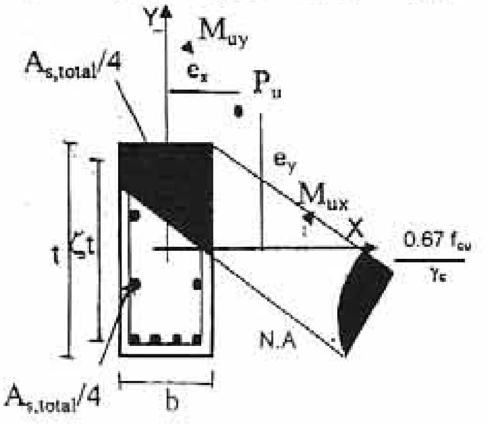
$$R_b = \frac{P_u}{f_{cu}\ b\ t}$$

$$\mu = \rho\ f_{cu}\ 10^{-4}$$

$$A_{s,total} = \mu\ b\ t$$

CHART (5-12): INTERACTION DIAGRAMS FOR BIAXIALLY LOADED SECTIONS





$$R_b = \frac{P_u}{f_{cu}\, b\, t}$$

$$\mu = \rho\ f_{cu}\ 10^{-4}$$

$$A_{s,total} = \mu\ b\ t$$

CHART (5-13): INTERACTION DIAGRAMS FOR BIAXIALLY LOADED SECTIONS

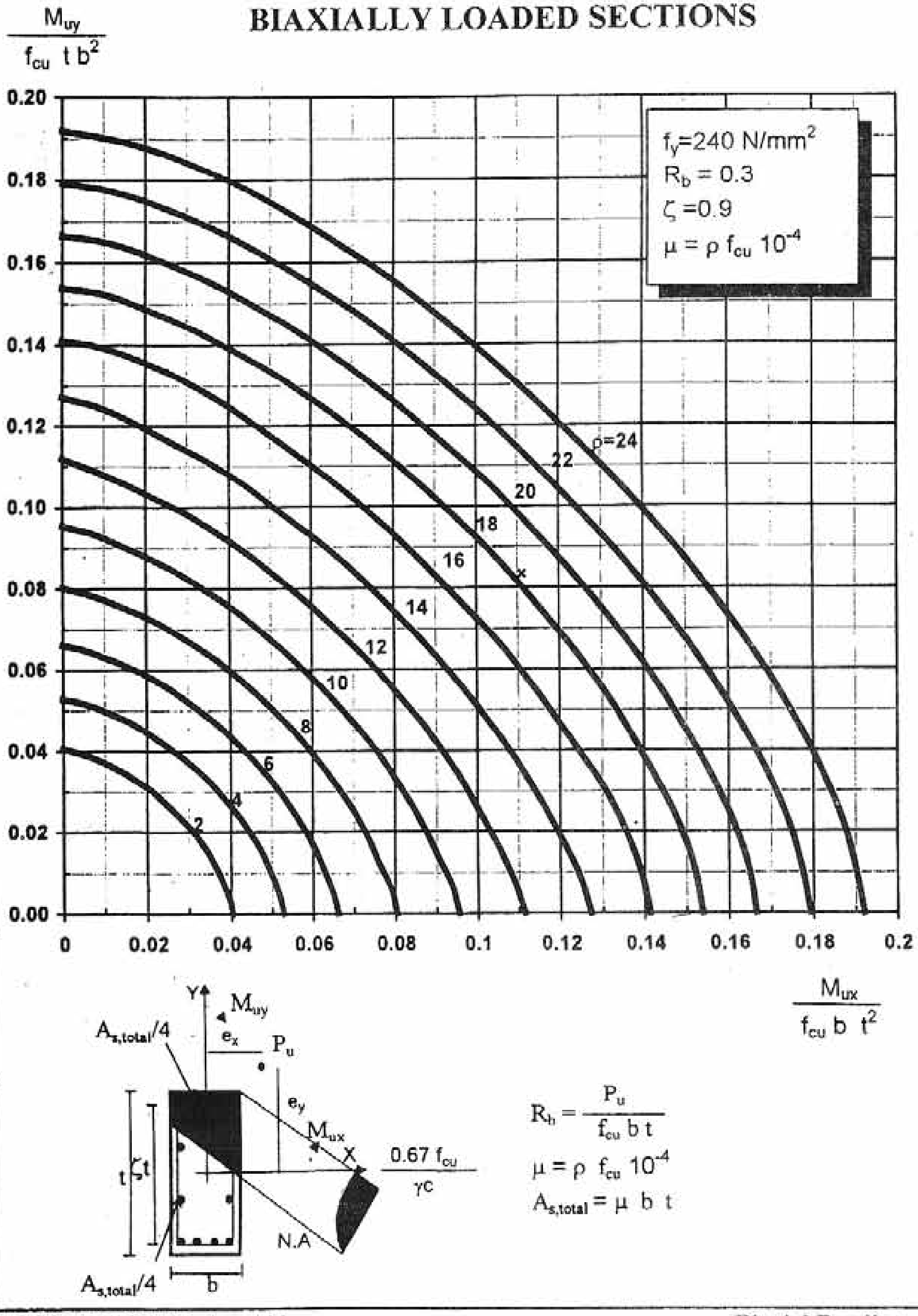
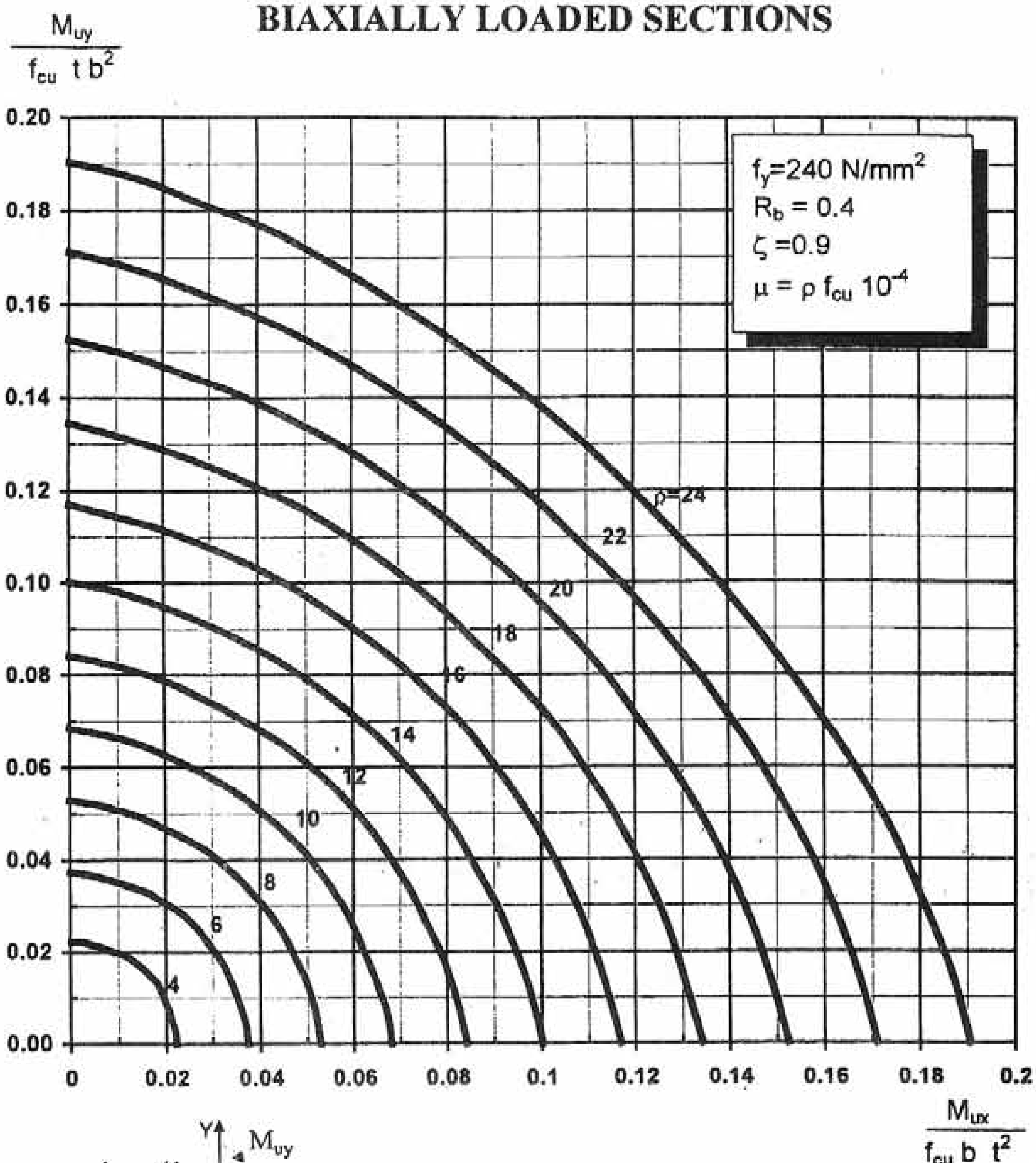
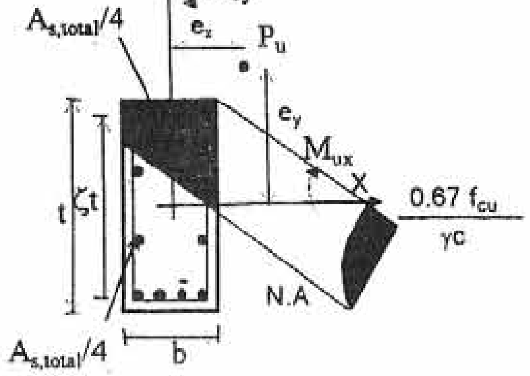


CHART (5-14): INTERACTION DIAGRAMS FOR BIAXIALLY LOADED SECTIONS



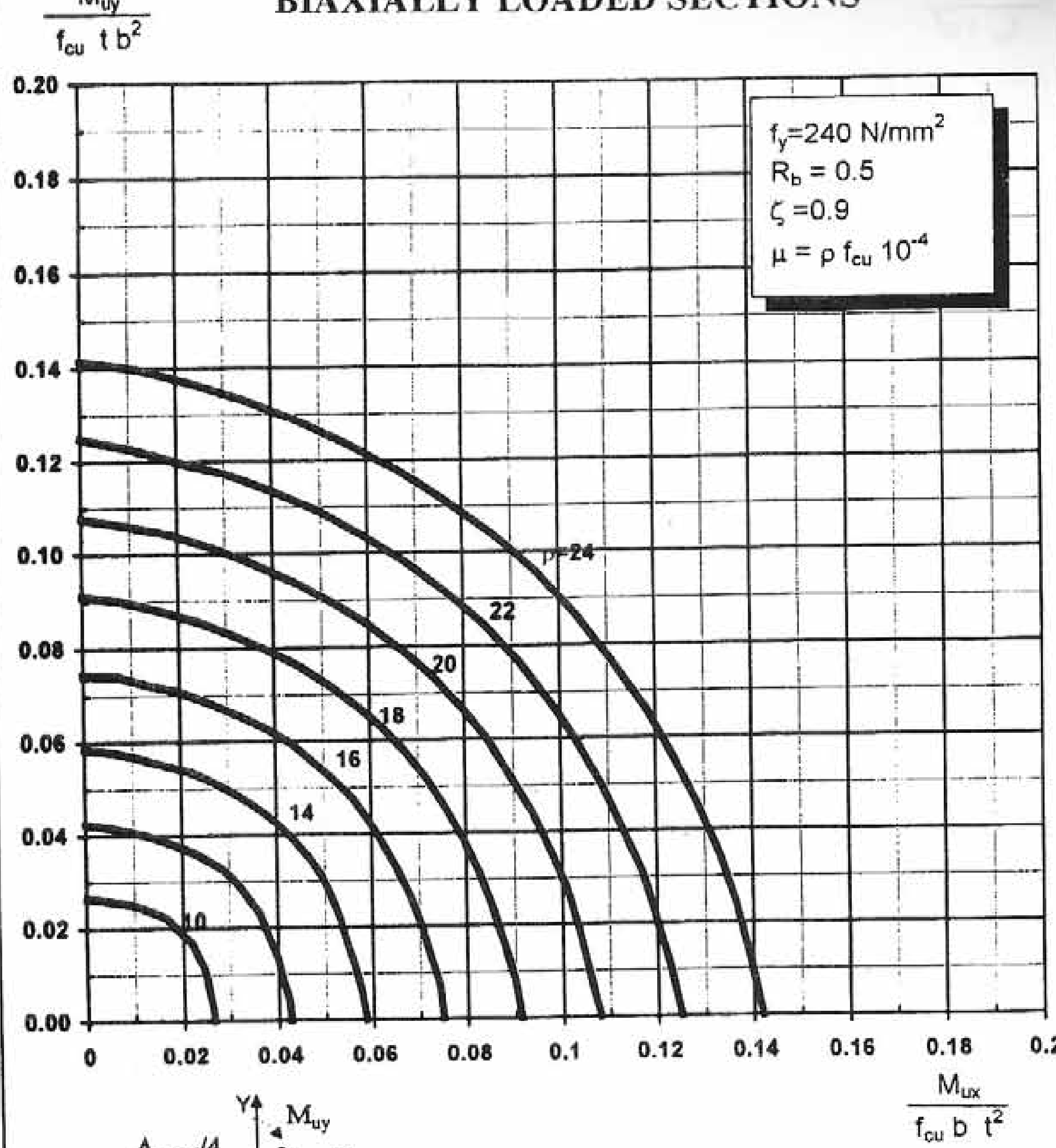


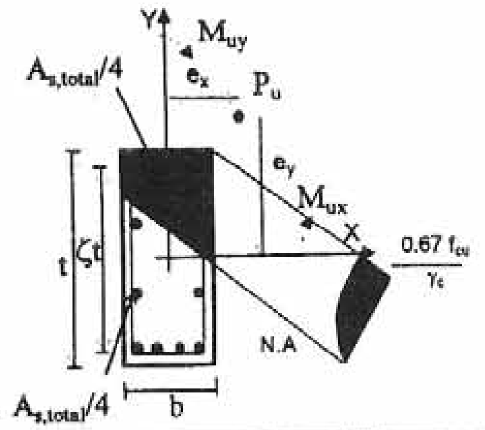
$$R_b = \frac{P_u}{f_{cu}\ b\ t}$$

$$\mu = \rho\ f_{cu}\ 10^{-4}$$

$$A_{s,total} = \mu\ b\ t$$

CHART (5-15): INTERACTION DIAGRAMS FOR BIAXIALLY LOADED SECTIONS



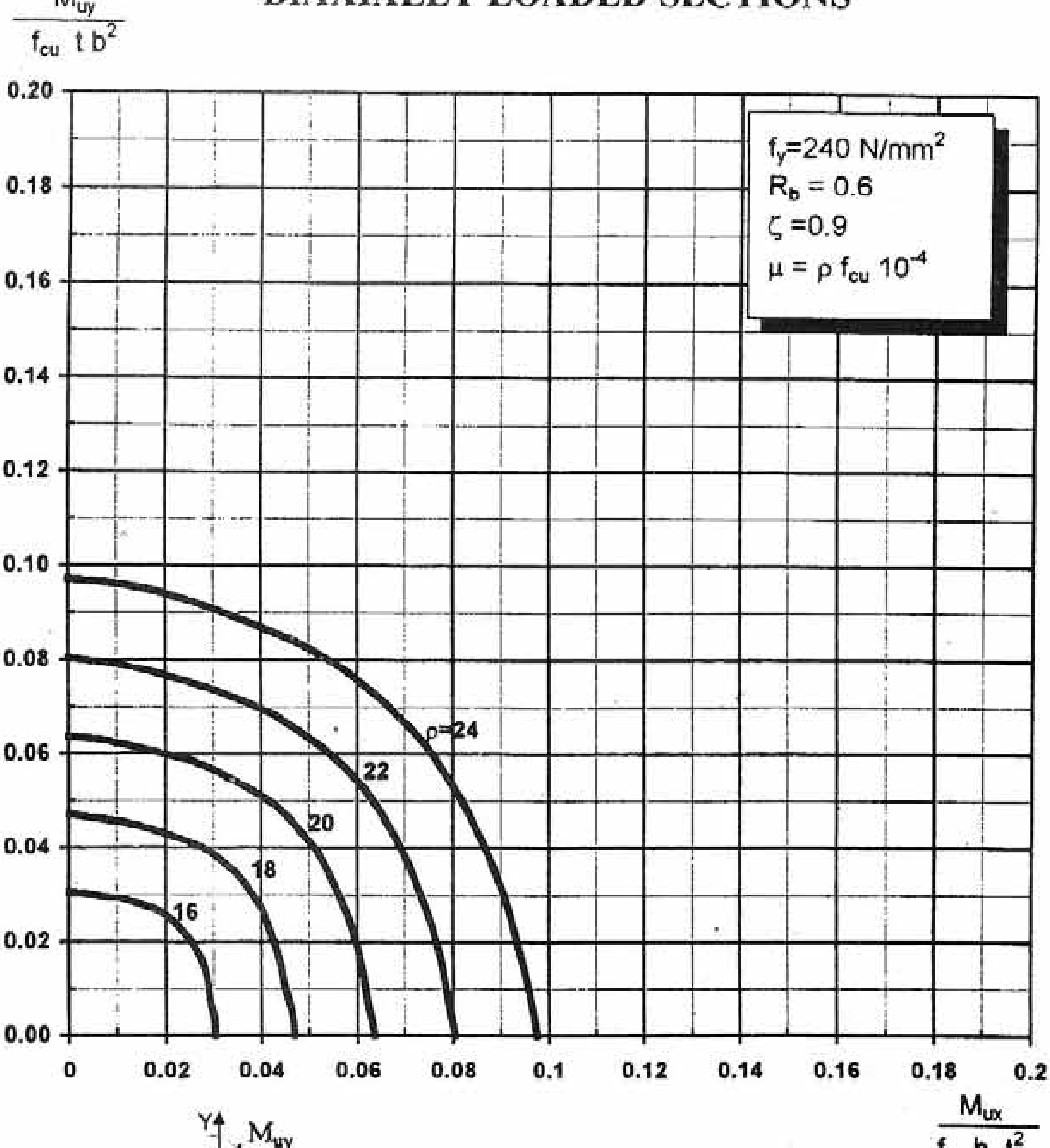


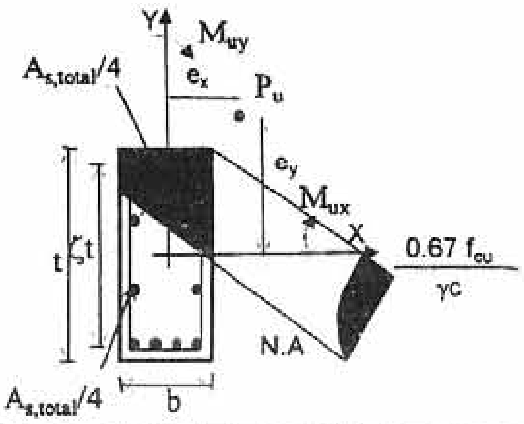
$$R_b = \frac{P_u}{f_{cu}\ b\ t}$$

$$\mu = \rho\ f_{cu}\ 10^{-4}$$

$$A_{s,total} = \mu\ b\ t$$

CHART (5-16): INTERACTION DIAGRAMS FOR BIAXIALLY LOADED SECTIONS





$$R_b = \frac{P_u}{f_{cu}\ b\ t}$$

$$\mu = \rho\ f_{cu}\ 10^{-4}$$

$$A_{s,total} = \mu\ b\ t$$

6. REINFORCED CONCRETE SLENDER COLUMNS

Columns are compression members that have:

1) Height greater than 5 times the smaller side (h>5b)

2) Length not more than five times the breadth (t>5b)

If length is greater than five times the breadth (t>5b) the member is considered as a wall

Braced and Unbraced Buildings

A Building can be considered braced if it is provided with walls extended to the full height of the building and connected to the foundation and the following equations are met:

For buildings that consist of 4 floors or more:

$$\alpha = H_b \sqrt{\frac{N}{\sum EI}} < 0.6 \qquad \text{Code Eq.(6.31.a)}$$

For buildings that consist of less than 4 floors:

$$\alpha = H_b \sqrt{\frac{N}{\sum EI}} < 0.2 + 0.1n \qquad \text{Code Eq.(6.31.a)}$$

H_b = Total height of building above the foundation level

N = Total unfactored vertical loads carried by all vertical elements

$\sum EI$ = Summation of the flexural rigidities of the walls sharing in supporting the building

n = Number of building floors

Slender Columns

According to the ECCS 203-2001, slender columns are defined as those that have slenderness ratios less than those mentioned in Table (6-7) but not more than those mentioned in Table (6-8)

Table (6-7) Limits of Slenderness Ratio for Short Columns

Column Status	λ_t or λ_b	λ_D	λ_i
Braced	15	12	50
Unbraced	10	8	35

Table (6-8) Limits of Slenderness Ratio for Slender Columns

Column Status	λ_t or λ_b	λ_D	λ_i
Braced	30	25	100
Unbraced	23	18	70

Slenderness Ratio (λ_b) = H_e/b

Slenderness Ratio (λ_i) = H_e/i

b =Smaller dimension of the section

i = Radius of gyration

= 0.25 D circular section

= 0.30 b rectangular section

H_e = Effective length

= $K*H_0$

H_o = Clear height of the column between end restraints

K =depends on column's end conditions and whether it is braced or unbraced

Calculation of K

For braced columns, K is the smaller of:

$$K = [0.7 + 0.05(\alpha_1 + \alpha_2)] < 1.0$$ Code Eq.(6.32.a)

$$K = [0.85 + 0.05(\alpha_{min})] < 1.0$$ Code Eq.(6.32.b)

For unbraced columns, K is the smaller of:

$$K = [1.0 + 0.15(\alpha_1 + \alpha_2)] > 1.0$$ Code Eq.(6.33.a)

$$K = [2.0 + 0.30(\alpha_{min})] > 1.0$$ Code Eq.(6.33.b)

$$\alpha = \frac{\sum(E_c I_c / H_o)}{\sum(E_b I_b / L_b)}$$ Code Eq.(6.34)

α_1, α_2 = Ratio of the sum of the column stiffness to the sum of the beam stiffness at column lower and upper ends

α_{min} = Smaller of α_1 & α_2

Special cases

- For a column fixed to the base α=1.0
- For a column hinged to the base α=10

Straining Actions for Slender Braced Columns

The effect of buckling can be considered as an additional moment

(1) Additional Moments

$$M_{add} = P\,\delta$$ Code Eq.(6.35)

For rectangular columns in the direction (t)

$$\delta = \frac{\lambda_t^2\, t}{2000}$$ Code Eq.(6.36.a)

For rectangular columns in the direction (b)

$$\delta = \frac{\lambda_b^2 \, b}{2000}$$ Code Eq.(6.36.b)

For circular columns of diameter (D)

$$\delta = \frac{\lambda_D^2 \, D}{2000}$$ Code Eq.(6.36.c)

(2) Design Moments

The Largest value of

a) M_2 b) $M_i + M_{add}$

c) $M_i + M_{add}/2$ d) $P.e_{min.}$ Code Eq. (6.37)

M_i is the moment at the critical section located near the mid-height and is calculated from:

$$M_i = 0.4M_1 + 0.6M_2 \geq 0.4M_2$$ Code Eq. (6.38)

For columns in double curvature, the sign of the moment M_1 is taken negative.

Straining Actions for Slender Columns in Unbraced Frames

(1) Additional moment

$$M_{add} = P \,.\, \delta_{av}$$ Code Eq. (6.39)

$$\delta_{av} = \sum \delta / n$$ Code Eq.(6.40)

δ_{av} = Average sway of the floor

n = the number of the columns in the floor

(2) Design moments

a) $M_2 + M_{add}$ b) $P.e_{min.}$

Example 1

Figure 1 shows a structural plan for a 10 story residential building. The following data are given:
Thickness of flat plate of all floors = 220 mm, the floor cover=1.5 kN/m^2, the equivalent wall loads =2 kN/m^2 , the live load = 3 kN/m^2, the height of the ground floor is 5000.0 mm. and the typical floor height is 3000.0 mm. The weight of walls and columns is equal to 18000 kN. Assume E=2.5x10^4 N/mm^2
It is required to check the bracing condition of the building in both directions.

<u>Clause 6.4.2 of the ECCS 203-2001 states that:</u>

The building can be considered braced in one direction if Equation (6-30-a) is satisfied for that direction and have structural walls

$$\alpha = H_b \sqrt{\frac{N}{\sum EI}} < 0.6 \qquad (6.30.a)$$

where

H_b The total height of the building above the foundation level
N Total unfactored vertical loads acting on all vertical elements
$\sum EI$ Summation of flexural rigidity of all vertical walls in the direction under consideration

Step 1 Calculation of N

Unit weight of floor = Own weight + Floor cover + Equivalent wall load + Live load
= 0.220*25 + 1.5 + 2.0 + 3.0
= 12 kN/m^2
Total Weight of floor = Area * Unit weight

= (25*33-5*15)*12
= 9000 kN/floor

Total Weight of Building, N
= No. of floors* Weight of floor + weight of walls and columns
= 10*9000+18000
= 108000 kN

Step 2 Calculation of H_b
Height of building
=9*3000.0+5000.0
=32000.0 mm

Step 3 Calculation of α_x

The lateral bracing of the building in X-direction is achieved by four walls each of them is 5000mm*300mm

$$\sum EI_x = 2.5*10^4 * \{\frac{1}{12} * (300) * (5000.0)^3 * 4\}$$

$$= 31.25*10^{16} \text{ N.mm}^2$$

$$\alpha_x = H_b \sqrt{\frac{N}{EI_x}}$$

$$\alpha_x = 32000.0 * \sqrt{\frac{108000*10^3}{31.25*10^{16}}}$$

$$= 0.59 < 0.60$$ Braced Structure in X-direction

Step 4 Calculation of α_y

The lateral bracing of the building in Y-direction is achieved by four walls each of them is 4000mm*300mm

$$\sum EI_y = 2.5*10^4 * \{\frac{1}{12} * (300) * (4000.0)^3 * 4\}$$

$$= 16.0*10^{16} \text{ N.mm}^2$$

$$\alpha_y = H_b \sqrt{\frac{N}{EI_y}}$$

$$\alpha_y = 32000.0 * \sqrt{\frac{108000*10^3}{16.0*10^{16}}}$$

$$= 0.83 > 0.60$$ Unbraced Structure in Y-direction

N.B.

It is highly recommended not to place shear walls a long the corners of the building, otherwise vertical loads induced due to wind or earthquake should be included in the design of the foundation.

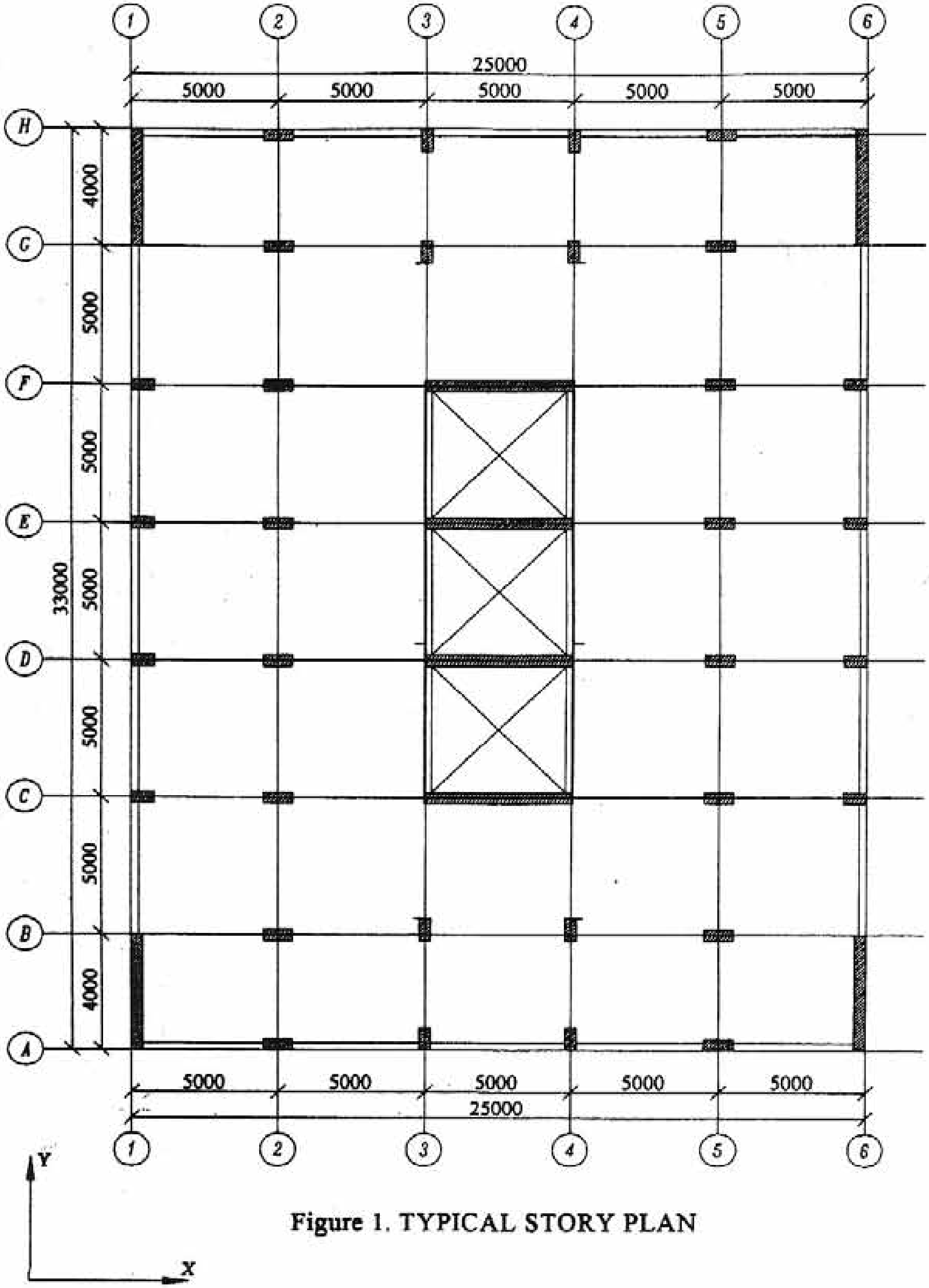


Figure 1. TYPICAL STORY PLAN

Example 2

Figure 2 shows a structural plan for a 15 story residential building. The total load at lower level is equal to 118823 kN and the floor height is equal to 3150 mm. It is required to check the bracing condition of the building in both directions. f_{cu}=25 N/mm^2

Clause 6.4.2 of the ECCS 203-2001 states that:

The building can be considered braced in one direction if Equation (6-30-a) is satisfied for that direction

$$\alpha = H_b \sqrt{\frac{N}{\sum EI}} < 0.6 \qquad (6.30.a)$$

where

H_b The total height of the building above the foundation level
N Total unfactored vertical loads acting on all vertical elements
$\sum EI$ Summation of flexural rigidity of all vertical walls in the direction under consideration

Step 1 Calculation of N

Total weight of building is equal to 118823 kN.

Step 2 Calculation of H_b

Height of building
=15*3150
=47250.00 mm

Step 3 Calculation of α_x

$E = 4400\sqrt{f_{cu}} = 4000\sqrt{25} = 22000\ N/mm^2$

The lateral bracing of the building in X-direction is achieved by the two cores

$$\sum I_x = 2 * 2.16 * 10^{13}\ \text{mm}^4$$
$$\sum EI_x = 22000 * \{2 * 2.16 * 10^{13}\}$$
$$= 95.04 * 10^{16}\ \ \text{N.mm}^2$$

$$\alpha_x = H_b \sqrt{\frac{N}{EI_y}}$$

$$\alpha_x = 47250 * \sqrt{\frac{118823 * 10^3}{95.04 * 10^{16}}}$$

$= 0.53 < 0.60$ Braced Structure in X-direction

Step 4 Calculation of α_y

The lateral bracing of the building in Y-direction is achieved by the two cores

$$\sum I_y = 2 * 0.34 * 10^{13} \text{ mm}^2$$

$$\sum EI_y = 22000 \times \{2 * 0.34 * 10^{13}\}$$

$$= 14.96 * 10^{16} \text{ N.mm}^2$$

$$\alpha_y = H_b \sqrt{\frac{N}{EI_y}}$$

$$\alpha_y = 47250 * \sqrt{\frac{118823 * 10^3}{14.96 * 10^{16}}}$$

$= 1.33 > 0.60$ Unbraced Structure in Y-direction

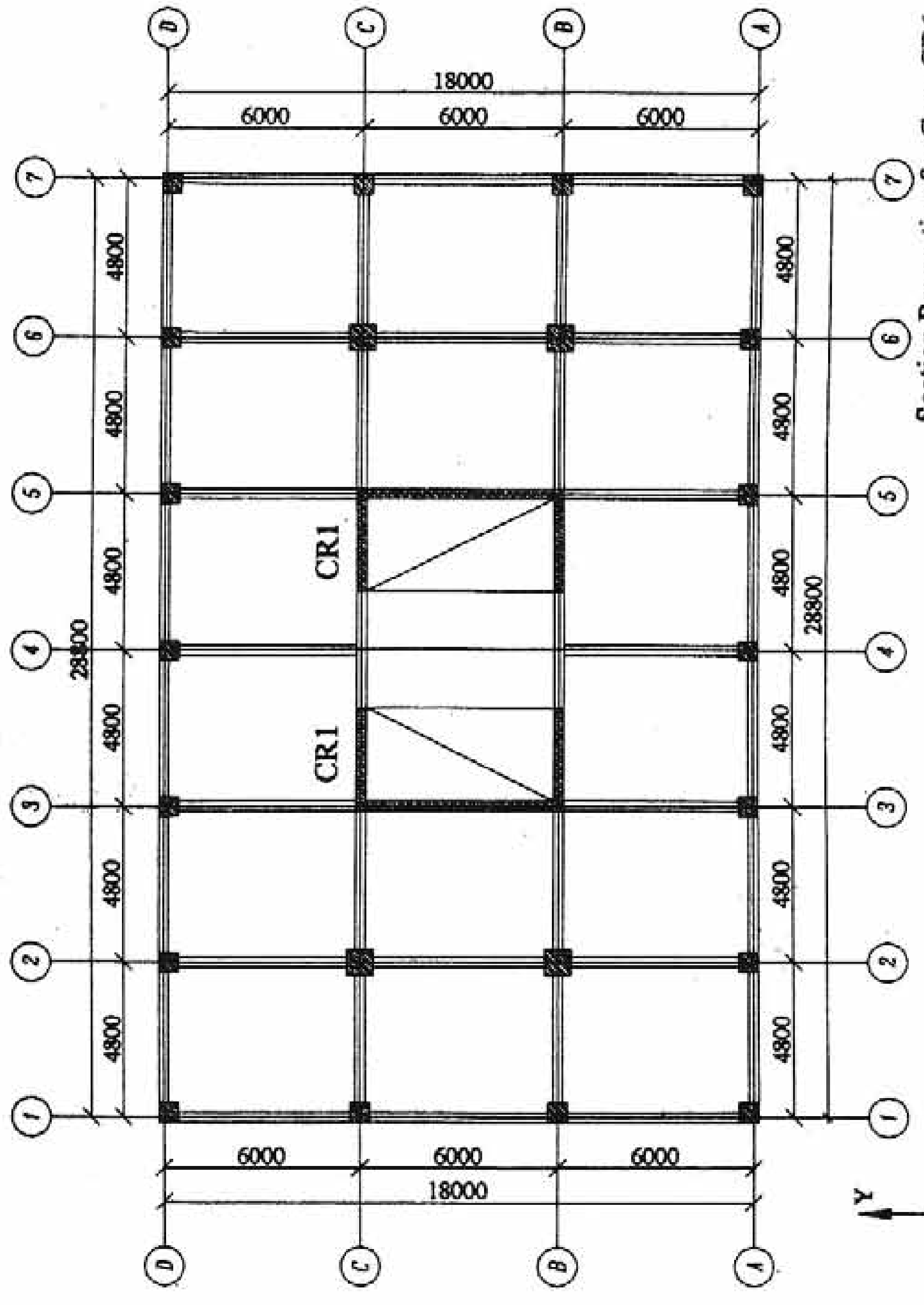


Figure 2 Typical Floor Plan

Section Properties for Core CR1

$A = 3600000\ mm^2$

$I_x = 2.16*10^{13}\ mm^4$

$I_y = 0.34*10^{13}\ mm^4$

7. SHEAR AND TORSION

7.1 Ultimate Shear Strength

Shear strength is evaluated in beams and slabs by calculating the nominal shear stress using the following relation:

$$q_u = \frac{Q_u}{bd} \text{ (N/mm}^2\text{)} \quad \text{....(Code 4-13)}$$

where:

Q_u is the ultimate shear force calculated at the critical section as shown in Figure (4-7) in the code. q_u is not an actual measurable stress but it represents the strength, including several components and factors affecting the strength, at a certain limit state. Two limits are usually recognizing the strength; a lower one to indicate the strength of a beam without web reinforcement and an upper one to indicate the capacity of beam with web reinforcement.

If the depth of the beam is variable, then the ultimate shear force is calculated using the following relation:

$$Q_{ur} = Q_u \pm \frac{M_u \tan\beta}{d} \quad \text{.... (Code 4-14)}$$

where: β is the inclination angle measured at the beam axis and tan β is not greater than 0.33. The minus sign in the equation refers to the case where the depth increases with the increase of the bending moment and vice versa. The nominal shear stress is obtained using the following relation:

$$q_u = \frac{Q_{ur}}{bd} \quad \text{(N/mm}^2\text{)}$$

In this section shear stresses are evaluated assuming that the shear stress resulting from torsion al moments can be ignored, i.e. they are less than the following:

$$q_{tu} = 0.06\sqrt{\frac{f_{cu}}{\gamma_c}} \quad \text{(N/mm}^2\text{)....(Code 4-17)}$$

7.1.1 Maximum shear strength

The maximum shear strength is obtained using the following equation:

$$q_u(\max.) = 0.70\sqrt{\frac{f_{cu}}{\gamma_c}} \quad \text{(N/mm}^2\text{).... (Code 4-15)}$$

but not more than 3 N/mm^2

7.1.2 Concrete shear strength

Concrete strength is obtained using the following equation:

$$q_{cu} = 0.24\sqrt{\frac{f_{cu}}{\gamma_c}} \quad (N/mm^2) \qquad \text{(Code 4-18)}$$

Between the two strength limits given in Table 7.1, reinforcement may be introduced in the form of stirrups or bent bars in addition to stirrups.

Table (7.1): Values of q_{cu} and $q_{u\,(max)}$. (N/mm^2)

f_{cu} (N/mm^2)	20	25	30	35	40
q_{cu}	0.88	0.98	1.07	1.16	1.24
$q_{u(max.)}$	2.56	2.86	3.00	3.00	3.00

If the section is subjected to compression, concrete shear strength is increased by multiplying q_{cu} by the following factor:

$$\delta_c = 1 + 0.07\frac{P_u}{A_c} \le 1.5 \qquad \text{(Code 4-19)}$$

where P_u is the axial compression force and A_c is the area of the concrete section.

Table (7.2): Concrete shear strength (q_{cu} with the presence of axial compression force)

$\frac{P_u}{A_c}$ N/mm^2	f_{cu} (N/mm^2)				
	20.00	25.00	30.00	35.00	40.00
0.00	0.88	0.98	1.07	1.16	1.24
1.00	0.94	1.05	1.15	1.24	1.33
2.00	1.00	1.12	1.22	1.32	1.41
3.00	1.06	1.19	1.30	1.40	1.50
4.00	1.12	1.25	1.37	1.48	1.59
5.00	1.18	1.32	1.45	1.57	1.67
6.00	1.24	1.39	1.52	1.65	1.76
7.00	1.32	1.46	1.60	1.73	1.85
8.00	1.32	1.47	1.61	1.74	1.86
9.00	1.32	1.47	1.61	1.74	1.86
10.00	1.32	1.47	1.61	1.74	1.86

If section is subjected to tensile force, q_{cu} is taken=0 or the concrete shear strength can be multiplied by the factor

$$\delta_t = 1 - 0.2\frac{P_u}{A_c} \qquad \text{..............................(Code 4-20)}$$

where P_u is the axial tensile force and A_c is the area of the concrete section.

Table (7.3): Concrete shear strength (q_{cu} with the presence of axial tensile force)

$\frac{P_u}{A_c}$ N/mm^2	f_{cu} (N/mm^2)				
	20	25	30	35	40
0	0.88	0.98	1.07	1.16	1.24
1	0.86	0.96	1.05	1.14	1.21
2	0.84	0.94	1.03	1.11	1.19
3	0.82	0.92	1.01	1.09	1.16
4	0.81	0.90	0.99	1.07	1.14
5	0.79	0.88	0.97	1.04	1.12
6	0.77	0.86	0.94	1.02	1.09
7	0.75	0.84	0.92	1.00	1.07
8	0.74	0.82	0.90	0.97	1.04
9	0.72	0.80	0.88	0.95	1.02
10	0.70	0.78	0.86	0.93	0.99

7.2 Design Criteria

In order to obtain a safe section , the ultimate shear stress should be less or equal to the maximum shear strength as follows:

$q_u \le q_{u(max.)}$

if $q_{cu} \le q_u \le q_{u(max.)}$ Design shear reinforcement

if $q_u \le q_{cu}$ Use minimum shear reinforcement as follows:

$$A_{st} = \mu_{min} bs = \frac{0.4}{f_y} b\, s \qquad \text{...(Code 4-28)}$$

where A_{st} is the area of the vertical branches of the stirrup, s is the spacing between stirrups and f_y is the yield stress (N/mm^2). Stirrups may not be less than 5ϕ6/m' μ_{min}=0.15% for ordinary mild steel and μ_{min}=0.10% for high tensile steel.

7.3 Shear Strength Of A Section With Shear Reinforcement

If the value of shear stress q_u exceeds the concrete shear strength q_{cu}, special web reinforcement should be introduced. The web (shear) reinforcement may take one of the following forms:

1- Vertical stirrups (perpendicular to the axis of member).
2- Inclined stirrups.
3- Perpendicular stirrups and bent bars and or inclined stirrups with an angle not less than 30^0 with beam axis.

$$q_{su} = q_u - 0.5\, q_{cu} \qquad \text{.........(Code 4-21)}$$

where q_u is the nominal shear strength, and q_{su} is the shear strength of the shear reinforcement.

$$q_{su} = q_{sus} + q_{sub} \qquad \text{.........(Code 4-24)}$$

where q_{sus} is the strength of the vertical stirrups and q_{sub} is the strength of the bent bars.

Ultimate shear resistance of vertical stirrups.

$$q_{sus} = \frac{A_{st}}{s.b}\,\frac{f_y}{\gamma_s} \qquad \text{.........(Code 4-22)}$$

Where b: the width of the section web.
A_{st}: The total area of available vertical branches of the stirrup .

$$A_{st} = n * A_{s\phi}$$

n : Number of the vertical branches of the stirrup
$A_{s\varphi}$: Cross sectional area of one branch of the stirrup.
s : Spacing of the stirrups.

- **Ultimate shear resistance of inclined stirrups or bent bars with more than one row:**

$$\frac{A_{sb}}{s.b} = \frac{q_{sub}}{\dfrac{f_y}{\gamma_s}\,(\sin\alpha + \cos\alpha)} \qquad \text{.........(Code 4-23)}$$

i.e. $$q_{sub} = \frac{A_{sb}}{s\,b} \times \frac{f_y}{\gamma_s}\,(\sin\alpha + \cos\alpha)$$

where α is the inclination angle of the bent bars or the inclined stirrups with respect to the center line of the member, A_{sb} is the cross sectional area of the bent bars.

If $\alpha=45^0$ then q_{sub} can be obtained by:

$$q_{sub} = \frac{A_{sb}}{s\, b} \times \frac{f_y}{\gamma_s} \, (\sqrt{2})$$

Ultimate shear strength in the case of one row of bent bars.

$$\frac{A_{sb}}{b.d} = \frac{q_{sub}}{\frac{f_y}{\gamma_s} \sin\alpha}$$ ....(Code 4-26)

i.e. $$q_{sub} = \frac{A_{sb}}{b.d} \times \frac{f_y}{\gamma_s} \sin\alpha$$

If $\alpha=45^0$ then q_{sub} can be obtained by:

$$q_{sub} = \frac{A_{sb}}{b.d} \times \frac{f_y}{\gamma_s} \, (\frac{1}{\sqrt{2}})$$

but $$q_{sub} \le 0.24\sqrt{\frac{f_{cu}}{\gamma_c}}$$ ....(Code 4-27)

Table (7-4): : Summary of Shear Equations

Design step	Ultimate Strength Design	Code Equation
1:Shear stress	$q_u = \frac{Q_u}{bd}$	(4-13)
2:Check	$q_u \le q_{u(max)}$ where: $q_{u(max.)} = 0.70\sqrt{\frac{f_{cu}}{\gamma_c}}$ (N/mm^2) ≤ 3 N/mm^2	(4- 15)
3: Check	*if* $q_{cu} \le q_u \le q_u$(max.) shear reinforcement is required	
4:Check	*if* $q_u \le q_{cu}$ Use minimum shear reinforcement as follows: $\mu_{min} = \frac{0.4}{f_y}$ $\therefore A_{st}(\min) = \frac{0.4}{f_y} b\, s$ and the percent ratio of μ_{min} must be not less than: 0.15 for mild steel. 0.10 for high strength deformed steel but not less than 5Ø6mm/m	(4- 28)
5: Strength of shear Reinforcement	$q_{su} = q_u - 0.5\, q_{cu}$	(4-21)
6: Concrete shear strength	$q_{cu} = 0.24\sqrt{\frac{f_{cu}}{\gamma_c}}$ (N/mm^2)	(4 – 18)
7: For vertical stirrups:	$q_{sus} = \frac{A_{st}\, f_y}{s.b\, \gamma_s}$	(4-22)
8: For bent bars and inclined stirrups:	$q_{sub} = \frac{A_{sb}}{s\,b} \times \frac{f_y}{\gamma_s}(\sin\alpha + \cos\alpha)$	(4 –23)
9: For bent bars with one row	$q_{sub} = \frac{Asb}{b.d} \times \frac{fy}{\gamma_s}\ \sin\ \alpha$	(4 –26)
10:Stirrups maximum spacing	Maximum spacing of vertical stirrups: 250mm or d/2	
11:Bent bars maximum spacing	Maximum spacing of bent bars s_{max}=d *if* $q_u \le 1.50\ q_{cu}\ \therefore s_{max} = 1.5d$ *if* $q_u \le 1.00\ q_{cu}\ \therefore s_{max} = 2.0d$	

7.4 Solved Examples

Example1

Determine the required vertical stirrups for a beam section with the following data:

Beam size: 300*600 mm and d=550mm

f_{cu}=25 N/mm^2 & f_y= 240 N/mm^2

Ultimate Shear Force=250 kN

Calculation Steps:

Step 1: Find ultimate shear stress.

$$q_u = \frac{Q_u}{bd} = \frac{250\times10^3}{300*550} = 1.515 \quad N/mm^2$$

Step 2: Compare with the maximum ultimate shear stress from table 7-1 :

Since $q_{u(max.)} = 0.70\sqrt{\frac{f_{cu}}{\gamma_c}}$

$\therefore q_u\ (max) = 2.86\ N/mm^2$ < **3.0 N/mm²**

$\therefore q_u \prec q_{u(max)}$

$q_{cu} = 0.24\sqrt{\frac{25}{1.5}} = 0.98\ N/mm^2$

& $q_u \succ q_{cu}$ Then shear reinforcement is required and should be designed

Step3: Find shear stress carried by shear reinforcement

$$q_{su} = q_u - 0.5\,q_{cu} = 1.51 - \frac{.98}{2} = 1.02 \quad N/mm^2$$

If the shear stress is required to be resisted by vertical stirrups only , then:

$$q_{sus} = q_{su} = \frac{A_{st}}{s.b}\cdot\frac{f_y}{\gamma_s}$$

$$1.02 = \frac{A_{st}}{s}\cdot\frac{240/1.15}{300}$$

therefore , $\frac{A_{st}}{s} = 1.48$

$$\frac{A_{st,min}}{s} = \frac{0.4}{f_y}b = \frac{0.4}{240}300 = 0.5 \prec \frac{A_{st}}{s} \quad \cdots o.k.$$

For 2 branch stirrups of diameter = 10 mm the total area is:

$$A_{st} = 2 * 78.5 = 157\ mm^2$$

$$\frac{157}{s} = 1.48 \qquad \therefore s = 106.6\ mm$$

The maximum allowable spacing of stirrups is d/2 or 250mm
S_{max} =550/2 or 250mm=250mm

Choose the spacing=100 mm < S_{max}

Example 2

For the section given in example 1, find the required area of bent bars if 2 branch stirrups with a diameter = 8 mm spaced at 100 mm were chosen. For calculation purposes consider the following. f_{cu}=25 N/mm^2 , f_y= 240 N/mm^2 , f_y for bent bars = 400 N/mm^2 and
Ultimate Shear Force=250 kN

Calculation Steps:

Step1: find ultimate shear stress :

$$q_u = \frac{250 * 10^3}{300 * 550} = 1.52 N/mm^2$$

Step 2: Compare with the maximum ultimate shear stress from table 7-1 :

$$q_{u(max)} = 2.86\ N/mm^2 \prec 3.0\ N/mm^2$$

$$\therefore q_u \prec q_{u(max)}$$

& $q_u \succ q_{cu}$. Then shear reinforcement is required and should be designed.

$$q_{su} = q_u - 0.5\, q_{cu} = 1.51 - \frac{0.98}{2} = 1.02\ N/mm^2$$

In this example, shear reinforcement consists of vertical stirrups and bent bars.

Step 3: Find shear carried by stirrups.

$$q_{sus} = \frac{Ast}{s} \cdot \frac{240/1.15}{300} = \frac{2 * 50}{100} \ \frac{240/1.15}{300} = 0.696 N/mm^2$$

Step 4: Find shear strength of bent bars.

$q_{sub} = q_{su} - q_{sur} = 1.02 - 0.696 = 0.32 \qquad N/mm^2$

this difference is carried by bent bars depending on the number of rows of bent bars

a) If more than one row of bent bars is available then:

$$q_{sub} = \frac{A_{sb} * \frac{f_y}{\gamma_s}}{s * b}(\sin\alpha + \cos\alpha)$$

if α=45 degrees and s=d=550mm then,

$$0.32 = \frac{A_{sb} * \frac{400}{1.15}}{550 * 300}(\sin 45 + \cos 45)$$

$A_{sb} = 107.3 \quad mm^2$

b) If only one row of bent bars is available then

Check if $q_{sub} \le 0.24\sqrt{\frac{f_{cu}}{\gamma_c}} = 0.98 \quad N/mm^2$

$$q_{sub} = 0.32 \le 0.24\sqrt{\frac{25}{1.5}} = 0.98 \quad N/mm^2 \qquad \text{O.K.}$$

Find the area of bent bars:

$$0.32 = \frac{A_{sb} * \frac{400}{1.15}}{550 * 300}(\sin 45)$$

$A_{sb} = 214.6 mm^2$ Therefore Use 2 bars of ϕ 12 mm diameter.

7.5 Shear Friction and Shear Resistance in Brackets and Corbels

7.5.1 Shear friction

This phenomenon is recommended for application by the code Clause 4-2-2-4 as follows:

a- This method is to be applied when shear forces is transferred by friction, this situation resembles the cases of construction and casting joints.

b- The concrete resistance to shear is neglected and the complete shear force transfers through the reinforcing steel which can be calculated as follows:

1- if steel reinforcement is placed perpendicular to the shear plane:

$$A_{sf} = \frac{Q_u}{\mu\,(\frac{f_y}{\gamma_s})} + \frac{N_u}{(\frac{f_y}{\gamma_s})} \qquad (Code\ 4-35)$$

Where; μ is the shear coefficient given in item (c) below.

And ; N_u is the force acting perpendicular to the shear plane and given (+ve) sign when tension and (–ve) sign when compression.

2- if steel reinforcement is placed inclined by an angle = α_f to the shear plane:

$$A_{sf} = \frac{Q_u}{[(\frac{f_y}{\gamma_s})\,(\mu\ \sin\alpha_f + \cos\alpha_f)]} + \frac{N_u}{[(\frac{f_y}{\gamma_s})\sin\alpha_f\,]} \qquad (Code\ 4-36\,)$$

c- the coefficient μ can be taken as follows:

- For monolithically cast concrete $\mu = 1.20$
- For concrete at construction or casting joint having surface roughness within 5mm size $\mu = 0.80$
- For cases similar to the above but with roughness having sizes < 5mm $\mu = 0.50$
- The shear stress Q_u / A_c must not exceed $0.15\ f_{cu}$ nor 4 N/mm^2 where A_c is the area of concrete section that resists the shear stress and f_y should not exceed 400 N/mm^2

7.5.2 Brackets and corbels:

A bracket is a cantilever member whose length does not exceed its effective depth at the face of the support.

$a \ \leq \ d$

Also the total height at the end of the bracket is not less than half its height at the face of the support.(See Code Figure (4-10))

Bracket reinforcement is divided into three types:

1- Main reinforcement.
2- Horizontal stirrups.
3- Vertical stirrups.

1- Main reinforcement :

Main reinforcement is calculated as the maximum value of the following:

- $A_s = A_n + A_f$ (Code 4-37-A)

- $A_s = A_n + \frac{2}{3} A_{sf}$ (Code 4-37-B)

- $A_{s\,min} = 0.03 \frac{f_{cu}}{f_y} b\, d$

where ;

A_f is the area of main reinforcement to resist the bending moment M_u

$$M_u = Q_u.a + N_u(t + \Delta - d) \quad \text{(Code 4-38)}$$

where Q_u is the ultimate shear force which should not be larger than:

$Q_u \le 0.15 f_{cu} b\, d$ but not more than (4 N/mm^2) $b\, d$.

A_n is the required steel area to resist the tensile force N_u

$$A_n = \frac{N_u}{f_y / \gamma_s} \quad \text{(Code 4-39)}$$

and A_{sf} is the area of steel reinforcement required to transfer the shear force by friction as:

$$A_{sf} = \frac{Q_u}{\mu f_y / \gamma_s} + \frac{N_u}{f_y / \gamma_s} \quad \text{(Code 4-35)}$$

and μ=1.2 for monolithic joints

2-Horizontal stirrups :

The area of the horizontal closed stirrups, A_h, which should be arranged within 2/3 the total height of the section in the tension zone is taken as:

$$A_h = 0.50(A_s - A_n) \quad \text{(Code 4-40)}$$

1- Vertical stirrups :

Vertical stirrups are used to resist torsional moments and should not be less than the minimum web reinforcement of beam sections given by:

$$A_{st} = \mu_{min} bs = \frac{0.4}{f_y} b\, s \text{ .... (Code 4-28)}$$

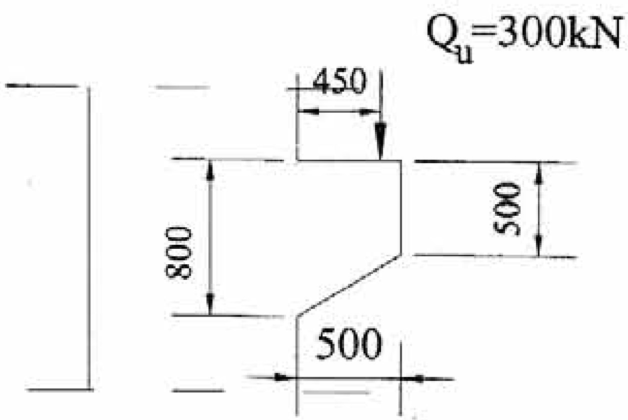
where A_{st} is the area of the vertical branches of the stirrup, s is the spacing between stirrups and f_y is the yield stress (N/mm^2). Stirrups may not be less than 5φ 6/m′

μ_{min}=0.15% for ordinary mild steel and μ_{min}=0.10% for high tensile steel.

7.5.3 Design example
Example 3

Determine the required reinforcement for the bracket shown in figure with the following data:

Bracket size: 250*800 mm and d=750mm
f_{cu}=25 N/mm^2
Ultimate Load=300 kN
f_y= 240 N / mm^2



Step 1: Check the bracket dimensions:
d=750mm> a (450mm)

Step 2: Check the ultimate shear friction value:

$$\frac{Q_u}{b\,d} \leq \ 0.15 f_{cu} \qquad \text{but not more than 4 N/mm}^2$$

$$\frac{300*1000}{250*750} = 1.6 \leq \ 0.15*25 = 3.75 \qquad \text{O.K.}$$

Step 3: Area of main reinforcement:

There are three equations to be evaluated;

- $A_s = \ A_n + A_f$ (Code 4-37-a)

$$M_u = \ Q_u.a = 300*45 = 135 kN.m$$

$$A_f = \ \frac{M_u}{(d - a/2) f_y / \gamma_s}$$

where a is the height of the equivalent stress block

$$M_u = 0.67\frac{f_{cu}}{\gamma_c}b.a.(d - a/2)$$

$$M_u = 135*10^6 = 0.67\frac{25}{1.5}.250.a.(750 - a/2)$$

$a = 67.5mm$ < 0.1 d, take a= 0.10 d =75 mm

$$A_f = \frac{135*10^6}{(750-75/2).240/1.15} = 907.9\ mm^2$$

$A_n = 0.0$

Then A_s=0.0+907.9 = 907.9 mm^2 from (Code 4-37-a)

The second equation is that of the Code (4-37-b)

$$- A_s = A_n + \frac{2}{3}A_{sf}$$

$$:A_{sf} = \frac{Q_u}{\mu f_y/\gamma_s} + \frac{N_u}{f_y/\gamma_s} = \frac{300*1000}{1.2\left(\frac{240}{1.15}\right)} + 0 = 1197.9\ mm^2 \qquad \text{(Code 4-35)}$$

then A_s=0+2/3*1197.9=799 mm^2

the third equation is :

$$- A_s = 0.03\frac{f_{cu}}{f_y}b\,d = 0.03\frac{25}{240}250*750 = 585.9mm^2$$

The maximum main reinforcement area is obtained from the first equation

Then:

$\boxed{A_s = 907.9\ mm^2 \quad \text{Use } 4\ \emptyset\ 18\ mm}$

Step 4: Find the area of the horizontal stirrups, as calculated using Code Eq. (4-40):

$$A_h = 0.50(A_s - A_n) = 0.50(907.9 - 0) = 453.95\ mm^2$$

This area is to be distributed over 2/3 of the effective depth,

Distance=2/3*750=500.0mm

Choose closed stirrups with diameter=10mm (two branches) spaced at 250 mm

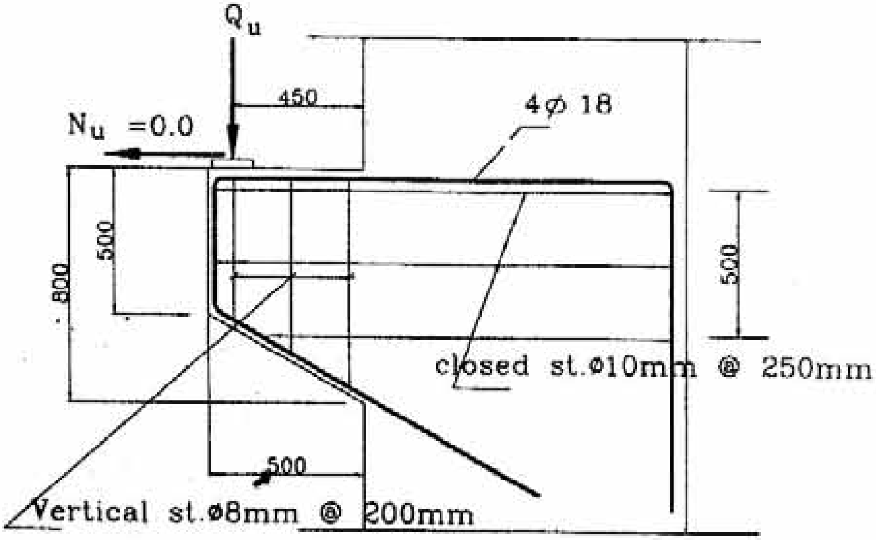
The available area=78.5*2*(500/250+1)=471 mm^2> 453.95 mm^2 (O.K.)

Step 5: Find the area of the vertical stirrups.

$$A_{st} = \mu_{min} b s = \frac{0.4}{f_y} b\, s = \frac{0.4}{240} * 250 * 200 = 83.3 mm^2$$

Choose vertical stirrups with diameter=8mm (two branches) spaced at 200mm

The available area=50*2=100mm^2>83.3mm^2 (O.K.)



Reinforcement details

7.6 Ultimate Torsion Strength

The critical section for torsion is at one of the following

- At section of maximum M_t
- If $M_{t\,max}$ is at the support, then the critical section is at distance d/2 from the face of the support

7.6.1 Nominal shear stress due to torsion

For rectangular section

$$q_{tu} = \frac{M_{tu}}{2A_o t_e} \qquad \text{.......(Code 4-47)}$$

$$A_o = 0.85 A_{oh}$$

$$t_e = \frac{A_{oh}}{P_h}$$

$$q_{tu} = \frac{M_{tu}}{1.7 A_{oh}^2 t_e}$$

Note:

1- Where Ao is the area bounded by the lateral outermost reinforcement that is resisting torsional moment. In the cases where there is no precise method for calculating A_o the value of A_o can be taken equal to (0.85 A_{oh}) where A_{oh} is the area bounded by the lateral outermost reinforcement that is resisting torsional moment, and te = A_{oh} / P_h where P_h is the length of the perimeter of the lateral outermost reinforcement that resisted torsional moments.

2- If the actual thickness of the wall of the hollow section is less than A_{oh} / P_h , then the actual wall thickness should be used in Eq.(Code 4-47).

For T & L sections:

a) The effective part of the slab may be neglected (i.e. section is treated as rectangular section)
b) If the effective part of the slab is considered , it must not exceed three times the slab thickness on each side , and the shear stress is calculated according to the following equation:

$$q_{tu} = \frac{M_{tu}}{2A_o t_e}$$

Where $(2A_o t_e)$ represents the areas of the section within the shear path {See hatched areas in Code Fig. (4-11-a & Fig.(4-11-b)}.

Important note:

When considering the effective slab part in calculating q_t ,then the slab must provided by web reinforcement to resist torsion .

The effect of torsional moment may be neglected, if the resulting shear stress due to torsion is less than the following value:

$$q_{tu\,min} = 0.06\sqrt{f_{cu}/\gamma_c} \quad \text{N/mm}^2 \quad \text{.................................(Code 4-17)}$$

If $q_{tu} > q_{tu\ min}$, the designer has to calculate the steel reinforcement to resist torsion stresses

7.6.2 Maximum shear strength

If the section is subjected to torsional moment only or shear force only :Then, the shear stress (q_{tu} or q_u) must be less than maximum shear stress
Where, maximum shear stress ($q_{tu(max)}$or $q_{u(max)}$) = $0.7\sqrt{f_{cu}/\gamma_c}$ N/mm^2

If the section is subjected to torsional moment and shear force
q_{tu} (torsion) ≤ $q_{tu}(\max) = 0.7\delta_{ti}\sqrt{f_{cu}/\gamma_c}$ N/mm^2 (Code 4-48-a)

q_u (shear) ≤ $q_u\ (\max) = 0.7\delta_{si}\sqrt{f_{cu}/\gamma_c}$ N/mm^2 (Code 4-48-b)

where δ_{ti} and δ_{si} are defined as follows
For rectangular sections

$$\delta_{ti} = \frac{1}{\sqrt{\{1+(q_u/q_{tu})^2\}}} \quad \text{(Code 4-49-a)}$$

$$\delta_{si} = \frac{1}{\sqrt{\{1+(q_{tu}/q_u)^2\}}} \quad \text{(Code 4-49-b)}$$

For box sections

$$\delta_{ti} = \frac{1}{\{1+\frac{q_u}{q_{tu}}\}} \quad \text{(Code 4-49-c)}$$

$$\delta_{si} = \frac{1}{\{1+\frac{q_{tu}}{q_u}\}} \quad \text{(Code 4-49-d)}$$

7.7 Torsion reinforcement

If $q_{tu\,(min)} < q_{tu} < q_{tu\,(max)}$ torsion reinforcement (Closed stirrups and longitudinal reinforcement) must be calculated and used to resist torsional moment. This reinforcement is added to flexural and shear reinforcement as in the following table.

Table (7-5):Stirrups Reinforcement for combined Shear and Torsion

	$q_{tu} \le q_{tu\,(min)}$	$q_{tu} > q_{tu\,(min)}$
$q_u <= q_{cu}$	Use min. shear reinforcement according to Eq.(Code 4-22)	Provide reinforcement to resist q_{tu}
$q_u > q_{cu}$	Provide reinforcement for (q_u - $q_{cu}/2$)	Provide reinforcement. to resist q_{tu} & (q_u - $q_{cu}/2$)

1-Closed stirrups resisting torsion

$$A_{str} = M_{tu}\, s / (\frac{f_{yst}}{\gamma_s}\ 2A_o) \quad \text{.......(Code 4-50)}$$

where A_o = 0.85 A_{oh} in the case of a rectangular section the above equation is replaced by the following equation:

$$A_{str} = M_{tu}\, s / 1.7 X_1 Y_1 (\frac{f_{yst}}{\gamma_s}) \quad \text{.......(Code 4-51)}$$

The spacing between closed stirrups should not exceed 200mm nor P_h/8.

2-longitudinal reinforcement (A_{sl}) resisting torsion

$$A_{sl} = \{A_{str} p_h / s\}(\frac{f_{yst}}{f_y}) \quad \text{.......(Code 4-53-a)}$$

But not less than,

$$A_{sl}\ \min = \frac{0.46\sqrt{\frac{f_{cu}}{\gamma_c}}\, A_{cp}}{f_y} - \{A_{str} p_h / s\}(\frac{f_{yst}}{f_y}) \quad \text{.......(Code 4-53-b)}$$

Where A_{cp} the total area of the section including the area of the openings and the value of $\frac{A_{str}}{s}$ must not be less $\frac{1}{6}\frac{b}{f_{yst}}$.

This longitudinal reinforcement must be distributed along the stirrups perimeter so that,

- spacing between bars ≤ 30 cm , with min. of one bar at each corner
- the min. bar diameter (ϕ_{min}) = 12 mm or (stirrups spacing (S) / 15)
- A_{sl} is added to flexural longitudinal reinforcement resisting B.M.
- Torsion reinforcement must be extended after their theoretical end points by a distance ≥ half stirrups perimeter.

7.8 General Recommendations Regarding Web Reinforcement

a-The stirrups required for resisting torsional moment and shearing force should not be less than that recommended by the following equation:-

$$2A_{str} + A_{st} \geq 0.35(s\ b) / \frac{fy}{\gamma_s} \quad \text{.......(Code 4-52)}$$

Where A_{str} & A_{st} are one branch area of torsion stirrups and shear stirrups respectively

b- Maximum spacing of vertical stirrups=200mm or P_h/8

c- In sections with stirrups having more than two branches . The outer two branches are the only ones resisting torsional moments .

1-Closed stirrups resisting torsion

$$A_{str} = M_{tu} \cdot s / (\frac{f_{yst}}{\gamma_s} \; 2A_o) \quad \text{..(Code 4-50)}$$

where A_o = 0.85 A_{oh} in the case of a rectangular section the above equation is replaced by the following equation:

$$A_{str} = M_{tu} \cdot s / 1.7 X_1 Y_1 (\frac{f_{yst}}{\gamma_s}) \quad \text{..(Code 4-51)}$$

The spacing between closed stirrups should not exceed 200mm nor $P_h/8$.

2-longitudinal reinforcement (A_{sl}) resisting torsion

$$A_{sl} = \{A_{str} p_h / s\} (\frac{f_{yst}}{f_y}) \quad \text{..(Code 4-53-a)}$$

But not less than,

$$A_{sl}\,min = \frac{0.46 \sqrt{\frac{f_{cu}}{\gamma_c}} A_{cp}}{f_y} - \{A_{str} p_h / s\} (\frac{f_{yst}}{f_y}) \quad \text{..................(Code 4-53-b)}$$

Where A_{cp} the total area of the section including the area of the openings and the value of $\frac{A_{str}}{s}$ must not be less $\frac{1}{6}\frac{b}{f_{yst}}$.

This longitudinal reinforcement must be distributed along the stirrups perimeter so that,

- spacing between bars ≤ 30 cm , with min. of one bar at each corner
- the min. bar diameter (ϕ_{min}) = 12 mm or (stirrups spacing (S) / 15)
- A_{sl} is added to flexural longitudinal reinforcement resisting B.M.
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7.8 General Recommendations Regarding Web Reinforcement

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Where A_{str} & A_{st} are one branch area of torsion stirrups and shear stirrups respectively

b- Maximum spacing of vertical stirrups=200mm or $P_h/8$

c- In sections with stirrups having more than two branches . The outer two branches are the only ones resisting torsional moments .

d- For box sections , it is allowed to use the transverse and longitudinal reinforcement at the outer and inner perimeter of the section for resisting torsional moment , as long as web thickness $t_u \leq$ section width/6 but if the two web's thickness is greater than this limits, the torsional moment is resisted only by the reinforcement exiting at the outer perimeter only.

e- For statically indeterminate structures where torsional moment is not necessarily required for equilibrium and produced torsion due to strain compatibility. The ultimate torsional moment can be reduced to the following value:

$$M_u = 0.316\left(\frac{A_{cp}^2}{P_{cp}}\right)\sqrt{\frac{f_{cu}}{\gamma_c}}$$

Where A_{cp} is the total area of the section including the opening if found and the P_{cp} is the external parameter

In this case redistribution of bending and shear is required .

N.B:

Torsion calculations in working stress method are the same in ultimate stress method that the shear strength are q_c and q_2 instead of q_{tcu} and $q_{tu(max)}$ respectively.

Table (7-6): Summary of Design Equations for Combined Shear and Torsion

Item	Ultimate Strength Design	Code Equation
1	$q_u = \frac{Q_u}{bd}$ $q_{tu} = \frac{M_{tu}}{2A_o t_e}$ $A_o = 0.85 A_{oh}$ $t_e = \frac{A_{oh}}{P_h}$	(4-13) (4.47)
2	$q_{tu\ min} = 0.06\sqrt{f_{cu}/\gamma_c}$ N/mm^2 $q_{cu} = 0.24\sqrt{f_{cu}/\gamma_c}$ N/mm^2	(4-17) (4-18)
3	$q_{tu}(\max) = 0.7\delta_{ti}\sqrt{f_{cu}/\gamma_c}$ $q_u(\max) = 0.7\delta_{si}\sqrt{f_{cu}/\gamma_c}$ N/mm^2	(4-48 a&b)
4	For solid sections : $\delta_{ti} = \frac{1}{\sqrt{\{1+(q_u/q_{tu})^2\}}}$ $\delta_{si} = \frac{1}{\sqrt{\{1+(q_{tu}/q_u)^2\}}}$	(4-49 a&b)
5	For box sections: $\delta_{ti} = \frac{1}{\{1+\frac{q_u}{q_{tu}}\}}$ $\delta_{si} = \frac{1}{\{1+\frac{q_{tu}}{q_u}\}}$	(4-49 c&d)
6	Area of closed stirrups: $A_{str} = M_{tu}\, s/(\frac{f_{yst}}{\gamma_s}\ 2A_o)$ in case of rectangular section $A_{str} = M_{tu}\, s/\{1.7 X_1 Y_1 (f_{yst}/\gamma_s\ )\}$	(4-50a) (4-50b)
7	Minimum area of stirrups for shear and torsion is given by: $2A_{str} + A_{st} \geq 0.35(s\ b)/\frac{f_{yst}}{\gamma_s}$	(4-52)

8	Maximum spacing s of vertical stirrups=200mm or $P_h/8$	
9	$A_{sl} = \{A_{str} P_h / s\}(\frac{f_{yst}}{f_y})$	(4-53- a)
	$A_{sl}\min = \frac{0.46\sqrt{\frac{f_{cu}}{\gamma_c}} A_{cp}}{f_y} - \{A_{str} P_h / s\}(\frac{f_{yst}}{f_y})$	(4-53-b)
10	$M_{tu} = 0.316(A_{cp}^2 / P_{cp})\sqrt{f_{cu}/\gamma_c}$	(4-54)
11	$C = \beta b^2 t \eta$ $\eta = 0.70$ for rectangular sections before cracking in which q_{tu} produced by twisting moment not more than $q_{tu} = 0.316\sqrt{f_{cu}/\gamma_c}$ $\eta = 0.2$ for rectangular section after cracking. β = parameter depending on (t/b) ratio to be taken from Table (4-6).	(4-55)

Table (7-7) Values of parameter β for calculating torsional stiffness

T/b	1	1,5	2	3	5	>5
β	0.14	0.20	0.23	0.26	0.29	0.33

For calculating the stiffness for(L) and (T) sections, divide section to rectangular sections and calculate the torsional stiffness as summation for the stiff nesses of rectangular sections.

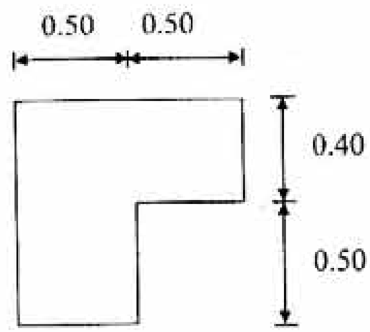
Example- 4.

Given:

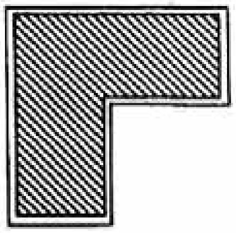
M_{tu}= 27.0 t.m. = 270.0 KN.m.
f_{cu} = 300 kg \ cm^2 = 30 N/mm^2
f_{yst} = f_y = 4000 kg \ cm^2 = 400 N/mm^2

Calculate: Required section reinforcement.

Calculation steps:



a-Cross section and dimensions



b-A_{oh}

Step – 1 calculate the shear stress due to torsion

- $$q_{tu} = \frac{M_{tu}}{2A_o t_e}$$
- A_o = 0.85 A_{oh}, $t_o = A_{ho} / P_h$

Assume concrete cover = 40 mm & diameter of stirrups = 16 mm

A_{oh} = [50 –2×4 – 1.6] [90 - 2×4 – 1.6]
+ [50 –4 – (1.6 / 2)][40 – 2 × 4 – 1.6] = 4622.24 cm^2 = 462224 mm^2

A_o = 0.85 Aoh = 0.85 × 4622.24 = 3928.9 cm^2 = 392890 mm^2

P_h = (100 – 2 × 4 –1.6) + (40 –2 × 4 –1.6) + (50 – 4 –1.6 / 2)
+ (50 –4 –(1.6 / 2) + (50 – 2 x 4 –1.6)
+ (90 –4 – 1.6) = 336.8 cm = 3368 mm

te = A_{oh} / P_h = 4622.24 / 336.8 = 13.72 cm = 137.2mm < t_{actual} = 200 mm OK

q_{tu} = 27 × 10^5 / 2 × 3928.9 × 13.72
= 25.04 kg /cm^2
= 2.504 N /mm^2

Step – 2 Shear stress due to torsion that causes cracking

$q_{tu\,min} = 0.06\ \sqrt{f_{cu}} / \gamma_c = 0.268$ N/mm
$= 2.68$ kg/cm^2

$q_{tu} > q_{tumin}$ Design for torsion should be considered and q_t must be less than q_{tumax}

Step – 3 design for torsion

3 – 1 check the adequacy of the concrete dimension

Since the shear force = 0.0 (Case of pure torsion)

$\delta_{si} = 0$, $\delta_{ti} = 1.0$

$q_{tu\,(max)} = 0.70\ \delta_{ti}\ \sqrt{f_{cu}/\gamma_c} = 3.13$ N/mm^2 > 3 N/mm^2
$= 3.0$ N/mm^2

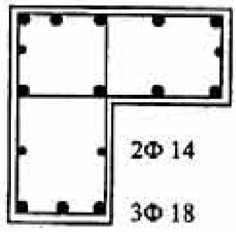
since $q_{tu} \approx q_{tu\,(max)}$ therefor concrete dimensions are adequate.

3 - 2 Design for torsion reinforcement.

3- 2-1 Stirrups.

$A_{str} = M_{tu}.\ S / (2A_o\ (f_{yst}/\gamma_s\ ))$
$A_{str} / S = 0.099$ cm
Using stirrups diameter of ϕ 12mm spaced 10 cm
$A_{str} = 0.099 \times 10 = 0.99$ cm^2

3.2-2 Longitudinal steel



Reinforcement Details

$$A_{sl} = \{A_{str}\ p_h / s\}(\frac{f_{yst}}{f_y})$$

$$= \mathbf{0.099 \times 336.8 = 33.3\ cm^2}$$

$$A_{sl}\min = \frac{0.46\sqrt{\frac{f_{cu}}{\gamma_c}}A_{cp}}{f_y} - \{A_{str} p_h / s\}(\frac{f_{yst}}{f_y})$$

$= 3085 - 3407 = -322$ mm^2 < $A_{sl} = 3380$ mm^2 required

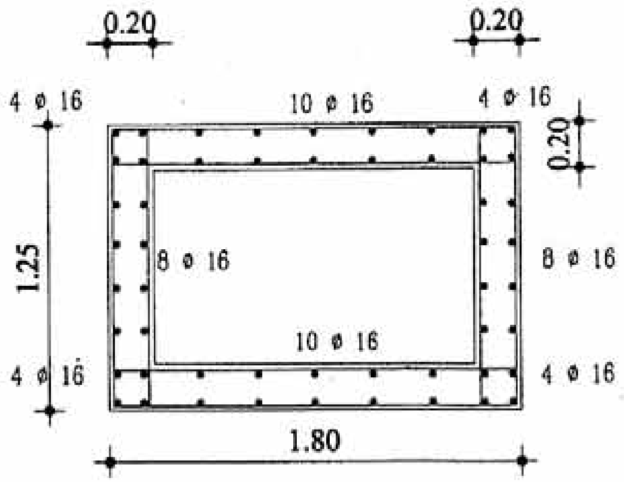
The chosen longitudinal steel is symmetrically distributed around the section perimeter as shown in the given reinforcement of the section.

N.B.: **The required area of longitudinal reinforcement for torsion has to be added to that needed for flexure steel if required.**

Example 5 .

Given:
f_{cu} = 35 N/mm^2 =350 kg/ cm^2 and f_y = 400 N/mm^2 =4000 kg/ cm^2, f_{yst}=400 N/mm^2
Concrete dimensions are shown and concrete cover = 40mm
M_{tu} = 1500 kN.m

Required: Torsional section reinforcement.



Calculation steps:

Step – 1 calculate the shear stress due to torsion

$$q_{tu} = \frac{M_{tu}}{2A_o t_e}$$

A_{ho} = (180 – 2 × 4 – 1.3) (125 – 2 × 4 – 1.3)

= 19750.0cm^2 = 1975000 mm^2

A_o = 0.85 A_{oh}

= 16787.5 cm^2 = 1678750 mm^2

P_h = 2 [(180 – 2 × 4 – 1.3) + (125 – 2 × 4 – 1.3] = 572.8 cm = 5728 mm

t_e = A_{ho} / P_h

= 34.48 cm = 344.8 mm

t_{actual} < t_e → t_e = 20 cm = 200 mm

q_{tu} = 1500 10^6 / (2 1678750 20)

= 2.233 N/mm^2 = 22.33 kg /cm^2

Step – 2 Shear stress due to applied torsion > shear stress that causes cracking

$$q_{tu\,min} = 0.06\sqrt{\frac{35}{1.5}} = 0.289\ N/mm^2$$

$q_{tu} > q_{tumin}$ (design for torsion stresses is required) and q_{tu} must be less than q_{tumax}

Step – 3 Design for torsion

3 – 1 check the adequacy of the concrete dimension

$$q_{tu(max)} = 0.70\sqrt{\frac{f_{cu}}{\gamma_c}} \le 3\ N/mm^2$$

Since the shear stress =0.0

$\delta_{si} = 0.0$ δ_{ti} =1.0

$q_{umax} = 3\ N/mm^2$

$q_{tu} < q_{tu\,(max)}$ → concrete dimensions are adequate.

3 - 2 Design for torsion reinforcement

3 – 2 – 1 Stirrups

$A_{str} = M_{tu}.\ s / (2A_o\,(f_{ys}/\gamma_s))$

$A_{str}/s = .128$ cm

for s= 10.0 cm, A_{str}=1.28 cm², take ϕ = 13mm

3 – 2 – 2 Longitudinal Reinforcement.

$Asl = A_{str}\ (p_h / s)\ f_{yst} / f_y$

$= 0.128 \times 572.8 = 73.3\ cm^2$

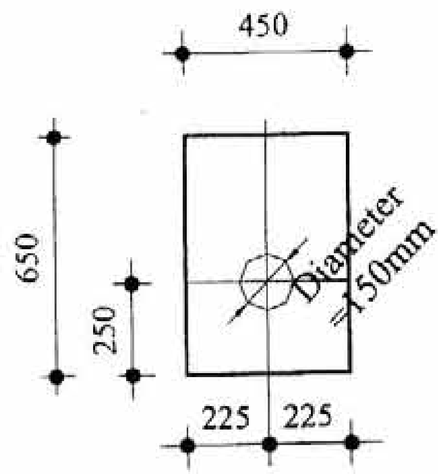
Choose (36 ϕ 16 +12 ϕ 16) =96 cm²

$$A_{sl}\min = \frac{0.46\sqrt{\frac{f_{cu}}{\gamma_c}}A_{cp}}{f_y} - \{A_{str}p_h/s\}\left(\frac{f_{yst}}{f_y}\right)$$

$= 12498.72 - 7331.84 = 5167\ mm^2 < A_{sl}$ required =7330 mm² ok

This amount of longitudinal steel , A_{sl} , has to be distributed uniformly along the section perimeter with maximum spacing between bars = 300 mm .

Design Example-6 .



Given a cross section of a precast beam of dimensions 450mm x 650mm. A circular opening of diameter 150 mm has to be provided at the location shown. Design the beam to resist an ultimate moment of 68.0 kN.m according to the following data:

$f_{cu} = 25$ N/mm^2
$f_{yst} = f_y = 360$ N/mm^2

calculation steps:

Step 1: Determine cross-sectional parameters

The cross-sectional parameters for torsion design are A_{oh} and P_h

P_h=2(450-2*40)+2(650-2*40)=1880 mm

A_{oh}=(450-2*40)(650-2*40)=210900mm^2

Step 2: Calculate the shear stresses due to torsion

Nominal ultimate shear stress due to torsion

$$q_{tu} = \frac{M_{tu}}{2A_o t_e}$$

For solid sections, the area enclosed by shear flow path, A_{oh} and the thickness of the equivalent thin walled tube, t_e, can be calculated from:

A_o = 0.85 A_{oh},

$$t_e = \frac{A_{oh}}{P_h} = \frac{210900}{1880} = 112mm$$

Note: Due to the existence of the opening, the minimum thickness of the section at the location of the opening is equal to:

$$t_{\min} = \frac{450-150}{2} = 150mm$$

Since $\left(t_e = \frac{A_{oh}}{P_h} \right) < 150mm,$ the nominal ultimate shear stress due to torsion is:

$$q_{tu} = \frac{M_{tu}}{1.7A_{oh}^{\ 2}/P_h}$$

$$= \frac{68.0*10^6}{1.7(210900^2/1880)} = 1.68N/mm^2 > 0.06\sqrt{\frac{f_{cu}}{\gamma_c}}$$

(We have to consider torsion in design)

Step 3: Check the adequacy of the concrete dimensions

$$q_{tu(\max)} = 0.70\sqrt{\frac{f_{cu}}{\gamma_c}}$$

(Note that since the section is subjected to pure torsion, δ_u=1.0 and , δ_u=0.0)

$$q_{tu(\max)} = 0.70\sqrt{\frac{25}{1.5}} = 2.86N/mm^2 \quad < 3.0N/mm^2$$

Since $q_{tu} < q_{tu(\max)}$, concrete dimensions of the section are adequate, but torsional reinforcement is required.

Step 4: Design of transverse reinforcement

The amount of closed stirrups required to resist the torsion is:

$$\frac{A_{str}}{s} = \frac{M_{tu}}{(1.7\,A_{oh}\,(f_{yst}/\gamma_s)}$$

$$\frac{A_{str}}{s} = \frac{68.0*10^6}{(1.7\,(210900)(360\ /1.15)} = 0.606mm^2/mm$$

Maximum spacing between stirrups is the smaller of 200mm and$\left(\frac{P_h}{8} = \frac{1880}{8} = 235mm\right)$

For a spacing between stirrups Of 200mm, the required are of one leg of closed stirrups is:

$A_{str} = 121mm^2$

Choose closed stirrups of 13mm(area=132mm^2)@200mm.

Step 5 – Design of longitudinal reinforcement.

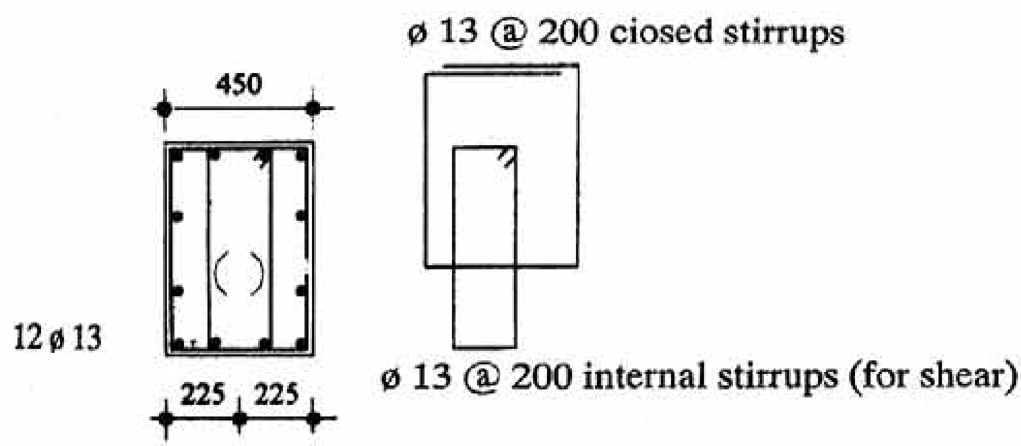
$$A_{sl} = \{A_{str} p_h / s\}(\frac{f_{yst}}{f_y}) = 0.606 \times 1880 = 1139.28\ \text{mm}^2$$

$$A_{sl}\min = \frac{0.46\sqrt{\frac{f_{cu}}{\gamma_c}}A_{cp}}{f_y} - \{A_{str} p_h / s\}(\frac{f_{yst}}{f_y})$$

$$= 1525.82 - 1139.28 = 386.54\ \text{mm}^2 \quad < A_{sl} \quad \text{OK}$$

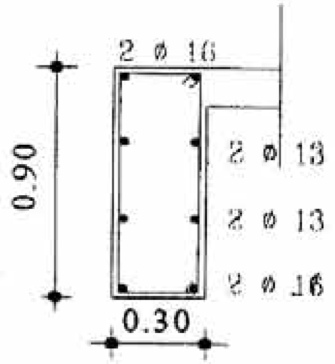
Choose 12 Φ 13 symmetrically distributed around the section

Note: Reinforcement around the opening is not shown



Example 7:

$f_{cu} = 30\ N/mm^2$
$f_{yst} = f_y = 360\ N/mm^2$
$Q_u = 106\ KN$ & $M_{tu} = 56\ kN.m$



CALCULATION STEPS:

Step – 1 Calculation of shear and torsion stresses.

Neglect slab contribution

$X_1 = 30 - 2 \times 4.5 = 21.0\ cm$

$Y_1 = 90 - 2 \times 4.5 = 81.0\ cm$

$A_{oh} = 21.0 \times 81.0 = 1701.0\ cm^2$

$A_o = 0.85\ A_{oh} = 1445.85\ cm^2$

$P_h = 204\ cm$ & $te = A_{oh} / P_h = 8.34\ cm$

$q_{tu} = M_{tu} / 2\ A_o\ t_e$

$= 2.32\ N / mm^2$

$= 23.22\ kg/cm^2$

$q_u = Q_u / bd$

$$= \frac{106 * 10^3}{300 * 850} = 0.416\ N / mm^2 = 4.16\ kg / cm^2$$

Step – 2 Check for the adequacy of the section dimensions .

$$\delta_{ti} = \frac{1}{\sqrt{\{1+(q_{u}/q_{tu})^{2}}} = 0.984$$

$$\delta_{si} = \frac{1}{\sqrt{\{1+(q_{tu}/q_{u})^{2}}} = 0.176$$

$q_{tu\ (max)} = 0.70\ \delta_{ti}\ \sqrt{f_{cu}}/\gamma_c \quad > 3\ N/mm^2$
$= 3.0\ N/mm^2 \qquad = 30.\ kg/cm^2$
$q_{su\ (max)} = 0.70\ \delta_{si}\ \sqrt{f_{cu}}/\gamma_c$
$= 0.\ 0.55\ N/mm^2$
$= 5.5\ kg/cm^2$

Since
$q_{tu} < q_{tu\ (max)}$
$q_{su} < q_{su\ (max)}$
Therefore, concrete dimensions are adequate

Step – 3 Check for the need for shear and torsion reinforcement

Since , $q_{tu} > q_{tumin} = 0.06\ \sqrt{f_{cu}}/\gamma_c$, then we need to satisfy torsion requirement .
Since contribution of concrete to torsion $= 0.0$
& contribution of concrete to shear $= q_{cu}$ where

$$q_{cu} = 0.24\sqrt{\frac{30}{1.5}} = 1.07\ N/mm^2$$

& Since $q_u < q_{cu}$, then we do not need shear reinforcement in this case .

4 – Design of reinforcement

4 -1 Stirrups.

$A_{str} = M_{tu}.\ s /[1.7\ (x1.y1)\ (f_{yst}/\gamma_s))]$
$A_{str}/s = 56 \times 10^6 / [1.7 \times 210 \times 810 \times 360 / 1.15] = 0.06$ mm

Choose stirrups of diameter ϕ 13 mm with spacing s =22.0 cm. Take s = 20.0 cm

$$\frac{A_{st,min}}{s} = \frac{0.4}{fy} b = \frac{0.4}{360} 30 = 0.033 \prec \frac{A_{str}}{s} \quad \cdots o.k.$$

4- 2 longitudinal reinforcement

$A_{sl} = A_{str}\ p_h / s\ (f_y / f_{yst})$

$= 0.06 \times 204 \times 1.0 = 12.24\ cm^2$

$$A_{sl}\min = \frac{0.46\sqrt{\frac{f_{cu}}{\gamma_c}}A_{cp}}{f_y} - \{A_{str}p_h/s\}\left(\frac{f_{yst}}{f_y}\right)$$

$= 1542.88 - 1262 = 280.88\ mm^2 < A_{sl}$ required ....o.k.

8. LIMIT STATE OF CRACKING

8.1 General

Reinforced concrete structures can be categorized according to there exposure to environmental effects as given in the following table

Table (8-1): Categories of Structures According to Exposure of Concrete Tension Surface to Environmental Effects

Category	Degree of exposure to environmental effects
One	Structures with *protected* tension side such as a) All protected internal members of ordinary buildings. b) Permanently submerged members in water (without harmful materials) or members permanently dry. c) Well isolated roofs against moisture and rains.
Two	Structures with *unprotected* tension side, such as: a) Structures in open air, e.g. bridges and roofs without good insulation. b) Structures of category one built nearbv seashores. c) Structures subjected to humidity such as open halls, sheds and garages.
Three	Structures with *severely exposed* tension side, such as: a) Members with high exposure to humidity. b) Members exposed to repeated saturation with moisture. c) Water tanks. d) Structures subjected to vapour, gases or weak chemical attacks.
Four	Structures with tension side *very severely* exposed to corrosive influences of strong chemical attacks which cause rusting of steel a) Structures subjected to conditions resulting in rust of steel such as gases and vapour including chemicals. b) Other tanks, sewerage, Structures subjected to seawater.

8.2 Satisfaction of Cracking Limit State

When designing reinforced concrete structures, one should fulfil the following relation:

$$w_k = \beta . s_{rm} . \varepsilon_{sm}$$

$$s_{rm} = \left(50 + 0.25 K_1 K_2 \frac{\phi}{\rho_r} \right)$$

$$\varepsilon = \frac{f_s}{E_s} \left(1 - \beta_1 \beta_2 \left(\frac{f_{sr}}{f_s} \right)^2 \right)$$

with the values of W_k less than or equal to the values w_{kmax} given in Table (8-2):

Table (8-2)Values of w_{kmax}

Category	One	Two	Three	Four
$w_{k(max)}$	0.3	0.2	0.15	0.1

where

β = Coefficient that relates the average crack width to the design crack width. It shall be taken as follows:

1.7 For cracks induced due to loading

1.3 For cracks induced due to restraining the deformation in cross sections having a width or depth (whichever smaller) less than 300mm.

1.7 For cracks induced due to restraining the deformation in cross sections having a width or depth (whichever smaller) more than 800mm.

For cross sections having a width or depth (whichever smaller) between value 300mm and 800mm, the coefficient β shall be proportionally calculated.

ϕ = Bar diameter in mm. In case of using more than one diameter in the cross section, the average diameter shall be used.

β_1 = A coefficient that reflects the bond properties of the reinforcing steel. It shall be taken equal to 0.8 for deformed bars and 0.5 for smooth bars.

β_2 = A Coefficient that takes into account the duration of loading. It shall be taken equal to 1.0 for short term loading and 0.50 for long term loading or cyclic loading.

k_1 = A Coefficient that reflects the type of steel of the reinforcing bars. It shall be taken equal to 0.8 for deformed bars and 1.6 for smooth bars.

In case of members subjected to imposed deformation, the values of k_1 shall be modified to kk_1 where the value of k is taken as follows:

a) k=0.80 for the case in which the tensile stresses are induced due to restraining the deformation. For rectangular cross section, the value of k is taken as follows:

   k=0.8 for rectangular section having thickness ≤ 300 mm.

   k=0.50 for rectangular sections having thickness ≤ 800 mm.

b) k=1.0 for cases in which the tensile stresses are induced due to restraint of extrinsic deformation.

k_2 = Coefficient that reflects the strain distribution over the cross section subjection. It shall be taken equal to 0.5 for sections subjected to pure bending and 1.0 for sections subjected to pure axial tension. For section subjected to combined bending and axial tension, k_2 shall be calculated from:

$$k_2 = \frac{\varepsilon_1 + \varepsilon_2}{2\varepsilon_1}$$

Where ε_1 and ε_2 are the maximum and minimum strain values to which the section is subjected, and shall be calculated based on the analysis of a cracked section.

ρ_r = effective tension reinforcement ratio.

$$\rho_r = \frac{A_s}{A_{cef}}$$

where

A_s = area of longitudinal tension steel within the effective tension area

A_{cef} = area of effective concrete section in tension.
= width of the section * t_{cef}

t_{cef} can be calculated according to Fig. (4.22) of ECCS 203-2001.

f_s = stress in longitudinal steel at the tension zone, calculated based on the analysis of cracked section under permanent loads.

f_{sr} = stress in longitudinal steel at the tension zone, calculated based on the analysis of cracked section due to loads causing first cracking.

Table (8-3) Working Stresses for Reinforcing Steel and Coefficients of reducing Bar Stresses β_{cr}

f_s (N/mm^2) W.S.D	Reduction factor L.S.D β_{cr}		Category one	Category two	Categories three & four
	36/52	40/60	Largest Bar diameter (ϕ_{max}) in mm		
220	1.00	0.92	18	12	8
200	0.93	0.83	22	16	10
180	0.85	0.75	25	20	12
160	0.75	0.67	32	22	18
140	0.65	0.58	--	25	22
120	0.56	0.50	--	--	28

Table (8-4) Minimum Concrete Cover (mm)

Category	All elements except slabs		Walls & Solid slabs	
	$f_{cu} \leq 25$ MPa	$f_{cu} > 25$ MPa	$f_{cu} \leq 25$ MPa	$f_{cu} > 25$ MPa
One	25	20	20	20
Two	30	25	25	20
Three	35	30	30	25
Four	45	40	40	35

8.3 Liquid Containing Structures

Liquid containing structures should be designed as non-cracked sections. In these structures the tensile stresses induced by loading should be less than f_{ctr}/η, i.e.:

$$f_{ct} = \left[f_{ct(N)} + f_{ct(M)}\right] \le \frac{f_{ctr}}{\eta}$$

where

f_{ctr} = the cracking strength of concrete and is given by the following equation

$$f_{ctr} = 0.6\sqrt{f_{cu}} \quad \text{N/mm}^2$$

$f_{ct(N)}$ = the tensile stresses due to axial tension force (negative sign is used for compressive stresses).

$f_{ct(M)}$ = tensile stresses due to bending moment.

η = a coefficient that depends on the virtual thickness of the section and can be obtained from table (8-5)

η The virtual thickness, t_v

$$t_v = t\left[1 + \left(\frac{f_{ct(N)}}{f_{ct(M)}}\right)\right]$$

where t is the thickness of the section

Table (8-5)Values of the Coefficient η

Virtual thickness, t_v (mm)		Coefficient η
Smaller than or equal to	100	1.00
	200	1.30
	400	1.60
Greater than or equal to	600	1.70

Application Of Equation (4-66) 203-2001

Design Step	Description	Code Reference
Step 1	Calculate the value of f_{ctr} & cracking moment M_{cr} of the section. If the section is to be designed as a water tight section; then equation (4-69) must be satisfied.	Equation (4-64)
Step 2	Calculate the cross sectional parameters, depth of neutral axis (c) and moment of inertia (I_{cr})	
Step 3	Calculate the value of steel stress at first crack f_{sr}	
Step 4	Calculate the value of the effective depth of the tensile area t_{cef}.	Fig. (4-22)
Step 5	Calculate the value of the effective tensile area of the concrete section A_{cef}.	
Step 6	Determine the value of the maximum allowed value of ω_k according to category of section.	Table (8-2)
Step 7	Determine the value of k_1 & β_1 according to type of steel.	Equation (4-66)
Step 8	Determine the value of β_2 according to period of loading.	Equation (4-66)
Step 9	Determine the value of k_2 according to kind of loading on section.	Equation (4-66)
Step 10	Solve the second degree equation to determine the value of allowed steel stress corresponding to each bar diameter.	Equation (4-66)

8.4 Examples

Example 1:

Figure (1) shows a section in a reinforced concrete water tank subjected to M= 70.0 kN.m and N = 100 kN. f_{cu} = 30 N/mm², f_y = 360 N/mm².

Step 1: Determine the concrete dimensions of the section to satisfy stage I (Uncracked section)

$M_u = 1.40 * 70 = 98$ kN.m
$N_u = 1.40 * 100 = 140$ kN [code Eq. (3-3)]

Assuming t =550 mm

$f_{ct}(N) = N/A_c$
$= 100*10^3/1000*550$
$= 0.182$ MPa

$f_{ct}(M) = 6M/bt^2$
$= 6*70*10^6/1000*550^2$
$= 1.39$ MPa

$t_v = t\{1+f_{ct}(N)/f_{ct}(M)\}$ [code Eq. (4-69)]

$= 550 * \{1+0.182/1.39\}$
$= 622$ mm

$\eta = 1.7$ [code Table (4-16)]

$f_{ctr} = 0.60\sqrt{f_{cu}}$ [code Eq. (4-64)]
$f_{ctr} = 0.60\sqrt{30}$
$= 3.28$ MPa

$f_{ctr}/\eta = 3.28/1.7$
$= 1.93$ MPa

$f_{ct} = N/A_c + M*Y/I$
$= 100*10^3/1000*550 + 6*70*10^6/1000*550^2$
$= 0.182+1.39$
$= 1.572$ MPa $< f_{ctr}/\eta = 1.93$ MPa OK.

Step 2: Determine the steel reinforcement according to stage II (cracked section)

$d = t - (\text{clear cover} + \phi/2)$

$d = 550 - (25+16/2)$
$= 517$ mm

$M_u = 98.0$ kN.m
$N_u = 140$ kN

$e = M_u/N_u$
$= 98 / 140$
$= 0.70$ m
$= 700$ mm

$e_s = e - t/2 + \text{cover}$
$= 700 - 550/2 + (25+16/2)$
$= 458$ mm

$M_{us} = N_u * e_s$
$= 140*458$
$= 64.12$ kN.m

For high tensile steel & $\phi = 16$ mm ..... $\underline{\beta_{cr} = 0.75}$ [code Table (4-15)]

$$A_s = \frac{M_{su}}{\beta_{cr} * f_y * j * d} + \frac{N_u}{\beta_{cr} * f_y / \gamma_s}$$

$$d = c_1 \sqrt{\frac{M_{us}}{b * f_{cu}}}$$

$$517 = c_1 \sqrt{\frac{64.12*10^6}{1000*30}}$$

c1 = 11.18 $\qquad$ $c/d < (c/d)_{min.}$

take $c/d = 0.125$ $\qquad$ $j = 0.826$

$$A_s = \frac{64.12*10^6}{0.75*360*0.826*517} + \frac{140*10^3}{0.75*360/1.15}$$

$= 556 + 596$

$= 1152$ mm^2 $\qquad$ Use 6 ϕ 16/m'

Example 2:

Figure (2) shows a section in a reinforced concrete raft **(categorized as category I)** of thickness 800 mm that is subjected to a working **bending moment of 643 kN.m (hogging** moment) $f_{cu} = 30$ N/mm^2, $f_y = 360$ N/mm^2

Step 1: Design the reinforcement to satisfy Stage 2 (cracked section)

-Minimum clear cover = 40.0 mm

*Ultimate Moment = Ultimate Factor * working Bending Moment

Assuming ultimate factor = 1.5 (clause 3.2.1.1.b.2)

Ultimate Moment = 1.5 * 643.0 = 964.5 kN.m

*Depth, d = total thickness – clear cover - $\phi/2$

d = 800 – 40 – 32/2

= 744.0 mm

$$*M_u = R * \frac{f_{cu}}{\gamma_c} b * d^2$$

$$964.5 * 10^6 = R * \frac{30}{1.5} * 1000 * 744^2$$

*From design charts with f_y =360 N/mm^2 , for R=0.087.... $\mu = 0.00608$

$A_s = \mu * b * d$

$A_s = 0.00608 * 1000 * 744.0$

$= 4520$ mm^2

using 6 ϕ 32

Step 2: Check the satisfaction of Equation (4-66 ECCS 203-2001)

2.1 Calculation of depth of neutral axis, c

The first moment of area about the neutral axis must be equal to zero assuming n=15

S = 0

$b * c^2/2 = n * A_s * (d-c)$

$1000 * c^2/2 = 15 * (6*800.0)*(744-c)$

solving the quadratic equation of c

The depth of the neutral axis from the extreme compression fiber, c = 263.14 mm

2.2 Calculation of cracked moment of inertia, I_{cr}

$$I_{cr} = b\frac{c^3}{3} + n * A_s * (d-c)^2$$

$$= 1000 * \frac{263.14^3}{3} + 15 * 4800.0 * (744.0 - 263.14)^2 \quad = 22.72 * 10^9 \text{ mm}^4$$

2.3 Calculation of Steel stresses f_s

$$f_s = n * \frac{M}{I_{cr}} * (d - c)$$

$$= 15 * \frac{643 * 10^6}{22.72 * 10^9} * (744.0 - 263.14)$$

$$= 204.13 \ MPa$$

2.4 Calculation of Cracking Limit Stress f_{ctr}

$$f_{ctr} = 0.60 * \sqrt{f_{cu}}$$

$$= 0.60 * \sqrt{30}$$

$$= 3.28 \ MPa$$

2.5 Calculation of Cracking Moment M_{cr}

$$M_{cr} = f_{ctr} * \frac{I_g}{y}$$

$$I_g = \frac{1}{12} * b * t^3$$

$$= \frac{1}{12} * 1000 * 800^3$$

$$= 42.66 * 10^9 \ mm^4$$

$$M_{cr} = 3.28 * \frac{42.66 * 10^9}{800/2}$$

$$= 349.81 \text{ kN-m}$$

2.6 Calculation of Steel Stress f_{sr}

For n=10 c= 223.53 mm I_{cr} = 16.72*10^9 mm^4

$$f_{sr} = n * \frac{M_{cr}}{I_{cr}} * (d - c)$$

$$= 10 * \frac{349.81 * 10^6}{16.72 * 10^9} * (744.0 - 223.53)$$

$$= 108.89 \ MPa$$

2.7 Calculation of ρ_r

$$\rho_r = \frac{A_s}{A_{cef}}$$

$A_{cef} = b^* \; t_{cef}$

$t_{cef} = 2.5$ (clear cover+ϕ/2)

$= 2.50*(40+32/2)$

$= 140.0$ mm < (t-c)/3 ok

$$\rho_r = \frac{6*800.0}{1000*140.0}$$

$= 0.03428$

2.8 Choice of other parameters appearing in Eq. (4-66) ECCS 203-2001

$k_1 = 0.80$ for deformed bars

$k_2 = 0.50$ for simple bending

$\beta_1 = 0.80$ for deformed bars

$\beta_2 = 1.0$ for short term loading

$\beta = 1.70$ for cracking due to loads

2.9 Application of Eq. (4-66) ECCS 203-2001

$$w_k = \beta . s_{rm} . \varepsilon_{sm}$$

$$s_{rm} = \left(50 + 0.25 k_1 \, k_2 \frac{\phi}{\rho_r}\right)$$

$$\varepsilon_{sm} = \frac{f_s}{E_s}\left(1 - \beta_1 \, \beta_2 \left(\frac{f_{sr}}{f_s}\right)^2\right)$$

$$s_{rm} = \left(50 + 0.25*0.80*0.50*\frac{32}{0.03428}\right)$$

$$s_{rm} = 143.35 \; mm$$

$$\varepsilon_{sm} = \frac{204.13}{2*10^5}\left(1 - 0.80*1.0*\left(\frac{108.89}{204.13}\right)^2\right)$$

$= 0.000788$

$w_k = 1.70*143.35*0.000788$

$w_k = 0.192 \, mm$ **< 0.30 mm OK.**

Example 3: Beam between two Structural Walls

Figure (3) shows a reinforced concrete beam that is spanning between two structural walls. The unfactored bending moments and normal forces due to loads and restrained deformations are 89 kN.m and 350 kN, respectively. The factored bending moments and normal forces due to loads and restrained deformations are 100 kN.m and 392 kN, respectively.

Step 1: Design the reinforcement to satisfy Stage 2 (cracked section)

Cross section of beam = 300* 700 mm^2

$$
\begin{aligned}
e &= M_u/N_u \\
&= 100.0/392.0 \\
&= 0.255 \text{ m} \\
&= 255 \text{ mm} < t/2 = 350 \text{ mm} \quad \text{........ eccentric tension force}
\end{aligned}
$$

$$
\begin{aligned}
e_{s1} &= t/2 - e - \text{cover} \\
&= 350 - 255 - 41 \quad \text{(Assuming that cover = 41.0 mm)} \\
&= 54 \text{ mm}
\end{aligned}
$$

$$
\begin{aligned}
e_{s2} &= t/2 + e - \text{cover} \\
&= 350 + 255 - 41 \\
&= 564 \text{ mm}
\end{aligned}
$$

$$A_{s1} = \frac{N_u * e_{s2}}{d - d'} / (f_y / \gamma_s)$$

$$A_{s1} = \frac{392.0 * 10^3 * 564}{659 - 41} / (360/1.15)$$

$= 1143.0$ mm^2 Use 3 ϕ 22

$$A_{s2} = \frac{N_u * e_{s1}}{d - d'} / (f_y / \gamma_s)$$

$$A_{s2} = \frac{392.0 * 10^3 * 54.0}{659 - 41} / (360/1.15)$$

$= 109.0$ mm^2 Use 2 ϕ 10

Step 2: Check the satisfaction of Equation (4-66 ECCS 203-2001)

2.1 Calculation of the steel stress f_s

$$f_{s1} = \frac{N * e_{s2}}{d - d'} / A_{s1}$$

$$=\frac{350.0*10^3*564}{659-41}/1140.0$$
$$= 279.45 \text{ MPa}$$

2.2 Calculation of Steel Stress f_{sr}

2.2.1 Calculation of Cracking Tension Force N_{cr}

In order to find the steel stress f_{str}, one has to calculate the combination M_{cr} and N_{cr} that result in first cracking of the section. It should be clear that one has to assume that the eccentricity of the tension force will be unchanged during the history of loading.

$$f_{ctr}=0.60*\sqrt{f_{cu}}/\eta$$
$$=0.60*\sqrt{25}/1.70$$
$$=1.76\ MPa$$

$$f_{ctr}=\frac{N_{cr}}{A}+\frac{6*M_{cr}}{b*t^2}$$

$$f_{ctr}=N_{cr}(\frac{1}{b*t}+\frac{6*e}{b*t^2})$$
$$1.760=N_{cr}*10^3(\frac{1}{300*700}+\frac{6*255}{300*700^2})$$

$$N_{cr}= 116.0 \text{ k.N}$$

2.2.2 Steel Stress f_{sr}

$$f_{sr}=\frac{N_{cr}*e_{s2}}{e_{s1}+e_{s2}}/A_{s1}$$

$$=\frac{116.0*10^3*564}{54+564}/1143.0$$

$$= 92.62 \text{ MPa}$$

2.3 Calculation of ρ_r

$$\rho_r = \frac{A_s}{A_{cef}}$$

$A_{cef} = b * t_{cef}$

$$\begin{aligned} t_{cef} &= 2.5\ (\text{clear cover} + \phi/2) \\ &= 2.50*(30+22/2) \\ &= 102.5\ \text{mm} \end{aligned}$$

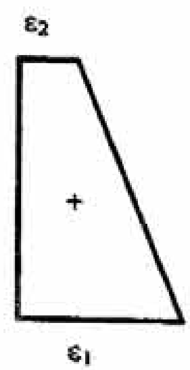
$$\rho_r = \frac{1143.0}{300*102.5} = 0.037$$

2.4 Calculation of k_2

$$k_2 = \frac{\varepsilon_1 + \varepsilon_2}{2\varepsilon_1}$$

The strains ε_1 and ε_2 are calculated through the analysis of the transformed section.

$$\begin{aligned} \text{Assuming } \varepsilon_2 &= \frac{f_y/\gamma_s}{E_s} \\ &= \frac{360/1.15}{2*10^5} \\ &= 0.00156 \end{aligned}$$



$$\begin{aligned} \varepsilon_1 &= \frac{e_{s2}}{e_{s1}} * \varepsilon_2 \\ &= \frac{565}{55} * 0.00156 \\ &= 0.016 \end{aligned}$$

$$k_2 = \frac{0.00156 + 0.016}{2*0.016} = 0.549$$

2.5 Choice of other parameters appearing in Eq. (4-66) ECCS 203-2001

$k = 0.8$	for deformed bars
$k_1 = 0.80$	for deformed bars
$\beta_1 = 0.80$	for deformed bars
$\beta_2 = 1.0$	for short term loading
$\beta = 1.70$	for case that includes loads

2.6 Application of Eq. (4-66) ECCS 203-2001

$$w_k = \beta . s_{rm} . \varepsilon_{sm}$$

$$s_{rm} = \left(50 + 0.25 k_1 k_2 \frac{\phi}{\rho_r} \right)$$

$$\varepsilon = \frac{f_s}{E_s} \left(1 - \beta_1 \beta_2 \left(\frac{f_{sr}}{f_s} \right)^2 \right)$$

$$s_{rm} = \left(50 + 0.25 * (0.80 * 0.8) * 0.549 \frac{22}{0.037} \right)$$

$$s_{rm} = 103 \ mm$$

$$\varepsilon = \frac{279.45}{2 * 10^5} \left(1 - 0.8 * 1.0 * \left(\frac{92.62}{279.45} \right)^2 \right)$$

$$\varepsilon = 0.00127$$

$$w_k = 1.70 * 10 * 0.00127$$

wk=0.222 mm < 0.30 o.k.

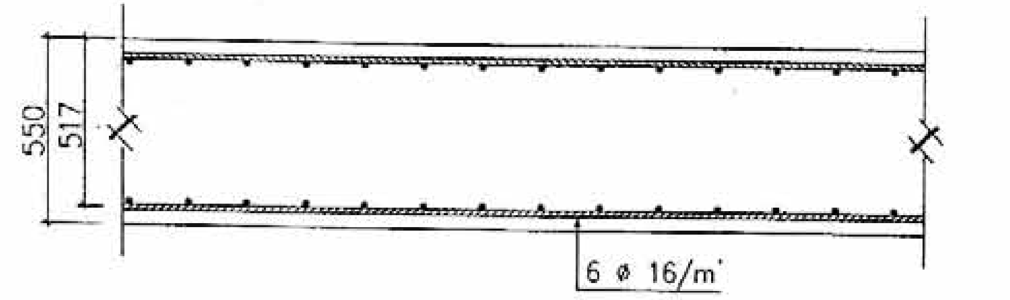


Fig. 1 Section in Water Tank

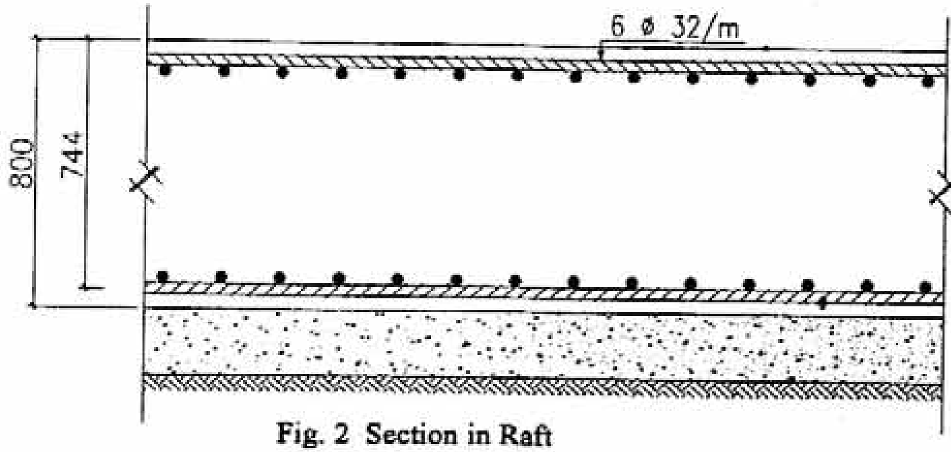


Fig. 2 Section in Raft

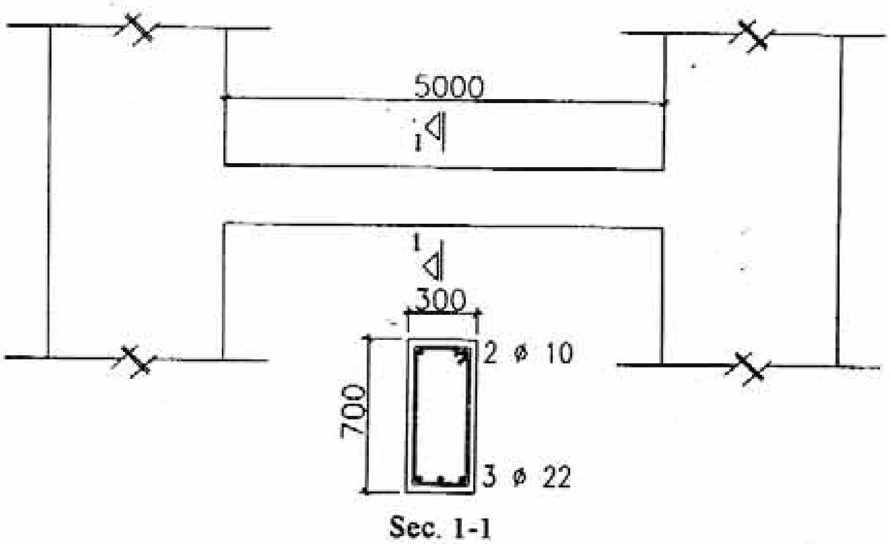


Fig. 3 Beam between Two Shear-Walls

Example (4)

Determine the allowable steel stresses for different bar diameters for the section shown in Fig. (4) to satisfy the limit-state of cracking according to the following data:

Required area of steel = 2000 mm^2
Section is designed according to category IV.
Clear concrete cover = 40 mm.
Steel 360/ 520 is used.
f_{cu} = 30 N/mm^2
Cracks occur due to applied loads

Step 1: Determine the value of the maximum tensile concrete stress f_{ctr} , and the cracking moment of the section M_{cr}

$f_{ctr} = 0.6\ \sqrt{f_{cu}} = 3.288$ N/ mm^2

$M_{cr} = (b\, t^2/ 6)\, (f_{ctr})$

$= [(300)\,(700)^2 / 6]\,[3.288]$
$= 80.6$ kN.m

Step 2: Calculation of cross sectional properties

Depth of section d = 622 mm

The first moment of area about the neutral axis must be equal to zero:

$b\,(c^2/ 2) - n\,(A_s)\,(d - c) = 0$
$300\,(c^2/ 2) - 15\,(2000)\,(622 - c) = 0$

Solving the quadratic equation for c, the depth of the neutral axis from the extreme compression fiber, c = 266.6 mm.

Second moment of inertia $I_{cr} = 568414.9 \times 10^4$ mm^4

Step 3: Calculate steel stress at first crack f_{sr}

$f_{sr} = n\ M_{cr}\,(d - c)/ I_{cr}$
$= (15)\,(8.06 \times 10^7)\,(355.4)/ (568414.9 \times 10^4)$
$= 75.6$ N/ mm^2

Step 4: Calculate value of effective tensile area of concrete

$t_{cef} = 2.5\,(t - d)$
$= 2.5\,(700 - 622) = 195$ mm

$A_{cef} = b\, t_{cef}$
$= (300)\,(195)$
$= 58.5 \times 10^3\ mm^2$

Step 5: Determine effective tensile reinforcement ratio

$\rho_r = A_s / A_{cef}$
$= 2000 / 58.5 \times 10^3 = 0.0342$

Step 6: Determine the value of the maximum allowed value of w_k according to category of section

$w_{k\,max} = 0.1$ for category IV according to table (8-2).

Step 7: Determine the value of k_1 and β_1 according to type of steel used

$k_1 = 0.8$ for ribbed bars
$\beta_1 = 0.8$ for ribbed bars

Step 8: Determine the value of β according to cross section dimensions and loading type

$\beta = 1.7$ for cracks due to applied loads

Step 9: Determine the value of β_2 according to period of loading

$\beta_2 = 0.5$ for permanent loads

Step 10: Determine the value of k_2 according to kind of loading on section

$k_2 = 0.5$ for sections under pure bending

Step 11: Solve the second degree equation to obtain the allowable steel stress corresponding to each bar diameter

$$w_{k\,max} \geq \beta\,[\,50 + 0.25\,k_1\,k_2\,(\phi / \rho_r)]\,[\,f_s / E_s\,]\,[\,1 - \beta_1\,\beta_2\,(\,f_{sr}\ / f_s\,)^2\,]$$

$$0.1 \geq 1.7\,[50 + (.25)(.8)(.5)(\phi / 0.0342)]\,[f_s\ / 2 \times 10^5]\,[1 - (.8)(.5)(75.6 / f_s)^2]$$

<u>If ϕ 16 is used:</u>

$f_s^2 - 121.55\, f_s - 2286 = 0$

$f_s = 138.1\ N/mm^2$

If ϕ 18 is used:

$$f_s^2 - 114.63\ f_s - 2286 = 0$$

$$f_s = 131.95\ N/mm^2$$

If ϕ 20 is used:

$$f_s^2 - 108.45\ f_s - 2286 = 0$$

$$f_s = 126.5\ N/mm^2$$

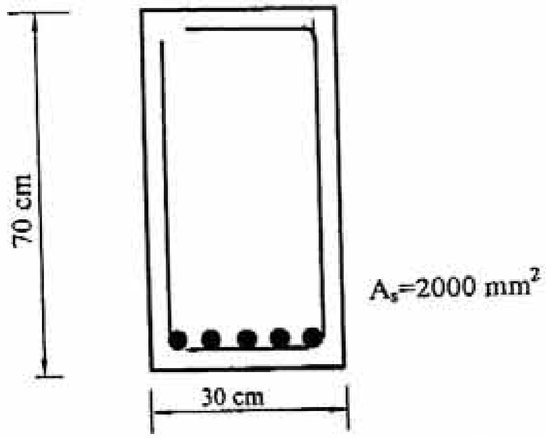


Fig. (4): Cross section Dimension and reinforcement of example 4

Example 5:

Design the section shown in Fig. (5) to satisfy the limit state of cracking according to the following data:

A_s = 10 $\#$ 20
Section is designed according to category IV.
Clear concrete cover = 40 mm.
Steel 360/ 520 is used.
f_{cu} = 30 N/mm²
Cracks occur due to loads

Step 1: Determine the value of the maximum tensile concrete stress f_{ctr}, and the cracking moment of the section M_{cr}

$f_{ctr} = 0.6\ \sqrt{f_{cu}} = 3.288\ N/mm^2$

$M_{cr} = (b\ t^2/6)\ (f_{ctr})$

$= [(400)\ (750)^2 / 6]\ [3.288]$
$= 1.233 \times 10^8\ N.mm$

Step 2: Calculation of cross sectional properties:

Depth of section d = 670 mm

The first moment of area about the neutral axis must be equal to zero.
$Q = 0$

$b\ (c^2/2) - n\ (A_s)\ (d - c) = 0$
$400\ (c^2/2) - 15\ (3140)\ (670 - c) = 0$

Solving the quadratic equation of c, the depth of the neutral axis from the extreme compression fiber c = 296 mm.

Second moment of inertia $I_{cr} = 1005909 \times 10^4\ mm^4$

Step 3: Calculate steel stress at first crack f_{sr}

$f_{sr} = n\ M_{cr}\ (d - c)/ I_{cr}$

$= (15)\ (1.233 \times 10^8)\ (670 - 296)/ (1005909 \times 10^4)$
$= 68.76\ N/mm^2$

Step 4: Calculate value of effective tensile area of concrete

$t_{cef} = 2.5\ (t - d)$
$= 2.5\ (750 - 670)$
$= 200\ mm$

$A_{cef} = b\ t_{cef}$
$= (400)\ (200)$
$= 80000\ mm^2$

Step 5: Determine tensile steel ratio

$\rho_r = A_s / A_{cef}$
$= 3140/ 80000$
$= 0.0393$

Step 6: Determine the value of the maximum allowed value of w_k according to category of section.

$w_{k\ max}$ = 0.1 for category IV according to Table (4-12) in the code.

Step 7: Determine the value of k_1 and β_1 according to type of steel used

k_1 = 0.8 for ribbed bars
β_1 = 0.8 for ribbed bars

Step 8: Determine the value of β according to cross section dimensions

β=1.7 for cracks due to loads

Step 9: Determine the value of β_2 according to period of loading

β_2 = 0.5 for long-term loading

Step 10: Determine the value of k_2 according to kind of loading on section.

k_2 = 0.5 for sections under pure bending

Step 11: Solve the second degree equation to obtain the allowable steel stress

$$w_k \geq \beta\,[\,50 + 0.25\,k_1\,k_2\,(\,\phi/\rho_r)]\,[\,f_s/E_s\,]\,[\,1 - \beta_1\,\beta_2\,(\,f_{sr}\ /f_s)^2\ ]$$

$$0.1 \geq 1.7\ [50 + (.25)(.8)(.5)(20/\ 0.0393)]\ [f_s\ /\ 200x1000]\ [1 - (.8)(0.5)(68.76/\ f_s)^2]$$

$$f_s^2 - 116.595\ f_s - 1891.2 = 0$$

f_s = 131 N/ mm^2

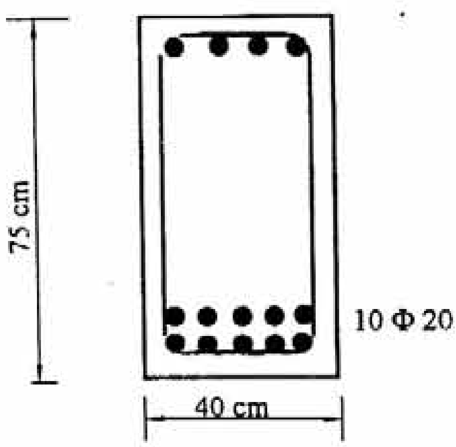


Fig. (5): Cross section Dimension and reinforcement of example 5

Representation of Code Equation (4-66)

Table (8-6) Application In Case Of Cracking Due To Applied Service Loads

	Internal Forces	Coefficient w_k
Ribbed Bars	Bending Moment *	$w_k = 0.85\,(500 + \dfrac{\Phi}{\rho_r}).[1 - 0.8\,\beta_2\,(f_{sr}/f_s)^2].f_s\,10^{-6}$
	Axial Tension	$w_k = 0.85\,(500 + 2\dfrac{\Phi}{\rho_r}).[1 - 0.8\,\beta_2\,(f_{sr}/f_s)^2].f_s.10^{-6}$
	Eccentric Tension	$w_k = 0.85\,(500 + 2\,k_2\dfrac{\Phi}{\rho_r}).[1 - 0.8\,\beta_2\,(f_{sr}/f_s)^2].f_s.10^{-6}$
Smooth Bars	Bending Moment *	$w_k = 0.85\,(500 + 2\dfrac{\Phi}{\rho_r}).[1 - 0.5\,\beta_2\,(f_{sr}/f_s)^2].f_s.10^{-6}$
	Axial Tension	$w_k = 0.85\,(500 + 4\dfrac{\Phi}{\rho_r}).[1 - 0.5\,\beta_2\,(f_{sr}/f_s)^2].f_s.10^{-6}$
	Eccentric Tension	$w_k = 0.85\,(500 + 4\,k_2\dfrac{\Phi}{\rho_r}).[1 - 0.5\,\beta_2\,(f_{sr}/f_s)^2].f_s.10^{-6}$

- In the above equations, steel modulus of elasticity (Es) = 200 000 N/mm^2.

* Equations for Bending moments apply also in the cases of eccentric tension or compression with big eccentricity [i.e. a triangular tensile strain distribution takes place over the whole or a part of the cracked concrete section]:

Where;

w_k is a coefficient and its value should be ≤ 0.3 in the first zone, ≤ 0.2 in the second zone, ≤ 0.15 in the third zone and ≤ 0.1 in the fourth zone; considering the zone definition in Code Table (4-11) and w_k limitations in Code Table (4-12).

Φ = the bar diameter in millimetres,
= the mean diameter in case of bars having different diameters, or
= the equivalent diameter in case of bundles; i.e. equals to 1.5 and 1.75 times the greatest diameter in a bundle consisting of 2 or 3 bars; respectively [Code clause 7-3-4]

$\rho_r = A_s / A_{cef}$

A_s = the tensile reinforcement located within A_{cef}.

A_{cef} is the effective concrete section subjected to tension. It equals to the section breadth multiplied by the effective depth (t_{cef}) subjected to tension.

t_{cef} ,as defined in Figure (4-22) in the code, is the effective depth subjected to tension. t_{cef} equals to 2.5 times the distance between the c.g. of A_s and the adjacent tensioned surface in the concrete section.

t_{cef} should not exceed the following limitations:

<u>In general</u>: t_{cef} should not exceed the distance between the neutral axis and the most tensioned surface in the section.

<u>In slabs</u>: $t_{cef} \leq (t - c) / 3$ where;
t is the section total depth.
c is the depth of the neutral axis measured within the compressed zone in the slab section.

<u>In sections subjected to pure tension or eccentric tension with small eccentricity</u>: $t_{cef} \leq$ half the section total depth (t).

β_2 is a coefficient = 0.5 in case of permanent or frequently repeating loads.
= 1.0 in case of short term loads.

f_s is the tensile steel stress, in N/mm^2; calculated on the basis of cracked section under applied service loads. Its value should not exceed the values defined in Code Table(5-1).

f_{sr} is the tensile steel stress in N/mm^2; calculated on the basis of cracked section under service loads initiating cracking.

k_2 is the ratio between the average and maximum tensile strains in the section.

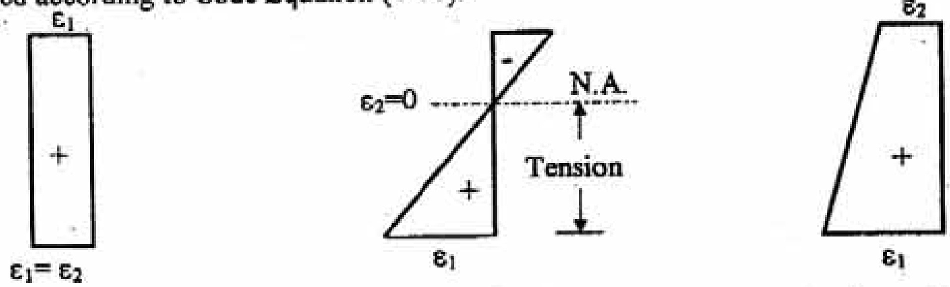
$k_2 = (\varepsilon_1 + \varepsilon_2) / 2\,\varepsilon_1$ Code Equation (4-67)

where ε_1 and ε_2 are the maximum and minimum tensile strains in the cracked concrete section respectively. Accordingly the extreme values of k_2 are:

k_2 = 1.0 in case of axial tension (Figure 6-a).

k_2 = 0.5 in the cases of pure B.M. and eccentric tension or compression with big eccentricity; where the tensile strain distribution is triangular (Figure 6-b).

The above mentioned values of k_2 are already merged in their respective equations. In the cases of eccentric tension with small eccentricity (Figire 6-c), k_2 should be calculated according to Code Equation (4-67).



k_2=1.0
a– Strain distribution due to axial tension.

k_2=0.5
b– Strain distribution due to B.M. or eccentric forces with big eccentricity.

$k_2=(\varepsilon_1+\varepsilon_2)/2\varepsilon_1$
c– Strain distribution due to eccentric tension with small eccentricity.

Fig. 6. Strain distribution for different loading conditions

Table (8-7): Application Of Equation (4-66) In Case Of Cracking Due Intrinsic Imposed Deformations [where $f_s = f_{sr}$]

	Internal Forces	Coefficient w_k
Ribbed Bars	Bending Moment *	$w_k = \beta^{\backslash} (500 + k \frac{\Phi}{\rho_r}).f_{sr}.10^{-6}$
	Axial Tension	$w_k = \beta^{\backslash} (500 + 2\,k \frac{\Phi}{\rho_r}).f_{sr}.10^{-6}$
	Eccentric Tension	$w_k = \beta^{\backslash} (500 + 2\,k\,k_2 \frac{\Phi}{\rho_r}).f_{sr}.10^{-6}$
Smooth Bars	Bending Moment *	$w_k = 1.25\,\beta^{\backslash} (500 + 2\,k \frac{\Phi}{\rho_r}).f_{sr}.10^{-6}$
	Axial Tension	$w_k = 1.25\,\beta^{\backslash} (500 + 4\,k \frac{\Phi}{\rho_r}).f_{sr}.10^{-6}$
	Eccentric Tension	$w_k = 1.25\,\beta^{\backslash} (500 + 4\,k\,k_2 \frac{\Phi}{\rho_r}).f_{sr}.10^{-6}$

- In the above equations; steel modulus of elasticity (Es) = 200 000 N/mm^2.

* Equations for Bending moments apply also in the cases of eccentric tension or compression with big eccentricity [i.e. a triangular tensile strain distribution takes place over the whole or a part of the cracked concrete section].

Where;

w_k, Φ, ρ_r, f_{sr} & k_2 are as defined in the case of cracking due to applied service loads.

Coefficient $\beta^{\backslash} = 0.39 + [(X - 300) . 2.4 . 10^{-4}]$ such that $0.39 \geq \beta^{\backslash} \geq 0.51$

where (X) is the section breadth (b) or total depth (t); whichever is smaller. The value of $\beta^{\backslash}$ should range between 0.39 (for $X \leq 300$ mm) and 0.51 (for $X \geq 800$ mm). $\beta^{\backslash}$ values corresponding to different (X) values are presented in Table (8-8).

In general; coefficient k equals to 0.8 for tensile stresses due to constrains. In the particular case of rectangular sections,

$k = 0.8 - [(X - 300) . 6 . 10^{-4}]$ such that $0.5 \leq k \leq 0.80$

where (X) is as defined above.

k value ranges between 0.8 (for $X \leq 300$ mm) and 0.5 (for $X \geq 800$ mm).

Table (8-8): Values of $\beta^\backslash$ and k for different (X) values in rectangular sections.

Breadth or total depth (whichever smaller)	≤300	350	400	450	500	550	600	650	700	750	≥800
$\beta^\backslash$	0.39	0.402	0.414	0.426	0.438	0.450	0.462	0.474	0.486	0.498	0.51
k	0.80	0.77	0.74	0.71	0.68	0.65	0.62	0.59	0.56	0.53	0.50

Where the breadth and total depth dimensions are in millimetres.

Table (8-9): Application Of Equation (4-66) In Case Of Cracking Of Walls Because Of Early Thermal Contraction

[Due to fixation at the wall base]

	Coefficient w_k
Ribbed Bars	$w_k = \beta^\backslash \cdot H \cdot f_{sr} \cdot 10^{-5}$
Smooth bars	$w_k = 1.25\ \beta^\backslash \cdot H \cdot f_{sr} \cdot 10^{-5}$

Where;

w_k , $\beta^\backslash$ & f_{sr} are as defined in the case of cracking due to intrinsic imposed deformations.

H = The wall height in millimetres.

In the above equations; steel reinforcement modulus of elasticity (Es) = 200 000 N/mm^2.

Table (9-1): Development Length (L_d) of Tension Bars With Straight Ends
(According to equations 4-56 & 4-57 in code clause 4-2-5-1)

* Code equations determining the development length (L_d) can be reduced to the following equation:

  L_d = coefficient x (the bar diameter) $\geq$ 400 mm for steel grades (240/350) & (280/450)
  $\geq$ 300 mm for steel grades (360/520) & (400/600)

The following Table gives the coeff. values for different concrete characteristic strengths (f_{cu}) and steel yield stresses (f_y) in case of <u>tension bars with straight ends</u>:

f_y (N/mm^2)	f_{cu} (N/mm^2) 18	20	25	30	35	40	45	50
240	51	48	43	39	36	34	32	31
280	59	56	50	46	42	40	37	36
360	57	54	48	44	41	38	36	34
400	63	60	54	49	45	43	40	40

Remarks:

In case of top bars having concrete depth below them >300 mm, multiply the given coefficient by η = 1.3.

The given values correspond to γ_c= 1.5 & γ_s= 1.15.

Bundles:

Bundles are only allowed for deformed bars [f_y= 360 & 400 N/mm^2] (Code clause 7-3-4).

Maximum diameter of bars in a bundle is 28 mm (Code clause 7-3-4).

In case of a bundle consisting of 2 bars, multiply the given coefficient by 1.47.

In case of a bundle consisting of 3 bars, multiply the given coefficient by 1.60 .

Examples:

<u>Date:</u> f_{cu} = 25 N/mm^2. Steel grade is (360/520)
Diameter of bars= 16 mm. Ends of bars are straight.

<u>A case of bottom tension bars:</u>

L_d = 48 times the bar diameter [i.e. L_d = 768 mm].

<u>A case of top tension bars in a (250x1000) R.C. beam:</u>

L_d = 48 x 1.3 = 62.4 times the bar diameter [i.e. L_d = 999 mm].

<u>A case of top tension bundle of 2 bars in a (250x1000) R.C. beam:</u>

L_d = 48 x 1.3 x 1.47 = 91.73 times the bar diameter [i.e. L_d = 1468 mm].

<u>A case of top tension bundle of 3 bars in a (250x1000) R.C. beam:</u>

L_d = 48 x 1.3 x 1.60 = 99.84 times the bar diameter [i.e. L_d = 1597 mm].

Table (9-2): Development Length (L_d) of Tension Bars With Hooked Ends

(According to equations 4-56 & 4-57 in code clause 4-2-5-1)

* Code equations determining the development length (L_d) can be reduced to the following equation:

  L_d = coefficient x (the bar diameter) $\geq$ 400 mm for steel grades (240/350) & (280/450)
  $\geq$ 300 mm for steel grades (360/520) & (400/600)

* The following Table gives the coefficient values for different concrete characteristic strengths (f_{cu}) and steel yield stresses (f_y) in case of <u>tension bars with hooked ends</u>:

f_y (N/mm^2)	f_{cu} (N/mm^2)							
	18	20	25	30	35	40	45	50
240	38	36	32	30	27	26	24	23
280	44	42	38	34	32	30	28	27
360	43	41	36	33	31	29	27	26
400	47	45	40	37	34	32	30	29

Remarks:

In case of top bars having concrete depth below them >300 mm, multiply the given coefficient by $\eta = 1.3$.

The given values correspond to $\gamma_c = 1.5$ & $\gamma_s = 1.15$.

Bundles:

Bundles are only allowed for deformed bars [f_y= 360 & 400 N/mm^2].
Maximum diameter of bars in a bundle is 28 mm [code clause 7-3-4].
In case of bundles consisting of 2 bars, multiply the given value by 1.47.
In case of bundles consisting of 3 bars, multiply the given value by 1.60 .

Examples:

<u>Date:</u> f_{cu} = 25 N/mm^2. Diameter of bars= 16 mm.
Steel grade is (360/520) Ends of bars are hooked.

<u>A case of bottom tension bars:</u>

L_d = 36 times the bar diameter [i.e. L_d = 576 mm].

<u>A case of top tension bars in a (250x1000) R.C. beam:</u>

L_d = 36 x 1.3 = 46.8 times the bar diameter [i.e. L_d = 749 mm].

<u>A case of top tension bundle of 2 bars in a (250x1000) R.C. beam:</u>

L_d = 36 x 1.3 x 1.47 = 68.8 times the bar diameter [i.e. L_d = 1101 mm].

<u>A case of top tension bundle of 3 bars in a (250x1000) R.C. beam:</u>

L_d = 36 x 1.3 x 1.60 = 74.9 times the bar diameter [i.e. L_d = 1198 mm].

Table (9-3): Development Length (L_d) of Compression Bars

(According to equations 4-56 & 4-57 in code clause 4-2-5-1)

* Code equations determining the development length (L_d) can be reduced to the following equation:

  L_d = coefficient x (the bar diameter) $\geq$ 400 mm for steel grades (240/350) & (280/450)

  $\geq$ 300 mm for steel grades (360/520) & (400/600)

* The following Table gives the coefficient values for different concrete characteristic strengths (f_{cu}) and steel yield stresses (f_y) in case of <u>compression bars</u> with or without hooked ends.

f_y (N/mm^2)	f_{cu} (N/mm^2) 18	20	25	30	35	40	45	50
240	36	34	30	28	26	24	23	22
280	41	39	35	32	30	28	26	25
360	38	36	32	30	27	26	24	23
400	42	40	36	33	30	29	27	26

Remarks:

In case of top bars having concrete depth below them >300 mm, multiply the given coefficient by η = 1.3.

The given values correspond to γ_c= 1.5 & γ_s= 1.15.

Bundles:

Bundles are only allowed for deformed bars [f_y= 360 & 400 N/mm^2].

Maximum diameter of bars in a bundle is 28 mm [code clause 7-3-4].

In case of bundles consisting of 2 bars, multiply the given value by 1.5.

In case of bundles consisting of 3 bars, multiply the given value by 1.6.

Examples:

<u>Date:</u> f_{cu} = 25 N/mm^2. Steel grade is (360/520)

Diameter of bars= 16 mm.

<u>A case of bottom Compression bars:</u>

L_d = 32 times the bar diameter [i.e. L_d = 512 mm].

<u>A case of top Compression bars in a (250x1000) R.C. beam:</u>

L_d = 32 x 1.3 = 41.6 times the bar diameter [i.e. L_d = 666 mm].

<u>A case of top Compression bundle of 2 bars in a (250x1000) R.C. beam:</u>

L_d = 32 x 1.3 x 1.5 = 62.4 times the bar diameter [i.e. L_d = 999 mm].

<u>A case of top Compression bundle of 3 bars in a (250x1000) R.C. beam:</u>

L_d = 32 x 1.3 x 1.6 = 66.56 times the bar diameter [i.e. L_d = 1065 mm].